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Cournot competition and the social value of information [☆]

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Abstract

In a differentiated-product Cournot model, each supplier receives informative signals about demand. The cross-industry correlations of the signals differ: more public signals have higher correlation coefficients. In equilibrium, information is used inefficiently. From the industry's perspective, information is over-used, and too much emphasis is placed on relatively public signals; from the consumer's perspective, information is under-used, and too much emphasis is placed on relatively private signals. Welfare is enhanced by increasing the use of information (as desired by consumers) but re-balancing that use away from public signals (as desired by the industry). If information is costly and endogenously acquired, then suppliers acquire too much new information, but they use it too little.

JEL classification: C72; D43; D83

Keywords: Cournot oligopoly; Uncertainty; Bayesian equilibrium; Information acquisition; Public and private information

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1. Introduction

Cournot suppliers' profits depend upon demand conditions and upon the output choices of their competitors. If demand conditions are uncertain then any information at the suppliers' disposal is used to form beliefs about those conditions. In addition, such information can and should be used to evaluate the likely information available to others, and so to form (higher order) beliefs about the output choices of a supplier's competitors.

In this paper, each Cournot supplier observes a set of informative signals about a demand shock. The signals differ not only in their precisions, but also in their correlations: they range from those which are perfectly public (they are commonly known, and so perfectly correlated across observers) to those which are perfectly private (conditional on the true state of demand, each supplier receives an independent observation).

Three questions arise. Firstly: how do the suppliers optimally use their information? Secondly: what are the welfare properties of equilibrium behaviour? Thirdly: how do the incentives of suppliers to improve the quality of their information (via costly information acquisition) differ from those of a social planner? To answer in brief: suppliers place relatively little weight on information that is relatively public in nature; nevertheless, from a social perspective, the emphasis on public information sources is still too great; and, finally, suppliers acquire too much information, but use it too little.

The model is developed in Section 2. Consumers' utility is quadratic in the profile of differentiated products consumed, and so prices are linearly related to outputs and a demand shock. Suppliers learn about that shock via a set of normally distributed signals. A signal is characterized by its precision and by the conditional correlation of its realizations across the industry. A signal that is more correlated is more public.

The linear-quadratic-normal specification generates an equilibrium in which suppliers' outputs respond linearly to their signal realizations (Section 3). Suppliers place greater weight on relatively private information: if two signals have the same precision, then a supplier places less weight on the signal with the higher cross-industry correlation coefficient. This is because Cournot outputs are strategic substitutes: suppliers avoid correlation with their competitors by shifting away from relatively public information.

As in any Cournot industry, outputs are inefficiently low. Suppliers also use information inefficiently. Section 4 investigates the externalities that generate this inefficiency.

One externality arises because a shift from relatively public to relatively private signals reduces the correlation of outputs. This benefits the entire industry but is not internalized by an individual supplier. The same logic applies to the overall use of information. A shift away from the use of informative signals and toward a supplier's prior also reduces output correlation, and so helps competitors. From an industry perspective, then, suppliers place too much emphasis on informative signals of the demand shock, and amongst those signals they put too much weight on relatively public signals.

A second externality concerns consumers. Marshallian consumer surplus is, of course, a convex function of prices. Consumers like riskiness in prices, and so they also like riskiness in outputs. Those outputs become more variable as suppliers place more weight on their informative signals. Similarly, a shift from private to public signals increases the covariance of their outputs and so increases the variance of aggregate production. Hence, from the perspective of consumers, there is too little use of new information and insufficient weight is placed on relatively public signals.

Marshallian welfare balances the interests of suppliers (industry profit) and consumers (consumer surplus). When a movement between the weights attached to signals is considered, the key factor is the balance between the variances and covariances of suppliers' outputs. Here, the profit externality is larger (in magnitude) than the consumer surplus externality, and this means that in equilibrium suppliers rely too strongly on relatively public signals. The overall use of information (the balance between the weight placed on new information and the weight placed on the prior) is also analysed. Here the key trade-off is between the variability of output and the covariance of output with demand conditions. The consumer surplus externality is larger, and so in equilibrium the suppliers make too little use of new information. In summary: too little new information is used, and that which is used is too public in nature.

In Section 5 the private and social incentives for information acquisition are discussed. A distinction is made between two sources of noise in an informative signal. Firstly, a common shock ("sender noise") moves all suppliers' signals. Secondly, an idiosyncratic shock ("receiver noise") is specific to an individual supplier's observation. By paying more (costly) attention to any particular signal, a supplier can reduce receiver noise, but cannot filter out any common sender noise.

Under the linear demand structure here, suppliers' profits respond linearly to their competitors' outputs. This means that any change in the idiosyncratic variance of a competitor's output has no effect on others in the industry: suppliers exert no direct externalities on competitors when varying their information acquisition decisions. On the other hand, an increase in information acquisition does reduce the variance of the supplier's overall output and this is actively disliked by consumers.

The upshot from these observations is that, although suppliers acquire the (socially) optimal mix of information, overall they acquire too much of it: fixing everything else, welfare is improved by a local decrease in the attention paid to all information sources. This is true even though the information acquired is actually under-used by suppliers. A clear message emerges: from a welfare perspective, Cournot competitors acquire too much new information about demand conditions, but they use it too little.

Beyond this finding, there are other results concerning the social value of information, and these reveal the incentives for industry and governmental bodies to influence information acquisition and use. Those results distinguish between the different ways in which information can be improved, and generalize various findings in the literature.

A fuller review of related research is postponed until Section 6. Briefly, this paper contributes to a literature (Palfrey [39], Vives [48]) which considered the efficiency of information use in large oligopolies with uncertain demand, and more recently within the context of supply-function competition (Vives [52,53]). Relative to that literature this paper focuses on the distinct (in)efficiency properties of information use versus acquisition when there are many (differently correlated) information sources. The game analysed has a similar structure to that found in the literature concerned with information sharing in oligopolies (Raith [41]). The informational framework follows in the tradition of the literature on the social value of information (Morris and Shin [35], Angeletos and Pavan [5–7], Angeletos, Iovino and La'O [4], Amador and Weill [2,3], Myatt and Wallace [36,37]). It joins other recent papers (Llosa and Venkateswaran [33], Colombo, Femminis and Pavan [15]) in evaluating the social value of information when that information is acquired endogenously.

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2. Cournot competition with uncertain demand

Demand There is a representative consumer and a continuum of products indexed by $\ell \in [0, 1]$. The consumption profile q yields quadratic gross utility

$$U(q) \equiv \int_0^1 u(q_\ell, Q)\, d\ell$$

$$\text{where } u(q_\ell, Q) \equiv q_\ell \left(\theta - \frac{\beta q_\ell + (1-\beta) Q}{2}\right) \quad \text{and} \quad Q \equiv \int_0^1 q_{\ell'}\, d\ell'.$$

The parameter $\beta \in [0, 1]$ indexes the extent of product differentiation: the products are undifferentiated if and only if $\beta = 0$, and they are independent if $\beta = 1$.[1] The "demand shifter" parameter θ is uncertain; more is said about this later in the section.

The consumer's net utility is quasi-linear in labour supply. The wage is the numéraire, and p_ℓ is price of product ℓ. Hence, demands are chosen to maximize $U(q) - \int_0^1 p_\ell q_\ell \, d\ell$. Equivalently, the market-clearing price of product ℓ is

$$p_\ell = \theta - \beta q_\ell - (1-\beta) Q.$$

Supply Manufacturer $m \in \{1, \ldots, M\}$ supplies a fraction $1/M$ of the products. The product space is partitioned into M intervals, where $L_m \subseteq [0, 1]$ is the product range supplied by m. The aggregate output and cost of manufacturer m are

$$Q_m = \int_{\ell \in L_m} q_\ell \, d\ell \quad \text{and} \quad C_m = \frac{c}{2} \int_{\ell \in L_m} q_\ell^2 \, d\ell,$$

where the cost is expressed in terms of the required labour input (the numéraire). Any linear component to a supplier's cost function can be absorbed into the representative consumers' willingness to pay, and so the parameter c captures the relative importance of the quadratic term in suppliers' costs rather than the size of costs overall. Equivalently, c is the extent to which a manufacturer experiences decreasing returns to scale.

Prices and profits Supplier m optimally spreads output equally and so $q_\ell = M Q_m$ if $\ell \in L_m$. Abusing notation, p_m is the common price of m's products. Hence:

$$p_m = \theta - \big(\beta M + (1-\beta)\big) Q_m - (1-\beta) \sum_{m' \neq m} Q_{m'} \quad \text{and} \quad C_m = \frac{c M Q_m^2}{2}.$$

The profit of supplier m and total industry profit, in terms of the numéraire, are

$$\text{Profit}_m = \theta Q_m - \left(\beta M + (1-\beta) + \frac{cM}{2}\right) Q_m^2 - (1-\beta) \sum_{m' \neq m} Q_m Q_{m'} \quad \Rightarrow$$

$$\text{Total Profit} = \theta \sum_{m=1}^{M} Q_m - \left(\beta M + (1-\beta) + \frac{cM}{2}\right) \sum_{m=1}^{M} Q_m^2 - (1-\beta) \sum_{m=1}^{M} \sum_{m' \neq m} Q_m Q_{m'}. \tag{1}$$

Total industry profit is decreasing in the variances and covariances of outputs.

[1] This quadratic specification is familiar from Dixit [20], Singh and Vives [45], and many others.

The M manufacturers simultaneously choose their supplies, their payoffs are their profits, and the information available to them is described below.

Consumer surplus and welfare The gross utility $U_m \equiv \int_{\ell \in L_m} u(q_\ell, Q)\, d\ell$ and consumer surplus $\mathrm{CS}_m \equiv U_m - p_m Q_m$ associated with the mth manufacturer's products are

$$U_m = Q_m\left(\theta - \frac{\beta M Q_m + (1-\beta)Q}{2}\right) \quad \text{and} \quad \mathrm{CS}_m = \frac{\beta M Q_m^2 + (1-\beta) Q Q_m}{2}.$$

Aggregating across the entire industry yields

$$\text{Consumer Surplus} = \frac{1}{2}\sum_{m=1}^{M}\left[\big(\beta M + (1-\beta)\big)Q_m^2 + (1-\beta)\sum_{m' \neq m} Q_m Q_{m'}\right]. \tag{2}$$

This is increasing in the variances and covariances of the manufacturers' outputs.

Finally, industry profits are returned to the price-taking representative consumer. Thus, (Marshallian) welfare is the sum of industry profit and consumer surplus:

$$\text{Welfare} = \sum_{m=1}^{M}\left[\theta Q_m - \frac{(\beta M + (1-\beta) + cM)Q_m^2}{2} - \frac{(1-\beta)\sum_{m' \neq m} Q_m Q_{m'}}{2}\right]. \tag{3}$$

This is decreasing in the variances and covariances of outputs.

