Bertrand Competition and Captive Customers

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Abstract. We study a Bertrand oligopoly with asymmetric costs in which each seller has some "captive" buyers. In the limit as captive buyers vanish, the lowest-cost firm sells to everyone at a price equal to the second-lowest marginal cost. However, the closest competing price arises from non-degenerate mixed strategies, firms play exclusively undominated strategies, and with positive probability all but one firm sets the monopoly price.

A received result in textbook Bertrand theory is that identical competitors ferociously undercut each other until their prices reach marginal cost. Of course, marginal-cost pricing is weakly dominated. In a symmetric oligopoly, Harrington (1989) showed how all equilibria have this feature.[1]

For strictly asymmetric marginal costs, standard classroom intuition is that the two most efficient firms compete down to the second-lowest cost. This price is dominated for that second-lowest-cost firm and so the issue persists. In a duopoly, Blume (2003) exhibited equilibria in undominated strategies in which the most efficient firm sets its price equal to its competitor's (higher) marginal cost, while that competitor mixes over an interval extending upward. The efficient firm serves everyone at a price equal to the inefficient firm's marginal cost. Kartik (2011) showed that for any Nash equilibrium with undominated strategies, this outcome is preserved. Nevertheless, there are many other equilibria, including those in which the efficient firm prices strictly below its nearest competitor's marginal cost, while a competitor mixes down to that price to dissuade any price rise.

For an asymmetric-cost duopoly, De Nijs (2012) elegantly resolved the multiplicity problem by adding "captive" customers for each firm (Varian, 1980; Narasimhan, 1988), and so studied the fully asymmetric captive-and-shopper duopoly of Golding and Slutsky (2000).[2] In the limit as the masses of captive customers vanish, the efficient firm serves everyone at a price equal to the inefficient marginal cost, while its competitor mixes all of the way up to (and places an atom at) the

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[1] He also showed this is approximately so when one replaces the usual continuous strategy space with a discrete one.

[2] Baye, Kovenock, and de Vries (1992) described the complete (large) set of equilibria of symmetric captive-shopper games, as well as the unique equilibrium when there are asymmetries in captive audiences. Golding and Slutsky (2000) allowed for asymmetric costs and captives in a duopoly, while recently Shelegia and Wilson (2021) allowed for asymmetric duopolies (and symmetric oligopolies) in which firms pay to advertise prices. Extending to richer price consideration, but with symmetric-cost firms, Armstrong and Vickers (2022) characterized equilibria in a triopoly, under symmetry, and for some natural special cases. In our own other work, we derived and studied "stable prices" for a variety of consideration specifications in place of single-stage Nash equilibria (Myatt and Ronayne, 2025b).

monopoly price. The outcome corresponds to the classroom description, but the observed prices do not: only the efficient firm prices at (the second lowest) marginal cost, while its competitor prices strictly higher and sets the monopoly price with strictly positive probability.

In recent work (Myatt and Ronayne, 2025a) we solved an oligopoly ($n > 2$) captive-shopper model allowing for asymmetries in both costs and captive audiences. Here we use our results to extend the findings of De Nijs (2012) to an oligopoly setting. In the zero-captives limit we find: (i) the most efficient firm serves all buyers at a price equal to the second-lowest marginal cost; (ii) inefficient firms exclusively use undominated prices, and set the monopoly price with strictly positive probability; and (iii) the distribution of the minimum price amongst inefficient firms has full support from the second-lowest marginal cost up to the monopoly price and coincides with the mixed strategy of the high-cost firm described by De Nijs (2012) in which the CDF at each price is equal to the ratio of profit margin enjoyed by the second-lowest-cost firm to that of the lowest-cost firm. The order in which the masses of captive buyers are taken to zero can matter (unlike in the duopoly case) so that mixing can involve all inefficient firms with supports that partition the full interval of prices so that they all "dance pairwise" with the efficient firm. Importantly, however, a key message from this strand of literature—that only one firm prices at (the second lowest) marginal cost, while other prices are higher—is maintained in an oligopoly.

Model. Each risk-neutral firm $i \in \{1, \ldots, n\}$ operates with a constant marginal cost $c_i \in [0, v)$ and simultaneously sets a price $p_i \in [0, v]$ where $v > 0$ is customers' common maximal willingness to pay. Costs are strictly asymmetric, and we order firms so that $c_1 < c_2 < \ldots < c_n$.