Information The parameter θ determines the market's demand conditions. It is unknown, and suppliers share a common prior: $\theta \sim \mathrm{N}(x_0, \kappa_0^2)$. Each supplier m receives n informative signals about θ. The ith signal received by m is

$$x_{im} = \theta + \eta_i + \varepsilon_{im}, \tag{4}$$

where $\eta_i \sim \mathrm{N}(0, \kappa_i^2)$, and $\varepsilon_{im} \sim \mathrm{N}(0, \xi_i^2)$, and where all the noise terms are uncorrelated.

η_i is a common shock; this is noise attributable to the sender of the information. ε_{im} is a supplier-specific shock to m's observation; this is noise attributable to the receiver.[2]

The specification of (4) induces a correlation structure for the observations. Conditional on θ, the correlation coefficient between the observations x_{im} and $x_{im'}$ made by two different suppliers is $\rho_i = \kappa_i^2/(\kappa_i^2 + \xi_i^2)$. The precision of signal i is $\psi_i = 1/(\kappa_i^2 + \xi_i^2)$. Signals differ in their correlation and in their precision. This captures the different qualities of the information sources. Loosely speaking, if the observations are more correlated then an information source is more public (publicity is taken to correspond to a higher value of ρ_i throughout). The prior is, in essence, a perfectly public signal.

Sections 3 and 4 restrict to this exogenous information structure. However, in Section 5 each supplier can engage in costly information acquisition. Specifically, in that section

$$\varepsilon_{im} \sim \mathrm{N}\left(0, \frac{\xi_i^2}{z_{im}}\right),$$

where z_{im} is the (costly) attention paid by supplier m to information source i, and where an information-acquisition cost $\hat{C}(z_m)$ is deducted from supplier m's profits.

[2] The "sender" and "receiver" terminology is from Myatt and Wallace [36], where the information structure is the same, albeit in a context where the focus is the endogenous acquisition of information in the context of a "beauty contest" quadratic-payoff coordination game.

3. Equilibrium

Optimal output Supplier m maximizes expected profit. The objective function is quadratic in Q_m; the associated first-order condition (for positive output) is

$$\underbrace{\mathrm{E}[\theta \mid x_m] - (1-\beta)\sum\nolimits_{m' \neq m} \mathrm{E}[Q_{m'} \mid x_m] - 2Q_m\big((1-\beta) + \beta M\big)}_{\text{expected MR}} = \underbrace{cMQ_m}_{\text{MC}}.$$

The unique solution is optimal if $Q_m \geq 0$, but if $\mathrm{E}[\theta \mid x_m] < (1-\beta)\sum_{m' \neq m} \mathrm{E}[Q_{m'} \mid x_m]$ then the supplier prefers to produce nothing. In the context of a symmetric equilibrium, $(1-\beta)\sum_{m' \neq m} \mathrm{E}[Q_{m'} \mid x_m] = (1-\beta)(M-1)\,\mathrm{E}[Q_{m'} \mid x_m]$. Summarizing,

$$Q_m = \max\left\{\frac{\mathrm{E}[\theta \mid x_m] - (1-\beta)(M-1)\,\mathrm{E}[Q_{m'} \mid x_m]}{2\beta M + 2(1-\beta) + cM}, 0\right\},$$

where expectations are conditional on the information m has and others' strategies.

The zero-output corner solution is a possibility when demand is expected to be weak. This is assumed away by allowing for negative solutions to each supplier's output and (relatedly) for the possibility that market-clearing prices may be negative: the focus is entirely on the first-order-condition solution for Q_m. Of course, if negative outputs and prices are disallowed then the strategies considered here sometimes specify infeasible actions. As noted by Vives [47, p. 77, fn. 2], "the probability of such an event can be made arbitrarily small by appropriately choosing the variances of the model".

This crude ignore-the-problem approach is not entirely satisfactory. Another approach would be to abandon the normal specification. The key advantage of the normal is the linearity of conditional expectations (Li [31], Li, McKelvey and Page [32]) which justifies strategies in which outputs are linearly related to signal realizations. If normality is abandoned and linearity is obtained via a different specification, or is directly imposed (perhaps by an appeal to simplicity), then the negativity problem can be circumvented.

Some have noted that results concerning information sharing in oligopolies can change if non-negativity constraints are respected.[3] Here, however, the focus is not on information sharing. Instead, the objective is to understand the use made of relatively private versus relatively public information and the efficiency properties of this decision.

Full-information benchmarks If $\theta > 0$ is known then the efficient output (which is equal across all products, and equates the price $p = \theta - Q$ to the marginal cost cQ) is $Q^\diamond = \theta/(1+c)$. In the full-information benchmark there is a unique and symmetric Nash equilibrium. Each supplier produces an equal share of aggregate equilibrium output $Q^\star$:

$$Q^\star = \frac{\theta M}{2\beta M + (1-\beta)(M+1) + cM} \quad \Rightarrow \quad \lim_{M \to \infty}\left(\frac{Q^\star}{Q^\diamond}\right) = \frac{1+c}{\beta + 1 + c}. \tag{5}$$

As usual, industry output $Q^\star$ falls below the efficient level $Q^\diamond$. Moreover, output remains inefficiently low even as $M \to \infty$ unless the products are homogeneous ($\beta = 0$). Some other

[3] Malueg and Tsutsui [34] and Lagerlöf [30] considered Cournot models with two demand states and showed that information sharing may reduce welfare; this contrasts with Vives [47]. They noted that the probability of negative output and negative price events is non-negligible if there is substantial uncertainty, and that (Lagerlöf [30, p. 862]) "the real world situations that the models are supposed to capture often involve a substantial amount of uncertainty". The results on the relative use of informative signals presented here are in any case of interest when there is rather less uncertainty.

items of interest in the benchmark case are the industry's total profit (each supplier obtains an equal share of this) and the surplus captured by the consumer:

$$\text{Industry Profit} = \frac{(2\beta M + 2(1-\beta) + cM)(Q^\star)^2}{2M} \quad \text{and} \quad \text{Consumer Surplus} = \frac{(Q^\star)^2}{2}.$$

Quadratic-payoff coordination games The Cournot game is strategically equivalently to a quadratic-payoff coordination game. To obtain this equivalence, define

$$\pi \equiv 1 + \frac{(1-\beta)(M-1)}{2\beta M + 2(1-\beta) + cM} \quad \Rightarrow$$
$$Q_m = \mathrm{E}\left[\frac{Q^\star}{M}\right] - (\pi - 1)\left(\mathrm{E}[Q_{m'}] - \mathrm{E}\left[\frac{Q^\star}{M}\right]\right), \tag{6}$$

where the full-information output $Q^\star$ from (5) is a scaled version of the demand shifter θ, and where the expectations are conditional on the signals x_m received by player m. This is the equilibrium condition from a quadratic-payoff "beauty contest" coordination game à la Morris and Shin [35]. In such a game each player seeks to minimize

$$\text{Quadratic Loss} = \pi(Q_m - \tilde{\theta})^2 + (1-\pi)\left(Q_m - \frac{1}{M-1}\sum_{m' \neq m} Q_{m'}\right)^2.$$

The first term represents a "fundamental" motive, whereas the second term represents a "coordination" motive. If $\pi \in (0, 1)$ then this game takes a coordination form: a player wishes to take an action that is close to the actions of others. However, if $\pi > 1$ (that is, if $\pi - 1 > 0$) then a player prefers to move away from others.

In the Cournot game, such an anti-coordination motive is present (so that $\pi > 1$). The results of this paper hold if the anti-coordination motive is not too strong, so that $\pi < 2$, or equivalently $\pi - 1 < 1$. This holds if and only if $(1-\beta)(M-3) < (c+2\beta)M$, which in turn holds for all M if $1 < c + 3\beta$; otherwise it holds if M is not too large.

The inequality $\pi < 2$ (equivalently: the anti-coordination motive satisfies $\pi - 1 < 1$) is imposed as a regularity condition throughout. As noted in Lemma 1 below, π is increasing in β (product differentiation) and c (which measures the extent of decreasing returns to scale) but decreasing in M (the number of competitors). These parameters are associated with the market power of a supplier. Thus, the condition $\pi < 2$ amounts to an assumption that the market is not too competitive. This condition guarantees the existence of an equilibrium. It is a critical sufficient condition in the following sense: if it fails then there are parameter values (specifically: when the receiver noise variances ξ_i^2 are small) for which a linear equilibrium does not exist. Furthermore, in the absence of uncertainty (when θ and so $Q^\star$ is known) it ensures the stability of a symmetric equilibrium in the context of an appropriate strategy-revision process.[4]

Equilibrium Consider a linear (Bayesian) equilibrium, so that $Q_m = \sum_{i=0}^{n} w_{im} x_{im}$ for weights $w_m \in \mathbb{R}^{n+1}$. Here, $x_{0m} \equiv x_0$ is the prior mean, so that $w_{0m} x_0$ is the intercept for an affine strategy. Owing to normality, $\mathrm{E}[\theta \mid x_m]$ is linear in x_m. Furthermore, if others use linear strategies then

[4] Suppose that θ is known, and suppose that all others choose output $Q_{m'}$. The best reply of supplier m is to choose output $Q_m = (Q^\star/M) - (\pi - 1)(Q_{m'} - (Q^\star/M))$ where $Q^\star/M$ is the equilibrium output for each supplier in a full-information world. Equivalently, $|Q_m - (Q^\star/M)| = |\pi - 1| \times |Q_{m'} - (Q^\star/M)|$. By inspection, a sequence of myopic best replies (defined in an obvious way) is explosive if $|\pi - 1| > 1$.

$\mathrm{E}[Q_{m'} \mid x_m]$ is also linear in x_m. It follows that m's best reply is linear in x_m; that is, the class of linear strategy profiles is closed under best reply.[5]

Proposition 1 *(Characterization of the Equilibrium). There is a unique linear equilibrium. Supplier m chooses $Q_m = \sum_{i=0}^{n} w_i^\star x_{im}$, where the weights $w^\star \in \mathbb{R}^{n+1}$ satisfy*

$$w_i^\star \propto \frac{1}{\pi\kappa_i^2 + \xi_i^2} \quad \text{and} \quad \bar{w}^\star \equiv \sum_{i=0}^{n} w_i^\star = \frac{1}{2\beta M + (1-\beta)(M+1) + cM}. \tag{7}$$

The expected output of the industry is equal to the expected output under full information:

$$\begin{aligned} \mathrm{E}\left[\sum_{m=1}^{M} Q_m\right] &= M\left(w_0^\star x_0 + \sum_{i=1}^{n} w_i^\star \mathrm{E}[\theta]\right) \\ &= \frac{M x_0}{2\beta M + (1-\beta)(M+1) + cM} = \mathrm{E}\big[Q^\star\big]. \end{aligned} \tag{8}$$

In terms of the precisions $\psi_i = 1/(\kappa_i^2 + \xi_i^2)$ and correlation coefficients $\rho_i = \kappa_i^2/(\kappa_i^2 + \xi_i^2)$,

$$\frac{w_i^\star}{w_j^\star} = \frac{\psi_i(1+(\pi-1)\rho_j)}{\psi_j(1+(\pi-1)\rho_i)} \quad \text{for } i, j \in \{0, 1, \ldots, n\}. \tag{9}$$

Fixing the correlation coefficients, relatively precise signals have relatively great influence; fixing the precisions, relatively correlated signals have relatively little influence.