A mass of $\lambda_S > 0$ of "shoppers" buy from the cheapest firm with ties broken arbitrarily. A mass of $\lambda_i > 0$ of "captives" buy exclusively from firm i. Our focus is industries that approximate a classic Bertrand environment so that the masses of captives are (vanishingly) small. Our exposition is smoothed (we explain just below) by placing an upper bound on captive masses from the outset: we set $\lambda_i < (\lambda_S/v)(c_{i+1} - c_i)$ for all i.

A firm's payoff is its profit, and we seek its Nash equilibria.

Equilibrium. Firm i sells to captives even when pricing at v and so its lowest undominated price, denoted $p_i^\dagger$, is that which earns the captive-only monopoly profit when it also sells to shoppers:

$$\lambda_i(v - c_i) = (\lambda_i + \lambda_S)(p_i^\dagger - c_i) \quad \Leftrightarrow \quad p_i^\dagger = \frac{\lambda_i v + \lambda_S c_i}{\lambda_i + \lambda_S}. \tag{1}$$

If captive masses are small then firms' costs determine the ranking of these lowest undominated prices: our assumption that $\lambda_i < (\lambda_S/v)(c_{i+1} - c_i)$ for all i implies that $p_1^\dagger < p_2^\dagger < \cdots < p_n^\dagger$.

We characterized the Nash equilibria of this game in an earlier paper (Myatt and Ronayne, 2025a).

Proposition 1 (Equilibrium). *A unique Nash equilibrium exists. It satisfies these properties.*

(1) At least two firms play mixed strategies. Any firm that does not mix sets the monopoly price v.

(2) The lowest-cost firm 1 mixes continuously over the support $S \equiv [p_2^\dagger, v]$.

(3) The lowest price amongst inefficient firms is distributed with full support S. Its CDF satisfies

$$\min_{i>1}\{p_i\} \sim \underline{F}(p) \quad \textit{where} \quad \underline{F}(p) = \frac{(p - p_2^\dagger)(\lambda_1 + \lambda_S)}{\lambda_S(p - c_1)} \quad \textit{for} \quad p \in [p_2^\dagger, v). \tag{2}$$

(4) If a firm $i > 1$ actively mixes, then it does so continuously over a single subinterval of S and then places remaining mass as a strictly positive atom at the monopoly price v. Any two such sub-intervals do not overlap, and all such sub-intervals form a partition of S.

(5) There is a $p^\ddagger \in (p_3^\dagger, v]$ such that the two lowest-cost firms mix continuously over $[p_2^\dagger, p^\ddagger)$ with

$$F_1(p) = \frac{(p - p_2^\dagger)(\lambda_2 + \lambda_S)}{\lambda_S(p - c_2)} \quad \textit{and} \quad F_2(p) = \underline{F}(p). \tag{3}$$

(6) At least three firms mix if and only if there is some $i \in \{3, \ldots, n\}$ and $p \in [p_2^\dagger, v)$ such that

$$(v - p)\lambda_i < (p - c_i)\lambda_S(1 - F_1(p))(1 - \underline{F}(p)). \tag{4}$$

(7) The expected profit of firm i is given by

$$\pi_i = \underbrace{\lambda_i(v - c_i)}_{\textit{captive-only profit}} + \begin{cases} (\lambda_1 + \lambda_S)(p_2^\dagger - p_1^\dagger) & \textit{if } i = 1, \textit{ and} \\ 0 & \textit{otherwise.} \end{cases} \tag{5}$$

Proof. These claims follow from Propositions 1–3 of Myatt and Ronayne (2025a). □

The statements describe a sequence of pairwise "dances" on a "dance floor" interval $[p_2^\dagger, v]$. The lowest-cost (or efficient) firm 1 mixes (or dances) continuously over the whole interval. Its first "dance partner" (commencing at $p_2^\dagger$) is its closest (in terms of cost) competitor firm 2. This may, if the inequality of eq. (4) fails for all $[p_2^\dagger, v]$ and all $i > 2$, be the only dance, such that only "two tango." Competition then reduces to a de facto duopoly (other firms simply charge the monopoly price to their captives) and the solutions of Golding and Slutsky (2000) and De Nijs (2012) apply.