The proof is in Appendix A; the argument is sketched here. Using m's profit from (1),

$$\begin{aligned} \mathrm{E}[\mathrm{Profit}_m \mid \theta] = \theta\, \mathrm{E}[Q_m \mid \theta] &- \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\left(\mathrm{E}[Q_m \mid \theta]\right)^2 + \mathrm{var}[Q_m \mid \theta]\right) \\ &- (1-\beta)\left(\sum_{m'\neq m} \mathrm{E}[Q_m \mid \theta]\,\mathrm{E}[Q_{m'} \mid \theta] + \sum_{m'\neq m} \mathrm{cov}[Q_m Q_{m'} \mid \theta]\right). \end{aligned} \tag{10}$$

Consider a linear strategy profile in which each supplier m' produces $Q_{m'} = \sum_{i=0}^{n} w_{im'} x_{im'}$ where $w_{m'} \in \mathbb{R}^{n+1}$. Define $\bar{w}_{m'} = \sum_{i=0}^{n} w_{im'}$. For such a profile,

$$\mathrm{var}[Q_m \mid \theta] = \sum_{i=1}^{n} w_{im}^2\left(\kappa_i^2 + \xi_i^2\right) \quad \text{and} \quad \mathrm{cov}[Q_m, Q_{m'} \mid \theta] = \sum_{i=1}^{n} w_{im} w_{im'} \kappa_i^2 \tag{11}$$

depend on the weights used across all signals (excluding the prior) but not on θ. The remaining elements of $\mathrm{E}[\mathrm{Profit}_m \mid \theta]$ depend on the signal weights only through their (symmetric) influence on $\bar{w}_m$ (the sum of the weights for all signals). For example, $\mathrm{E}[Q_m \mid \theta] = \bar{w}_m \theta + w_{0m}(x_0 - \theta)$ depends on $\bar{w}_m$ and w_{0m} (the weight on the prior). So,

$$\begin{aligned} \mathrm{E}[\mathrm{Profit}_m \mid \theta] = \text{other terms} &- \left(\beta M + (1-\beta) + \frac{cM}{2}\right) \mathrm{var}[Q_m \mid \theta] \\ &- (1-\beta) \sum_{m'\neq m} \mathrm{cov}[Q_m Q_{m'} \mid \theta], \end{aligned} \tag{12}$$

[5] Linear strategies are not restrictive. Dewan and Myatt [18] established that any equilibrium amongst those that involve strategies that are bounded above and below by linear strategies is itself linear.

where the "other terms" depend on the weights w_{im} for $i \in \{1, \ldots, n\}$ only via the totals $\bar{w}_m$. This means that supplier m's optimally chosen weights must solve

$$\min\left\{\operatorname{var}[Q_m \mid \theta] + \frac{(1-\beta)\sum_{m' \neq m} \operatorname{cov}[Q_m, Q_{m'} \mid \theta]}{\beta M + (1-\beta) + cM/2}\right\}$$

$$\text{subject to} \quad \sum_{i=1}^{n} w_{im} = \text{constant}.$$

Recall the definition of π from (6). The objective of supplier m is equivalently to minimize $\operatorname{var}[Q_m \mid \theta] + 2(\pi - 1)\operatorname{cov}[Q_m, Q_{m'} \mid \theta]$ subject to the constraint on $\bar{w}_m$.[6]

The variance penalizes any deviation from the ideal full-information best reply. This comes about because both revenue and costs are quadratically related to m's output. The covariance penalizes a supplier for any correlation with competitors. The supplier prefers to shift output to situations where the price is higher, which is when competitors' outputs are lower: a supplier wishes to avoid correlation with the competition.

Substituting in the variance and covariance terms from (11), and evaluating at a candidate symmetric equilibrium (that is, where $w_{im'} = w_i^\star$ for all i and all m'):

$$\frac{\partial(\operatorname{var}[Q_m \mid \theta] + 2(\pi - 1)\operatorname{cov}[Q_m, Q_{m'} \mid \theta])}{\partial w_{im}} \propto w_i^\star\left(\pi\kappa_i^2 + \xi_i^2\right).$$

From this, (7) in Proposition 1 is readily obtained.

The noise associated with the common-across-suppliers shock η_i from information source i (measured by its variance κ_i^2) has a scaled effect on the weight chosen by the supplier different from its effect on the signal's variance (or precision). It is adjusted by $\pi > 1$ relative to the effect of ξ_i^2. Suppliers shift away from signals which have disproportionately large sender noise. This, of course, is because such signals are relatively correlated, whereas a supplier has a preference for output to correlate negatively with others.

Comparative-static exercises The equilibrium quantities reported in Proposition 1 depend on the exogenous parameters of the model M (the number of competitors), β (product differentiation), and c (the importance of the quadratic component of each supplier's cost function). Often the impact of a change in one of these parameters will make itself felt via its impact on π. It is useful therefore to summarize the effects of these parameters on the motive for suppliers to hit the target $Q^\star/M$ relative to their desire to coordinate. An inspection of π straightforwardly yields this lemma.

Lemma 1 *(Response of the Coordination Motive to the Parameters). π is increasing in M and decreasing in β and c. Equivalently, the size $|\pi - 1|$ of the anti-coordination motive is increasing in the number of competitors, but decreasing in product differentiation and in the importance of the quadratic component of suppliers' costs.*

The more suppliers there are the more advantageous it becomes to take an uncorrelated action. On the other hand, the higher is β, the lower is π. Greater product differentiation reduces the necessity to avoid correlation with competitors' output levels.

[6] The importance of both the variance and covariance terms to a supplier's profit was noted by Novshek and Sonnenschein [38] and others who studied information sharing in oligopolies.

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Using these facts, the following proposition describes the comparative-static results for various equilibrium quantities of interest: in particular, the weight attached to each information source $w_i^\star$, the ratio of the weights attached to different information sources $w_i^\star/w_j^\star$, and the total weight attached to information in equilibrium $\bar{w}^\star$. This last object is related to expected industry output, indeed $\mathrm{E}[Q^\star] = Mx_0\bar{w}^\star$ from (8).

Proposition 2 *(Comparative-Static Predictions). The total weight placed on information $\bar{w}^\star$ is decreasing in M, β, and c. The weight placed on signal i relative to signal j is decreasing in M and increasing in β and c if and only if i is more public than j. That is*

$$\frac{\partial[w_i^\star/w_j^\star]}{\partial M} < 0 \quad \Leftrightarrow \quad \frac{\partial[w_i^\star/w_j^\star]}{\partial \beta} > 0 \quad \Leftrightarrow \quad \frac{\partial[w_i^\star/w_j^\star]}{\partial c} > 0 \quad \Leftrightarrow \quad \rho_i > \rho_j. \tag{13}$$

Expected industry output is increasing in M, but decreasing in β and c. Finally, $w_i^\star$ is decreasing in M and c for all i. $w_i^\star$ is increasing in β if and only if i is sufficiently public:

$$\frac{\partial w_i^\star}{\partial \beta} > 0 \quad \Leftrightarrow \quad \pi^\dagger(\hat{\rho}_i - \hat{\rho}) > 1, \quad \text{where } \hat{\rho}_i = \frac{\pi\kappa_i^2}{\pi\kappa_i^2 + \xi_i^2},$$

$\pi^\dagger$ is given in (14), and $\hat{\rho}$ is a measure of "average publicity":

$$\hat{\rho} \equiv \frac{\sum_{i=0}^n \hat{\psi}_i \hat{\rho}_i}{\sum_{i=0}^n \hat{\psi}_i}, \quad \text{where } \hat{\psi}_i = \frac{1}{\pi\kappa_i^2 + \xi_i^2}.$$

Summarizing: lower product differentiation or reduced importance of the quadratic component of costs, or an increase in competition, results in (i) an increase in expected industry output and (ii) a shift from relatively public to relatively private signals.

Note that the condition required for $w_i^\star$ to be increasing in β will fail if β is high enough (an inspection of the quantity $\pi^\dagger$ given in (14) will confirm this). Only when products are relatively homogeneous is it possible for an increase in product differentiation to result in an increase in the weight placed on the most public of signals (and the prior in particular, which has $\rho_0 = \hat{\rho}_0 = 1$). At the same time, the relative weight on public versus private signals is certainly increasing in the extent of product differentiation, whilst the total weight placed on all information sources is decreasing.

Whilst an increase in the size of the industry, the extent of product differentiation, or costs all reduce the total weight attached to the informative signals, they affect the balance of information use in a more subtle way. An increase in the size of the industry makes correlation with competitors' outputs more of a problem (there are more of them) and so, even though the weight attached to each and every signal is reduced, this is done in a way that favours relatively private sources at the expense of relatively public ones. On the other hand, an increase in the extent of product differentiation increases the monopoly power of each supplier, and whilst this means less can be produced (and so total weight is reduced), it also means that correlation with competitors' outputs is less of an issue: more use can be made of relatively public signals. Indeed, if a signal i is sufficiently public it may even be that the weight attached to it rises in absolute terms.

4. Industry profits, consumer surplus, and social welfare

This section explores the externalities that suppliers impose upon one another (Proposition 3) and upon consumers (Proposition 4). It also characterizes the socially optimal use of information and how it differs from equilibrium use (Proposition 5), and offers results on how the various externalities are affected by industry characteristics (Proposition 6). Finally, it investigates the social value of improved information (Propositions 7 and 8).

Industry profits From the collective viewpoint of the suppliers, output is too high. Ideally, they would collusively lower output to keep prices higher. In general, this means they would prefer to choose lower weights on all of their signals.

Here, however, the focus is not on the level of output but rather on the use of information. To explore the inefficiencies of such use, fix the total weight used by each supplier $\bar{w}_{m'}$ (this fixes expected output) and consider moving weight from signal j to signal i.

Consider again the expected profit of supplier m reported in (12). The "other terms" depend only on $\bar{w}_{m'}$ and $w_{0m'}$ for each $m' \neq m$, and so they are unaffected by a shift in weight from $w_{jm'}$ to $w_{im'}$. Similarly, $\text{var}[Q_m \mid \theta]$ depends only on the weights used by supplier m. From this it follows that

$$\begin{aligned}
&\frac{\partial\, \mathrm{E}[\text{Profit}_m \mid \theta]}{\partial w_{im'}} - \frac{\partial\, \mathrm{E}[\text{Profit}_m \mid \theta]}{\partial w_{jm'}} \\
&\quad = (1-\beta)\left(\frac{\partial \operatorname{cov}[Q_m, Q_{m'} \mid \theta]}{\partial w_{jm'}} - \frac{\partial \operatorname{cov}[Q_m, Q_{m'} \mid \theta]}{\partial w_{im'}}\right) \\
&\quad = (1-\beta)\big(w_{jm}\kappa_j^2 - w_{im}\kappa_i^2\big).
\end{aligned}$$

Supplier m' exerts a positive externality on his competitors if weight is shifted away from information sources that have, relative to their use, a higher "sender noise" variance κ_j^2, and that have higher covariances. This is because Cournot outputs are strategic substitutes: suppliers help competitors if outputs are less correlated. This is achieved by placing less emphasis on relatively correlated information.

At an equilibrium strategy profile where every supplier puts weight $w_i^\star$ on the ith signal, supplier m' benefits others by shifting weight from signal j to signal i if and only if

$$\frac{\partial\, \mathrm{E}[\text{Profit}_m \mid \theta]}{\partial w_{im'}} > \frac{\partial\, \mathrm{E}[\text{Profit}_m \mid \theta]}{\partial w_{jm'}} \quad \Leftrightarrow \quad \frac{\kappa_j^2}{\kappa_i^2} > \frac{w_i^\star}{w_j^\star} = \frac{\pi\kappa_j^2 + \xi_j^2}{\pi\kappa_i^2 + \xi_i^2} \quad \Leftrightarrow \quad \rho_i < \rho_j.$$

This conclusion is reached when a shift between signals is considered. The story is different when considering a shift between the prior and a signal. Here, the benefit of moving weight from a relatively public (and hence correlated) signal to a relatively private (and hence uncorrelated) signal remains: the prior is "perfectly" public, it has $\rho_0 = 1$, so moving weight away from it is beneficial. However, this is always outweighed by the additional cost of moving away from the prior: the value of the prior is known ex ante, and by moving weight to an unknown information source a supplier exerts a negative externality by increasing the variance of output. Suppliers do not like variance, and this effect dominates local to equilibrium, as the next proposition confirms.

Proposition 3 *(Equilibrium Properties of Industry Profit). (i) From the perspective of suppliers there is too much emphasis on relatively public information: if $\rho_j > \rho_i$ then a supplier exerts a positive externality on other suppliers by shifting from the relatively public signal j to the relatively private signal i.*

(ii) Too much new information is used: other suppliers' profits are increased by shifting weight from any signal i to the prior.

(iii) The signal weights which (collusively) maximize industry profits are, for all i,

$$w_i^\dagger \propto \frac{1}{\pi^\dagger \kappa_i^2 + \xi_i^2}$$

$$\text{with } \pi^\dagger = \frac{(2+c)M}{2\beta M + 2(1-\beta) + cM} \quad \text{and} \quad \bar{w}^\dagger = \sum_{i=0}^{n} w_i^\dagger = \frac{1}{(2+c)M}. \tag{14}$$

More weight is placed on signal i relative to j at the collusive optimum than in equilibrium if and only if i is more private than j; collusion favours uncorrelated signals.

The final part of the proposition reveals the collusively optimal weights the suppliers would use. The total weight placed on the signals and prior $\bar{w}$ falls relative to the equilibrium, so that expected output is lower at the collusive optimum (naturally): $\bar{w}^\dagger < \bar{w}^\star$. Moreover, π in (7) is replaced with $\pi^\dagger > \pi$ in the optimal weights. The (inverse) role of κ_i^2 is enhanced in the weight on signal i used by the supplier. It follows that, in comparison to the non-cooperative equilibrium, relatively more emphasis is placed on relatively uncorrelated information sources. Collusion favours private information, and the prior is again treated exactly as a perfectly public signal.

If the suppliers were (for whatever reason) restricted to use a total weight equal to the equilibrium value, so that $\sum_{i=0}^{n} w_i = \bar{w}^\star$, but were able to choose these weights collusively (as in the third part of the proposition), they would choose weights for $i \neq 0$ exactly as in (14), so that $w_i = w_i^\dagger$, whilst increasing the weight on the prior. In fact, $w_0 = w_0^\dagger + (\bar{w}^\star - \bar{w}^\dagger) > w_0^\dagger$. Suppliers increase the weight on the prior simply to ensure that expected output reaches its equilibrium level.

Consumer surplus Expected consumer surplus is increasing in the variance of the various Q_m and in their covariances. Furthermore, the balance between the covariance and variance terms is different for consumers than for suppliers. In particular,

$$\begin{aligned} &\mathrm{E}[\text{Consumer Surplus} \mid \theta] \\ &\quad = \text{other terms} + \frac{(\beta M + (1-\beta))\,\mathrm{var}[Q_m \mid \theta]}{2} + (1-\beta) \sum_{m' \neq m} \mathrm{cov}[Q_m, Q_{m'} \mid \theta], \end{aligned} \tag{15}$$

where the "other terms" are those which do not depend on shifts between w_{im} and w_{jm}. Compared to (12) note that the balance between these terms differs. This is easiest to see by summing the profit of supplier m with consumer surplus. Doing so,

$$\begin{aligned} &\mathrm{E}[\text{Profit}_m \mid \theta] + \mathrm{E}[\text{Consumer Surplus} \mid \theta] \\ &\quad = \text{other terms} - \frac{(\beta M + (1-\beta) + cM)\,\mathrm{var}[Q_m \mid \theta]}{2}. \end{aligned} \tag{16}$$

Upon internalizing the concerns of consumers, supplier m's objective would become:

$$\min \mathrm{var}[Q_m \mid \theta] \quad \text{subject to} \quad \sum_{i=1}^{n} w_{im} = \text{constant}.$$

The solution to this reverses the previous bias against public signals; beginning from equilibrium, consumers would prefer greater emphasis on relatively public information.

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This discussion concerns the balance between various information sources. The allocation of weight between the signals and the prior is also important. Consumers prefer variable output and so they would like greater overall use of new information.

Proposition 4 *(Equilibrium Properties of Consumer Surplus). (i) From the perspective of the consumers there is too much emphasis on relatively private information: if $\rho_j < \rho_i$ then a supplier exerts a positive externality on consumers (consumer surplus rises) by shifting weight from the relatively private signal j to the relatively public signal i.*

(ii) Too little new information is used: consumer surplus is increased by shifting weight from the prior to any signal i.

There is no analogue to the third part of Proposition 3: if consumers could choose any weights, then they would wish to increase unboundedly the aggregate weight to increase both the expectation of output and its variability. Moreover, fixing total weight $\bar{w}$, the convexity of consumer surplus ensures that they would prefer suppliers to place maximal weight on a single (and highest variance) information source.

Social welfare Aggregating industry profits and consumer surplus yields the expression for social welfare introduced in (3).

Proposition 5 *(Equilibrium Properties of Welfare). (i) From the perspective of welfare there is too much emphasis on relatively public information: if $\rho_j > \rho_i$ then welfare is increased by shifting weight from the relatively public signal j to the relatively private i.*

(ii) Too little new information is used: welfare is increased by shifting weight from the prior to any signal i.

(iii) The weights on the signals which maximize social welfare are, for all i,

$$w_i^\diamond \propto \frac{1}{\pi^\diamond \kappa_i^2 + \xi_i^2}$$
$$\text{with } \pi^\diamond = \frac{(1+c)M}{\beta M + (1-\beta) + cM} \quad \text{and} \quad \bar{w}^\diamond = \sum_{i=0}^{n} w_i^\diamond = \frac{1}{(1+c)M}. \tag{17}$$

More weight is placed on signal i relative to j at the social optimum than in equilibrium if and only if i is more private than j. The social optimum favours uncorrelated signals.

From the perspective of industry profits, too much use is made of relatively public information; from the perspective of consumer surplus too little use is made of relatively public information. Proposition 5 reveals that from a welfare perspective the first effect outweighs the second. Insufficient weight is given to relatively private information.

A natural conjecture is that the greater size of the industry-profit externality carries over to the overall level of information use. However, this is not so: the consumer surplus effect (that too little weight is placed on new information) outweighs the industry-profit effect (that too much weight is placed on new information), and so welfare would be enhanced with less weight on the prior and more on new information. The key here is that consumers do not care about the covariance of output with the unknown demand shifter. Once their focus on increasing variance is incorporated, welfare places less emphasis (compared with profit) on minimizing the (conditional) variance of output and its covariance with others' production. This can be seen by considering the case of a monopolist, so that $M = 1$. Here the output covariance terms disappear, and so

$$\text{Profit}_m = \theta Q_m - \left(1 + \frac{c}{2}\right) Q_m^2 \quad \text{and} \quad \text{Consumer Surplus} = \frac{Q_m^2}{2}.$$

The combination of these two terms (welfare) moves emphasis towards the maximization of $\mathrm{E}[\theta Q_m]$; this is achieved by increasing the weight on new information.

The third part of Proposition 5 identifies the socially optimal weights which, once again, in comparison with the equilibrium, attach relatively more weight to relatively private signals. The weight on any signal i (including the prior) should be proportional to the inverse of $\pi^\diamond \kappa_i^2 + \xi_i^2$, where $\pi^\dagger > \pi^\diamond > \pi$. The conclusions of the third part of Proposition 3 go through; albeit to a lesser extent. The (ratios of the) socially optimal weights involve moving somewhat towards those that maximize industry profits, but not all the way.

Industry characteristics and efficiency It has been established that information is used inefficiently. This section considers the relationship between the nature of this inefficiency and the industry's characteristics.

Consider first the difference between the equilibrium use of information and the industry's preferred use. From Proposition 3, there is (from the perspective of suppliers) too much use of public information relative to private information. To explore this distortion, order the information sources in decreasing order of publicity, so that $\rho_1 \geq \cdots \geq \rho_n$. Furthermore, suppose that the most public signal is perfectly public (so that $\xi_1^2 = 0$) and the most private signal is perfectly private (so that $\kappa_n^2 = 0$). If this is so, then

$$\frac{w_1^\star}{w_n^\star} = \frac{\xi_n^2}{\pi \kappa_1^2} \quad \text{and} \quad \frac{w_1^\dagger}{w_n^\dagger} = \frac{\xi_n^2}{\pi^\dagger \kappa_1^2} \quad \Rightarrow \quad \frac{w_1^\star / w_n^\star}{w_1^\dagger / w_n^\dagger} = \frac{\pi^\dagger}{\pi}.$$

Thus, the equilibrium over-reliance on public information, relative to private information, is determined by the ratio $\pi^\dagger/\pi$. This is from the perspective of industry profits. Using Proposition 5, a similar exercise confirms that the over-reliance on public information from a welfare perspective is determined by the ratio $\pi^\diamond/\pi$.