If the inequality reported in eq. (4) can be satisfied, then at some price $p^\ddagger \in (p_3^\dagger, v)$ which solves $(v - p^\ddagger)\lambda_i = (p^\ddagger - c_i)\lambda_S(1 - F_1(p^\ddagger))(1 - \underline{F}(p^\ddagger))$, another firm $i > 2$ replaces firm 2 as firm 1's dance partner. As we move up through the interval of prices, firm 1 may have multiple partner swaps. As this suggests, the equilibrium strategies are best specified in detail by an algorithm, which is not necessary for the current analysis and so we omit a full technical description.[3]

[3]We refer the interested reader to the full details in (Myatt and Ronayne, 2025a, Online Appendix Sections S.1-S.3).

Vanishing Captives. Mimicking the approach of De Nijs (2012), we now fix both the number of firms and their marginal costs, and we consider a sequence of captive-shopper pricing games in which their captive masses vanish to zero so that $\max_{i\in\{1,\ldots,n\}}\{\lambda_i\}\downarrow 0$ and so $p_i^\dagger \downarrow c_i$ for all i.[4]

Along that sequence, Proposition 1 only describes fully the equilibrium when the condition in eq. (4) fails so that only two firms dance. Nevertheless, the proposition characterizes firms' profits, the distribution of the minimum price amongst inefficient firms (because they must mix to keep the lowest-cost firm indifferent across the whole support) and the distribution used by the efficient firm when evaluated at lower prices (which coincides with established duopoly analyses).

Proposition 2 (Prices and Profits as Captivity Vanishes). *Taking the limit as $\lambda_i \downarrow 0$ for all i,*

$$F_1(p) \to 1 \text{ and } \underline{F}(p) \to \frac{p-c_2}{p-c_1} \text{ for } p \in (c_2, v) \text{ and so } \Pr\left[p_1 < \min_{i>1}\{p_i\}\right] \to 1. \tag{6}$$

The efficient firm serves shoppers at a price equal to the second-lowest cost and earns $\lambda_S(c_2-c_1)$. The lowest price amongst its competitors is v with probability $(c_2-c_1)/(v-c_1)$. They earn nothing.

Proof. Any $p \in (c_2, v)$ satisfies $p > p_2^\dagger$ if λ_2 is sufficiently small, so that the solutions for $F_1(p)$ and $\underline{F}(p)$ stated in Proposition 1 apply. Taking the limit as $\lambda_2 \downarrow 0$ (for $F_1(p)$ when $p \in (c_2, c_3)$, which then underpins the same limit for larger p) and also as $\lambda_1 \downarrow 0$ (for $\underline{F}(p)$, which depends on both λ_1 and λ_2, and where the solution applies for any $p \in (c_2, v)$) generates the claimed results. □

The key claims of De Nijs (2012) are maintained, but the price distribution of the high-cost firm in a duopoly is replaced by the distribution of the minimum price of inefficient firms. Also as in De Nijs (2012), this does not depend on the order in which the captive masses vanishes.

However, that order does matter for the nature of the equilibrium. In the limit, the key inequality of eq. (4) which determines whether firm $i > 2$ wishes to "step on to the dance floor" becomes

$$\frac{\lambda_i}{\lambda_2} < \frac{(p-c_i)(p-c_2)}{(p-c_1)(c_2-c_1)}. \tag{7}$$

Whether this holds or not depends on the relative (rather than absolute) size of captive audiences.

Conclusion. Intuition suggests that Bertrand rivals undercut to marginal cost, yielding a single market price. Our analysis reinforces that of De Nijs (2012): in the vanishing-captive limit, only the most efficient firm sets the second-lowest cost, while others often set the monopoly price. Thus, rather than a "law of one price," equilibrium reflects a "law of monopoly pricing" where most firms sit (far) above cost while exactly one anchors the (low) transaction price at which the market clears.

[4] There are some applied motivations to consider reductions in captivity. For example, increasingly efficient search technology might allow navigation to the best deal via search engines, comparison websites, or switching services (see, for example, Baye and Morgan, 2001; Ronayne, 2021; Garrod, Li, and Wilson, 2023).

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