Lemma 2 *(Relative Coordination Motives and Industry Parameters). The ratio $(\pi^\dagger/\pi)$ is increasing M, and decreasing in β and c. If (β/c) is sufficiently large, $2\beta(\beta + c) \geq c$, then the ratio $(\pi^\diamond/\pi)$ shares the same properties. However, if (β/c) is sufficiently small and M is sufficiently large then $(\pi^\diamond/\pi)$ is decreasing in M and increasing in β.*

Proposition 6 *(Inefficient Information Use and Market Power). From the perspective of industry profits, the distortion in the use of a perfectly public signal relative to a perfectly private signal is decreasing in indicators of an individual supplier's market power: that is, the distortion is increasing in M, and decreasing in β and c.*

Clearly, a related claim can be made from the perspective of social welfare. No such similar result can be produced for consumer surplus, since consumers do not have a well-defined optimal weight ratio between each of the signals. The consumers would like all weight to be placed on some high-variance (suitably measured) signal.

Abstracting from other sources of inefficiency, for β sufficiently large, increasing the number of suppliers moves information use away from the industry and social optimum. In fact, a monopolist uses information efficiently in the sense that the weight placed on a signal is inversely proportional to its precision: $\pi^\diamond = \pi^\dagger = \pi = 1$ when $M = 1$. The differentiation between the goods, as measured by β, plays a similar role. If the goods are entirely independent, again $\pi^\diamond = \pi^\dagger = \pi = 1$ and information is used efficiently (monopolistic competition). On the other hand when β is low, so that goods are relatively close substitutes, these comparative statics are reversed in the case of the socially optimal relative use of information. More suppliers and lower β can (locally) improve the efficiency properties of the relative use of information if M is high enough.

The social value of information Equilibrium profits, consumer surplus, and social welfare all vary with the precisions of the signals. A natural question is whether more information (greater precision) is better for the suppliers, for the consumers, and for welfare as a whole. Given the distinction between receiver noise and sender noise, Lemma 4 (relegated to Appendix A) investigates how changing the receiver-noise precision $(1/\xi_j^2)$ or the sender-noise precision $(1/\kappa_j^2)$ of any signal j affects profits, surplus, and welfare.

Recall that industry profit, for example, is influenced directly by the variances and correlation coefficients of the signals. Profit is also influenced by the equilibrium weights placed on the various information sources by the suppliers. These weights are in turn affected by the parameters ξ_i^2 and κ_i^2. These direct and indirect effects may act in opposite directions. For instance, whilst more information (a decrease in ξ_i^2, say) reduces variance and is therefore good for profit; a more informative signal will also attract more weight in equilibrium. If this signal is relatively public this may result in a decrease in profit (and indeed a decrease in ξ_i^2 increases ρ_i and thus the publicity of signal i), hence counteracting the direct effect. Similarly, increasing a signal's precision raises the use of new information versus the prior, and adversely affects profit.

Increasing the precision of a perfectly private signal (with $\rho_i = 0$ or equivalently $\kappa_i^2 = 0$) has no effect on its publicity. From the above discussion it appears likely that the direct effect will dominate, and more information should be better for profit. The following proposition confirms this is indeed the case. It also reports the analogous results for surplus (where the latter effect dominates via the increased use of new information, see part (ii) of Proposition 4) and for welfare (see part (ii) of Proposition 5).

Proposition 7 *(Social Value of Private and Public Information). (i) Let $\kappa_i^2 = 0$, so that signal i is perfectly private. An increase in the precision of signal i increases total profit (so long as $\pi \le 2$), increases consumer surplus, and consequently increases social welfare.*

(ii) Let $\xi_j^2 = 0$ so that signal j is perfectly public. An increase in the precision of signal j may decrease either total profits or consumer surplus, but always increases social welfare.

Part (ii) of Proposition 7 reports a similar finding for a perfectly public signal (i.e. $\rho_i = 1$ or equivalently $\xi_i^2 = 0$). If, say, a social planner could (perhaps at some cost) increase the precision $1/\kappa_i^2$ of a perfectly public announcement, the planner would have an incentive to do so. Welfare increases from the variance reduction and increase in new information use; although consumers and suppliers may disagree about such a policy's utility.

Part (i) of Proposition 7 is reminiscent of a result in Vives [49]. There, a single (perfectly private) signal is available to suppliers in an industry, and a comparison is made between the use of this private information versus the possibility of the suppliers sharing their information. As $M \to \infty$, the shared information becomes perfectly informative of demand conditions, and

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indeed (in the "common value" Cournot case, which corresponds to the framework here) information sharing is always better for consumers and welfare. Vives [49] shows that profits may be adversely affected by information sharing: in fact, for this to happen it must be that $\pi > 2$, as alluded to in Proposition 7.[7]

Of course, the focus of this paper is not perfectly private nor, for that matter, perfectly public signals. Signals with intermediate levels of publicity (where $\rho_i \in (0,1)$, or equivalently $\kappa_i^2 > 0$ and $\xi_i^2 > 0$) are the focus of the analysis. The next proposition reports the results for changes in the characteristics of such an information source.

Proposition 8 *(Social Value of Information). (i) A reduction in the receiver noise (ξ_i^2) of any signal $i \in \{1, \dots, n\}$ is always good for social welfare, and is always good for consumer surplus. Moreover, if the anti-coordination motive is not too strong ($\pi \leq \frac{4}{3}$), then a reduction in receiver noise is always good for total profits.*

(ii) A reduction in the sender noise (κ_i^2) of any signal $i \in \{1, \dots, n\}$ is always good for social welfare. If the anti-coordination motive is not too strong ($\pi \leq \frac{3}{2}$), then a reduction in sender noise is always good for total profits; it is always good for consumer surplus if

$$\pi \leq \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{c}}\right).$$

Essentially, the message is this: if the anti-coordination motive is not too strong then more information is better for all. Only when the market is very competitive (high M, low β, or low c) can more information have perverse effects.[8]

5. The incentives for information acquisition

Endogenous information acquisition So far, the characteristics of the signals have been exogenous. Sender noise, measured by the κ_i^2, is interpreted as a common error in the observation of θ by the information provider and is beyond the control of suppliers. However, ξ_i^2 is arguably endogenous: increased attention raises the precision with which the signal is observed, and so reduces receiver noise. To capture this idea, let

$$\varepsilon_{im} \sim \mathrm{N}\left(0, \frac{\xi_i^2}{z_{im}}\right),$$

where z_{im} is the (costly) attention paid by supplier m to information source i, and where an (increasing) acquisition cost $\hat{C}(z_m)$ is deducted from supplier m's profits. That is,

[7] A full discussion of this point is relegated to the supplementary Appendix B.

[8] Such perverse effects (in a perfectly-private/perfectly-public two-signal world) were documented iby Ui and Yoshizawa [46]. In an application to the Cournot setting with $\beta = c = 0$, so that $\pi = (M+1)/2$, Ui and Yoshizawa [46, Section 4.1] reported that for $M = 2$ more public information is good for profits (and for welfare). Now $M = 2 \Rightarrow \pi = \frac{3}{2}$, and part (ii) of Proposition 8 confirms. For $\pi > \frac{3}{2}$ (or $M = 3 \Rightarrow \pi = 2$) it can go either way. The Cournot application described by Ui and Yoshizawa [46] is nested in the more general informational setting here (see the supplementary Appendix B for further details). Part (i) of Proposition 7 above moreover confirms the result reported by Ui and Yoshizawa [46], that more private information is always good for welfare (and is also always good for profits, as long as $M \leq 3 \Leftrightarrow \pi \leq 2$).

$$\text{Profit}_m = \theta Q_m - \left(\beta M + (1-\beta) + \frac{cM}{2}\right) Q_m^2 - (1-\beta) \sum_{m' \neq m} Q_m Q_{m'} - \hat{C}(z_m).$$

For the welfare results presented here, the precise form taken by $\hat{C}(\cdot)$ does not need to be specified.[9] Nevertheless, it is instructive to report conditions under which a unique equilibrium exists and some of the key properties of this equilibrium.

Existence of a unique equilibrium Suppose the suppliers simultaneously choose both the weights placed on their signal realizations and also the attention paid to each information source. For this discussion, define $\bar{Q}_m \equiv w_{0m} x_0 + \sum_{i=1}^n w_{im}(\theta + \eta_i)$. This is the supplier's expected output conditional on the underlying average signal realizations. Using this notation, supplier m's expected profit is

$$\begin{aligned} \mathrm{E}[\text{Profit}_m] = {} & \mathrm{E}[\theta \bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\pi\, \mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\right) \\ & \underbrace{- \frac{1-\beta}{2} \sum_{m' \neq m} \mathrm{E}\big[\bar{Q}_{m'}^2\big]}_{\text{externality term}} + \underbrace{\frac{1-\beta}{2} \sum_{m' \neq m} \mathrm{E}\big[(\bar{Q}_m - \bar{Q}_{m'})^2\big]}_{\text{interaction term}} - \hat{C}(z_m). \end{aligned} \tag{18}$$

The externality term depends only upon others' moves and so is irrelevant to the supplier's decision. The interaction term does depend on w_m. However, local to a symmetric strategy profile w_m has no first-order effect on this interaction term. Specifically,

$$\mathrm{E}\big[(\bar{Q}_m - \bar{Q}_{m'})^2\big] = (\bar{w}_m - \bar{w}_{m'})^2 \big(x_0^2 + \kappa_0^2\big) + \sum_{i=1}^n (w_{im} - w_{im'})^2 \kappa_i^2 \quad \Rightarrow$$

$$\left.\frac{\partial\, \mathrm{E}[(\bar{Q}_m - \bar{Q}_{m'})^2]}{\partial w_{im}}\right|_{w_m = w_{m'}} = 0.$$

This means that the interaction term can also be neglected (at least to first order) when considering the optimal decision of supplier m in the context of a symmetric equilibrium. Therefore, a symmetric equilibrium $(w^\star, z^\star)$ must be a local maximizer of

$$\mathrm{E}[\theta \bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\pi\, \mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\right) - \hat{C}(z_m).$$

The various expectations which appear here are as follows:

$$\mathrm{E}[\theta \bar{Q}_m] = \bar{w}_m x_0^2 + (\bar{w}_m - w_{0m}) \kappa_0^2,$$

$$\mathrm{E}\big[\bar{Q}_m^2\big] = \bar{w}_m^2 x_0^2 + (\bar{w}_m - w_{0m})^2 \kappa_0^2 + \sum_{i=1}^n w_{im}^2 \kappa_i^2, \quad \text{and}$$

$$\mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big] = \sum_{i=1}^n \frac{w_{im}^2 \xi_i^2}{z_{im}}.$$

[9] Myatt and Wallace [36] imposed convexity on $\hat{C}(\cdot)$ and (in the context of a quadratic-loss coordination game) found a unique equilibrium with symmetric information acquisition decisions.

Substituting in these various expressions, this means that, at a symmetric equilibrium, the choice of a supplier must be a local maximizer of this objective function:

$$\bar{w}_m x_0^2 + (\bar{w}_m - w_{0m})\kappa_0^2 - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\pi\left(\bar{w}_m^2 x_0^2 + (\bar{w}_m - w_{0m})^2\kappa_0^2\right)$$
$$- \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\sum_{i=1}^{n} w_{im}^2\left(\pi\kappa_i^2 + \frac{\xi_i^2}{z_{im}}\right) - \hat{C}(z_m). \tag{19}$$

Absent any acquisition cost, Lemma 3 confirms that (19) is jointly concave in w_m and z_m. This is the point at which the maintained assumption that $\pi < 2$ is crucial: the proof provides further details and is contained in Appendix A.

Lemma 3 *(Concavity of a Supplier's Profit). Ignoring the cost of information acquisition, the expected profit of supplier m is a concave function of* (w_m, z_m). *Excluding the interaction term of (18), this expected profit is also a concave function of* (w_m, z_m).

This means that the objective function in (19) has a unique local maximizer $(w^\star, z^\star)$ if $\hat{C}(z_m)$ is convex. This unique local maximizer is the only candidate for a symmetric linear equilibrium. Moreover, given that profit (once the interaction term is included) is globally (and jointly) concave in w_m and z_m, this candidate forms an equilibrium. This discussion is concerned with the existence and uniqueness of a symmetric equilibrium. However, the proof of Proposition 9 confirms that any equilibrium must be symmetric if $3\beta + c > 1$, which is equivalent to requiring $\pi < 2$ when the number of suppliers is large.

Proposition 9 *(Equilibrium with Endogenous Information Acquisition). (i) If the information acquisition cost function* $\hat{C}(z_m)$ *is convex then there is a unique symmetric equilibrium in which supplier m devotes attention* $z_i^\star$ *to information source i and produces output* $Q_m = \sum_{i=0}^{n} w_i^\star x_{im}$, *where* $w_i^\star \propto 1/(\pi\kappa_i^2 + \xi_i^2/z_i^\star)$. *If* $3\beta + c > 1$ *(which must hold if* $\pi < 2$ *for large* M*) then there are no asymmetric equilibria.*

(ii) In equilibrium, an information source has influence if and only if it receives attention: $w_i^\star > 0 \Leftrightarrow z_i^\star > 0$. *Amongst those that have influence, attention satisfies*

$$z_i^\star \propto \frac{\xi_i(K_i - \xi_i)}{\pi\kappa_i^2} \quad \textit{where } \frac{1}{K_i^2} = \frac{\partial\hat{C}(z^\star)}{\partial z_{im}},$$

while for those signals that are ignored $\xi_i \geq K_i$.

A case of interest arises when information acquisition cost is an increasing (and convex) function of the sum $\sum_{i=1}^{n} z_{im}$.[10] Here z_{im} may be interpreted as the time devoted to observing the ith information source, which naturally might correspond to the sample size (and hence precision) obtained from that source. In this context, the parameter ξ_i measures the difficulty of observing information source i. Hence, if $\xi_i < \xi_j$ then i is "clearer" than j. With additive attention costs, K_i is the same for all i and so the signals that are ignored are those which

[10] This is precisely the cost technology assumed in much of the literature (Li, McKelvey and Page [32], Vives [48], Hwang [27,28], Hauk and Hurkens [24]), albeit with just one signal.

satisfy $\xi_i > K$ for some K; that is, the least clear information sources. Moreover, the clearest information sources are also (endogenously) the most highly correlated, and so the most public.[11]

Corollary *(To Proposition 9). (i) If the cost of information acquisition is a function of $\sum_{i=1}^{n} z_{im}$, then the information sources that receive strictly positive attention are the clearest: that is, $z_i^\star > 0$ implies that $z_j^\star > 0$ if $\xi_j \leq \xi_i$. Moreover, the clearest signals are also the most highly correlated: $\xi_j \leq \xi_i$ implies that $\rho_j \geq \rho_i$.*

(ii) If all information sources are equally clear, so that $\xi_i = \xi$ for all i, then the attention paid to an information source is proportional to its underlying precision: $z_i^\star \propto 1/\kappa_i^2$. Moreover, the cross-industry correlation coefficients of such signals are identical: $\rho_i = \rho$.

(iii) If the conditions of (i) and (ii) are met, then a local shift in influence between two signals i and j has no first-order effect on either industry profit or consumer surplus, and so (conditional on their overall use) signals are used efficiently.

The claims of (iii) hold because the (endogenously acquired) informative signals are equally public. The inefficiencies documented in Section 4 came about because of the differences in relative publicity across the set of signals.

Externalities in information acquisition The analysis which follows does not require uniqueness: it focuses on the implications of local shifts away from an equilibrium.

Given that, in the (symmetric) equilibrium, weights w_{im} and precisions z_{im}/ξ_i^2 are chosen optimally, a small change in the value of ξ_i^2/z_{im} has no first-order effect on the profit of supplier m. Nor does it have any impact upon any other supplier m'. Consider profits as written in (12). The impact of the decisions of m are felt only through the final covariance term, which, from (11), does not contain any ξ_i^2 terms. Thus, fixing weights w_{im}, other suppliers are unaffected by changes in ξ_i^2/z_{im} local to equilibrium.

Consumers, however, are affected. Consider the expression for consumer surplus in (15). The variance term, as can be seen from (11), incorporates ξ_i^2 directly. An increase in ξ_i^2/z_{im}, holding everything else constant, improves consumer surplus. As a result, reducing attention local to an equilibrium increases welfare. That is, suppliers are obtaining too much costly information relative to the social optimum. They do this to reduce the noise associated with the information they receive, whereas, as discussed previously, consumers are in favour of more variance. The next proposition summarizes.

Proposition 10 *(Efficiency and Information Acquisition). (i) Fixing the weights on the signals, suppliers have an incentive to acquire too much costly information from the perspective of consumer surplus and therefore from the perspective of social welfare.*

(ii) In equilibrium, the amount of costly information acquired from source i relative to source j is nevertheless socially optimal.

Part (ii) of the proposition says that suppliers acquire the right mix of information. To see this, consider the "marginal rate of substitution" between (the inverse of) z_{im} and z_{jm}. Recalling

[11] Many of these observations (or closely related results) were reported by Myatt and Wallace [36], although part (iii) of the corollary (which is specific to the model considered here) is new.

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expected profits from (12), noting the definitions in (11), and fixing $w_{im} = w_i$,

$$\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m \mid \theta]}{\partial[1/z_{im}]} = -\left(\beta M + (1-\beta) + \frac{cM}{2}\right) w_i^2 \xi_i^2, \quad \text{so} \quad \mathrm{MRS}_{ij}^m = \frac{w_i^2 \xi_i^2}{w_j^2 \xi_j^2}.$$

Now consider the consumers "marginal rate of substitution" between the two sources. Once again returning to (15), using (11), and fixing $w_{im} = w_i$,

$$\frac{\partial\, \mathrm{E}[\text{Consumer Surplus} \mid \theta]}{\partial[1/z_{im}]} = -\frac{(\beta M + (1-\beta)) w_i^2 \xi_i^2}{2}, \quad \text{so} \quad \mathrm{MRS}_{ij}^{\mathrm{CS}} = \frac{w_i^2 \xi_i^2}{w_j^2 \xi_j^2}.$$

Clearly then $\mathrm{MRS}_{ij}^{\mathrm{CS}} = \mathrm{MRS}_{ij}^m$. The consumers and the suppliers agree about the relative amount of information to acquire; they differ only in their preferences about how much in total to obtain. This observation justifies a key message of the paper.

Corollary *(To Propositions 5 and 10). From a welfare perspective, the Cournot suppliers acquire too much new information, but use it too little.*

The results presented above complement the findings of Colombo, Femminis and Pavan [15] and Pavan [40]. These papers also document the divergence of the (efficient) use of information from its acquisition. Relative to the former of these papers, the informational environment here is richer; it is identical to that of the latter. However, the two key innovations here are that, in a Cournot setting, (i) it is natural to consider social welfare and consumer surplus as well as total profit (which is the measure of welfare upon which these two papers focus) and (ii) the case of interest is precisely one to which many of the propositions in those papers do not immediately apply.[12]

Information manipulation and welfare The previous discussion has identified the efficiency properties of the use and acquisition of information. Putting aside the balance between new information and the prior for a moment, suppliers use the new information they receive inefficiently (Section 4, and particularly Proposition 5). Unless $\rho_i = \rho_j$ for all $i \neq j$, social welfare may be improved by shifting weight from some $i \neq 0$ to some other $j \neq 0$. On the other hand (Section 5, Proposition 10) suppliers acquire new information in a socially efficient way: the quantity of costly attention devoted to source i relative to j is optimal, even though the total amount of new information acquired may not be.

This leads to a natural question: how could a social planner manipulate information acquisition to improve welfare? Fixing the total amount of information acquired and the influence (weight) attached to each source in equilibrium the acquisition decisions are efficient. The weights themselves are part of the suppliers' strategies, and as such it would be unnatural to think of them as directly observable or verifiable. Nonetheless a social planner may wish to distort acquisition decisions (which are actual actions taken by the suppliers) since changing $z_i^\star$ changes $w_i^\star$: if more information is acquired from source i then more weight is attached to i in

12 In particular, and in the notation of Colombo, Femminis and Pavan [15] or Pavan [40] (which follows that of Angeletos and Pavan [6]), $\kappa_1 > \kappa_1^*$ and $\alpha > \alpha^*$ in the Cournot context. Many of results reported in those papers do not consider this particular parameter constellation. Further details can be found in the supplementary Appendix B, which maps the notation of these papers to this one.

equilibrium. Thus there may be a "second best" argument for manipulation of the information acquisition decision process.

Taking this further, suppose that the planner can distort acquisition decisions by taxing or subsidizing information sources. Suppose further that i is less public than j, so that $\rho_i < \rho_j$. A (small) tax on source j along with a (small) subsidy on source i can be implemented to shift z_j down and z_i up by the same amount. Locally there is no first-order effect on welfare via the acquisition decisions directly (these were socially optimal to begin with). However, the weights w_i and w_j were being chosen away from the social optimum, and so there is a first-order effect on welfare. Indeed, weight will be shifted from the more public j to the less public i: this is welfare enhancing by Proposition 5.

A planner may wish to intervene not to change acquisition decisions directly, but rather to distort the influence these signals have in the quantity setting process. The planner would do so by taxing relatively public signals whilst subsidizing relatively private ones. In so doing, the planner alters the publicity properties of the signals. In particular since

$$\rho_i = \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2/z_i}$$

is increasing in z_i, subsidizing a relatively private signal i and taxing a relatively public signal j drives ρ_i closer to ρ_j. The information sources become more "averagely public".

From the corollary to Proposition 9, when the cost of acquisition is a function of $\sum_{i=1}^{n} z_i$, the most public signals are also the clearest ($\xi_i \geq \xi_j$ if and only if $\rho_i \leq \rho_j$). Therefore, the planner's desire to equalize the publicity of the various signals may be seen as a desire to correct distortions arising as a result of the disparate clarities associated with the information sources. As a consequence, insofar as affecting the relative influence of new information goes, there is no role for a planner when $\xi_i = \xi_j$ for all $i, j \neq 0$ in this case.

Of course, there may still be a role for a social planner even when all the information sources have equal clarity. As the corollary to Propositions 5 and 10 states, suppliers acquire too much new information, but use it too little. Relative to the prior there is inefficiency in both information acquisition and information use even when $\xi_i = \xi_j$ for all $i, j \neq 0$. However, the inefficiency is not all in one direction: any tax on new information overall (for instance) will reduce its acquisition (good) but also reduce its use (bad). In general, the optimal policy will depend upon the particular circumstances at hand.

6. Related literature

Three strands of related literature are discussed here: the use of public and private information in a class of quadratic-payoff coordination games; information sharing in oligopoly models; and, finally, the information aggregation properties of large oligopolies.

Public and private information Starting with Morris and Shin [35] a large literature has considered information use in quadratic-payoff coordination games. Initially, this literature sharply distinguished between public and private information. Each player sees two signals of an unknown parameter (θ): one is perfectly private (conditionally uncorrelated with others' observations), and the other is perfectly public (common knowledge to all the players). Using this information, players minimize a weighted sum of the expected distance of their action from θ and from the average action. One message is that players place greater emphasis on public information, because a public signal plays a greater role in higher-order expectations; a second message is that the use of public information may sometimes be socially excessive, and so welfare may sometimes be improved by destroying public information.[13]

This framework has been applied extensively: Angeletos and Pavan [5] studied an investment game; Angeletos and Pavan [6] developed applications to business cycles and large oligopoly games (retaining the public-private distinction); and Hellwig [25] applied a similar structure to a model with monopolistically competitive firms. There have also been applications to political leadership (Dewan and Myatt [18,19]), financial markets (Allen, Morris and Shin [1]), and much more.[14]

This paper joins several others that have admitted multiple information sources.[15] The information structure used here follows the one introduced in Dewan and Myatt [18], and developed in a Lucas-Phelps island setting in Myatt and Wallace [37]. Partial correlation of observations have also been incorporated in the work of Angeletos and Pavan [7], Baeriswyl [8], and Baeriswyl and Cornand [9,10].

Endogenous information acquisition, of the sort considered in Section 5, was considered in a political science setting by Dewan and Myatt [18,19], in a simple "beauty contest" setting by Myatt and Wallace [36], and in a different but related model by Hellwig and Veldkamp [26]. In other recent work, and in the same spirit, the models of Colombo, Femminis and Pavan [15], Llosa and Venkateswaran [33], and Pavan [40] allow players to choose the precision of a private signal.

The key contribution relative to this first strand of literature is to highlight, in the specific context of a micro-founded Cournot oligopoly setting, the (in)efficiency of information use and acquisition in a framework which admits a general correlation structure.

Information sharing in oligopolies An extensive literature has examined the incentives of oligopolists to share information. The canonical model is a Cournot industry with linear demand, where suppliers receive a single private signal about a common demand shock. Shared information aligns outputs with demand conditions, but also induces correlation of output choices. This latter effect is harmful in a quantity-setting (strategic substitutes) environment. The latter (negative) effect outweighs the (positive) former and so shared information generally reduces industry profits (Novshek and Sonnenschein [38], Clarke [14], Vives [47,49], Li [31], Gal-Or [22]).

Others have focused on information sharing about private cost (or equivalently, supplier-specific demand) conditions. Contrary to the conclusion above, in a Cournot oligopoly, suppliers benefit from sharing such information by reducing the positive correlation between their output choices (Fried [21], Li [31], Gal-Or [23], Shapiro [44]).[16]

An insightful unification of this literature (along with the extensions and variants contained in Sakai [42], Kirby [29], Sakai and Yamato [43]) was provided by Raith [41]. The deciding factor

[13] Outside the simple framework of a static quadratic-payoff beauty-contest coordination game, others have evaluated the effect of public information in various settings. For example, Amador and Weill [2] found, using a micro-founded macroeconomic model, that public information release can increase uncertainty about a monetary shock. Amador and Weill [3] considered a dynamic model in which players wish to coordinate with the state of the world and learn from the ongoing actions of others. They showed that public information can harm welfare when a significant private learning channel is present.

[14] On a different but related tack, other authors have used the coordination game structure to study endogenous communication on networks (Calvó-Armengol and de Martí Beltran [11,12], Calvó-Armengol, de Martí Beltran and Prat [13]). Recent work in this dimension, including an application to a Cournot model, may be found in Currarini and Feri [17].

[15] Indeed, a step in this direction was taken in the supplementary material to Morris and Shin [35].

[16] Many of these conclusions are reversed in a price-setting environment (Vives [47], Gal-Or [23]).

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is whether the information the Cournot oligopolist shares perfectly reveals supplier-specific information or not. The negative impact of information sharing arises from any positive correlation between outputs that it induces: this can happen if supplier m learns from $\hat{m}$ something about m's own demand (or cost) conditions; whereas if m learns from $\hat{m}$ something about $\hat{m}$'s own demand (or cost) conditions only, then this instead induces (beneficial) negative correlation.

A feature of these models is that there is only a single (perfectly private) signal. Here, there are many ($n > 1$) and the information is endogenously acquired.[17] How this more general structure would affect the conclusions of this literature is an open question.[18]

Information aggregation in oligopolies Building upon work by Palfrey [39], Vives [48] investigated the efficiency properties of an industry with dispersed private information about demand conditions; the paper by Li, McKelvey and Page [32] reported a closely related model. The specification of Vives [48] is a homogeneous-product oligopoly with linear demand where each supplier acquires a single, perfectly private, signal about demand conditions; this corresponds to $\beta = 0$ (no differentiation), $n = 1$ (one signal), and $\kappa_1^2 = 0$ (no sender noise, and so $\rho_1 = 0$). The focus was on the efficiency or otherwise of information acquisition and use: in particular, in the competitive limit (as $M \to \infty$, where the economy is replicated) information is both acquired and used efficiently. The relative contribution here is threefold: first, results are obtained away from the competitive limit for a differentiated-product oligopoly; second, suppliers may have access to $n > 1$ generally correlated signals, rather than a single (perfectly private) one; and finally, this more general structure throws light on the distinct (in)efficiency properties of the equilibrium use of information versus its acquisition.

This literature developed in several directions. For example, Hwang [27] studied a duopoly with asymmetric firms; Hwang [28] compared oligopolistic to competitive behaviour for fixed M; Hauk and Hurkens [24] specified secret information acquisition (that is, suppliers do not observe the precisions chosen by their competitors); and Vives [50] compared the efficiency losses from informational incompleteness versus the usual losses associated with market power. Further work by Vives [52,53] has also considered associated market games in which competitors submit supply schedules to an equilibrating market maker. These and other contributions are helpfully unified in a textbook treatment (Vives [51]). Naturally, an open question (and the subject of ongoing research by the authors of this paper) is to investigate the properties of a multiple-information-source model in the context of such a market game.

Appendix A. Proofs of propositions

Proof of Proposition 1. The expected profit of supplier m is

$$\mathrm{E}[\mathrm{Profit}_m] = \mathrm{E}[\theta Q_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\mathrm{E}\big[Q_m^2\big] - (1-\beta)\sum_{m' \neq m} \mathrm{E}[Q_m Q_{m'}].$$

If strategies are linear, so that $Q_m = \sum_{i=0}^n w_{im} x_{im} = \sum_{i=0}^n w_{im}(\theta + \eta_i + \varepsilon_{im})$, then $\mathrm{E}[\mathrm{Profit}_m]$ is quadratic in w_m and $w_{m'}$. It is concave in w_m, and so m's best reply is characterized by $n + 1$

[17] Others have considered the interplay between information sharing and acquisition in specific settings. For example, Creane [16] did so in a learning-by-doing environment.

[18] The insight of Raith [41] was that a shared signal hardwires together information about demand conditions (yielding positively correlated outputs) and about others' actions (yielding negative correlation). Multiple signals with different precisions and publicities may well link these effects in different ways.

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linear (in w_m and $w_{m'}$) first-order conditions. (The concavity in w_m is confirmed in the proof of Lemma 3.) The full set of $M(n+1)$ such conditions has full rank, and solves to yield a unique linear equilibrium. To characterize it, define $\bar{Q}_m \equiv \sum_{i=0}^{n} w_{im}(\theta+\eta_i)$. The errors are uncorrelated and so $\mathrm{E}[Q_m^2] = \mathrm{E}[\bar{Q}_m^2] + \mathrm{E}[(Q_m - \bar{Q}_m)^2]$, $\mathrm{E}[Q_m Q_{m'}] = \mathrm{E}[\bar{Q}_m \bar{Q}_{m'}]$, and $\mathrm{E}[\theta Q_m] = \mathrm{E}[\theta \bar{Q}_m]$. Moreover:

$$\mathrm{E}[\bar{Q}_m \bar{Q}_{m'}] = \frac{\mathrm{E}[\bar{Q}_m^2] + \mathrm{E}[\bar{Q}_{m'}^2] - \mathrm{E}[(\bar{Q}_m - \bar{Q}_{m'})^2]}{2}.$$

Using this and the other expressions derived above, the expected profit of supplier m is

$$\begin{aligned}\mathrm{E}[\mathrm{Profit}_m] = {} & \mathrm{E}[\theta \bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\pi\, \mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\right) \\ & - \underbrace{\frac{(1-\beta)\sum_{m'\neq m} \mathrm{E}[\bar{Q}_{m'}^2]}{2}}_{\text{externality term}} + \underbrace{\frac{(1-\beta)\sum_{m'\neq m} \mathrm{E}[(\bar{Q}_m - \bar{Q}_{m'})^2]}{2}}_{\text{interaction term}} \\ & \text{where } \pi = \frac{2\beta M + (1-\beta)(M+1) + cM}{2\beta M + 2(1-\beta) + cM} = \frac{\beta M + \frac{(1-\beta)(M+1)}{2} + \frac{cM}{2}}{\beta M + (1-\beta) + \frac{cM}{2}}. \end{aligned} \tag{20}$$

From the perspective of supplier m, the externality term is exogenous. The dependence of the interaction term on w_m is second order local to a symmetric strategy profile:

$$\frac{\partial}{\partial w_{im}} \mathrm{E}\left[\frac{(\bar{Q}_m - \bar{Q}_{m'})^2}{2}\right]\Bigg|_{w_m = w_{m'}} = \mathrm{E}\big[(\bar{Q}_m - \bar{Q}_{m'})(\theta + \eta_i)\big] = 0,$$

where the final equality holds because $\bar{Q}_m = \bar{Q}_{m'}$ if $w_m = w_{m'}$. Hence the externality and interaction terms in (20) do not matter (for any first order condition with respect to w_m) local to a symmetric profile. Using the notation $\bar{w}_m \equiv \sum_{i=0}^{n} w_{im}$,

$$\begin{aligned} & \mathrm{E}[\theta \bar{Q}_m] = w_{0m} x_0^2 + (\bar{w}_m - w_{0m})\big(x_0^2 + \kappa_0^2\big), \\ & \mathrm{E}\big[\bar{Q}_m^2\big] = \bar{w}_m^2 x_0^2 + \kappa_0^2 (\bar{w}_m - w_{0m})^2 + \sum\nolimits_{i=1}^{n} w_{im}^2 \kappa_i^2, \quad \text{and} \\ & \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big] = \sum\nolimits_{i=1}^{n} w_{im}^2 \xi_i^2, \quad \Rightarrow \\ & \mathrm{E}[\mathrm{Profit}_m] = \text{other terms} + w_{0m} x_0^2 + (\bar{w}_m - w_{0m})\big(x_0^2 + \kappa_0^2\big) \\ & \qquad - \left(\beta M + (1-\beta) + \frac{cM}{2}\right) \\ & \qquad \times \left(\pi \bar{w}_m^2 x_0^2 + \pi (\bar{w}_m - w_{0m})^2 \kappa_0^2 + \sum\nolimits_{i=1}^{n} w_{im}^2 \big(\pi \kappa_i^2 + \xi_i^2\big)\right). \end{aligned}$$

Differentiating with respect to w_{0m}:

$$\begin{aligned} & \frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{0m}}\bigg|_{w_m = w_{m'}\ \forall m' \neq m} = x_0^2\big(1 - \big(2\beta M + 2(1-\beta) + cM\big)\pi \bar{w}_m\big) = 0 \\ & \Leftrightarrow \quad \bar{w}_m = \frac{1}{\pi(2\beta M + 2(1-\beta) + cM)} = \frac{1}{2\beta M + (1-\beta)(M+1) + cM}. \end{aligned}$$

This determines the total $\bar{w}_m^\star$ of the equilibrium weights attached to the prior and to the signals, and so in turn determines the expected output of each supplier. That is,

$$\mathrm{E}[Q_m] = \bar{w}_m^\star \,\mathrm{E}[\theta] = \frac{x_0}{2\beta M + (1-\beta)(M+1) + cM} = \frac{\mathrm{E}[Q^\star]}{M},$$

where $Q^\star$ is the full-information equilibrium industry output. For $i \in \{1, \dots, n\}$,

$$\begin{aligned}
&\left.\frac{\partial\,\mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im}}\right|_{w_m = w_{m'}\ \forall m' \neq m} \\
&\quad = x_0^2 + \kappa_0^2 - \big(2\beta M + 2(1-\beta) + cM\big) \\
&\qquad \times \big(\pi \bar{w}_m \big(x_0^2 + \kappa_0^2\big) + w_{im}\big(\pi\kappa_i^2 + \xi_i^2\big) - w_{0m}\big(\pi\kappa_0^2 + \xi_0^2\big)\big),
\end{aligned}$$

where $\xi_0^2 = 0$ so that the weight on the prior mimics the weight on a signal. Incorporating $\bar{w}_m$,

$$\begin{aligned}
&\left.\frac{\partial\,\mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im}}\right|_{w_m = w_{m'}\ \forall m' \neq m} \\
&\quad = \big(2\beta M + 2(1-\beta) + cM\big)\big(w_{0m}\big(\pi\kappa_0^2 + \xi_0^2\big) - w_{im}\big(\pi\kappa_i^2 + \xi_i^2\big)\big) = 0,
\end{aligned}$$

which implies $w_{im}^\star = w_i^\star \propto 1/(\pi\kappa_i^2 + \xi_i^2)$, as required. The final claims hold by inspection. □

Proof of Lemma 1. The proof follows from a direct inspection of π in (6). □

Proof of Proposition 2. The first observation of the proposition follows immediately from an inspection of the expression for $\bar{w}^\star$ in (7). The weight placed on signal i relative to j is

$$\frac{w_i^\star}{w_j^\star} = \frac{\pi\kappa_j^2 + \xi_j^2}{\pi\kappa_i^2 + \xi_i^2}, \quad \text{so} \quad \frac{\partial[w_i^\star/w_j^\star]}{\partial\pi} = \frac{\partial}{\partial\pi}\left[\frac{\pi\kappa_j^2 + \xi_j^2}{\pi\kappa_i^2 + \xi_i^2}\right] = \frac{\kappa_j^2\xi_i^2 - \kappa_i^2\xi_j^2}{(\pi\kappa_i^2 + \xi_i^2)^2}. \tag{21}$$

So once again, $\partial[w_i^\star/w_j^\star]/\partial\pi > 0$ if and only if $\kappa_j^2\xi_i^2 > \kappa_i^2\xi_j^2$, or $\rho_j > \rho_i$. Using this fact in conjunction with Lemma 1 establishes the four equivalences in (13). The observation concerning expected industry output follows straightforward differentiation of the expression in (8).

The final part of the proposition concerning $w_i^\star$ is more involved. First, it is useful to record the relevant derivatives of π with respect to M, β, and c. They are

$$\begin{aligned}
&\frac{\partial\pi}{\partial M} = (1-\beta)(2+c)\big[\bar{w}^\star\pi\big]^2, \qquad \frac{\partial\pi}{\partial\beta} = -M(M-1)(2+c)\big[\bar{w}^\star\pi\big]^2, \quad \text{and} \\
&\frac{\partial\pi}{\partial c} = -M\bar{w}^\star\pi(\pi-1).
\end{aligned} \tag{22}$$

Second, it is useful to record the associated derivatives of $\bar{w}^\star$, which are respectively

$$\begin{aligned}
&\frac{\partial\bar{w}^\star}{\partial M} = -(1+\beta+c)\big[\bar{w}^\star\big]^2, \qquad \frac{\partial\bar{w}^\star}{\partial\beta} = -(M-1)\big[\bar{w}^\star\big]^2, \quad \text{and} \\
&\frac{\partial\bar{w}^\star}{\partial c} = -M\big[\bar{w}^\star\big]^2.
\end{aligned} \tag{23}$$

Consider first the derivative of $w_i^\star$ with respect to M. Note that

$$w_i^\star = \bar{w}^\star\left[\sum_{j=0}^{n} \frac{\pi\kappa_i^2 + \xi_i^2}{\pi\kappa_j^2 + \xi_j^2}\right]^{-1}, \tag{24}$$

and so

$$\begin{aligned}\frac{\partial w_i^\star}{\partial M} &= \frac{\partial \bar{w}^\star}{\partial M}\left[\sum_{j=0}^{n}\frac{\pi\kappa_i^2+\xi_i^2}{\pi\kappa_j^2+\xi_j^2}\right]^{-1} - \bar{w}^\star\left[\sum_{j=0}^{n}\frac{\pi\kappa_i^2+\xi_i^2}{\pi\kappa_j^2+\xi_j^2}\right]^{-2}\sum_{j=0}^{n}\frac{\partial}{\partial\pi}\left[\frac{\pi\kappa_i^2+\xi_i^2}{\pi\kappa_j^2+\xi_j^2}\right]\frac{\partial\pi}{\partial M}\\ &= \left[\sum_{j=0}^{n}\frac{\pi\kappa_i^2+\xi_i^2}{\pi\kappa_j^2+\xi_j^2}\right]^{-1}\left\{\frac{\partial\bar{w}^\star}{\partial M} - w_i^\star\frac{\partial\pi}{\partial M}\sum_{j=0}^{n}\frac{\kappa_i^2\xi_j-\kappa_j^2\xi_i^2}{(\pi\kappa_j^2+\xi_j^2)^2}\right\}\\ &= -w_i^\star\bar{w}^\star\left\{(1+\beta+c)+(1-\beta)(2+c)\pi\underbrace{\left(w_i^\star\pi\sum_{j=0}^{n}\frac{\kappa_i^2\xi_j-\kappa_j^2\xi_i^2}{(\pi\kappa_j^2+\xi_j^2)^2}\right)}_{A}\right\},\end{aligned}$$

where the second equality follows Eqs. (21) and (24), and the third equality from (24) and then substitution of the differentials from Eqs. (22) and (23). Consider the final term of the left-hand side of the bottom line (labeled A). Using the definitions for $\hat{\psi}_i$ and $\hat{\rho}_i$ given in the text of the proposition, this expression can be reformulated as follows

$$\begin{aligned}A &= w_i^\star\left\{\pi\kappa_i^2\sum_{j=0}^{n}\frac{\xi_j^2}{(\pi\kappa_j^2+\xi_j^2)^2} - \xi_i^2\sum_{j=0}^{n}\frac{\pi\kappa_j^2}{(\pi\kappa_j^2+\xi_j^2)^2}\right\}\\ &= w_i^\star\left\{\pi\kappa_i^2\sum_{j=0}^{n}\hat{\psi}_j(1-\hat{\rho}_j) - \xi_i^2\sum_{j=0}^{n}\hat{\psi}_j\hat{\rho}_j\right\}\\ &= \bar{w}^\star\left[\sum_{j=0}^{n}\hat{\psi}_j\right]^{-1}\left\{\frac{\pi\kappa_i^2}{\pi\kappa_i^2+\xi_i^2}\sum_{j=0}^{n}\hat{\psi}_j(1-\hat{\rho}_j) - \frac{\xi_i^2}{\pi\kappa_i^2+\xi_i^2}\sum_{j=0}^{n}\hat{\psi}_j\hat{\rho}_j\right\}\\ &= \bar{w}^\star\left[\sum_{j=0}^{n}\hat{\psi}_j\right]^{-1}\left\{\hat{\rho}_i\sum_{j=0}^{n}\hat{\psi}_j(1-\hat{\rho}_j) - (1-\hat{\rho}_i)\sum_{j=0}^{n}\hat{\psi}_j\hat{\rho}_j\right\}\\ &= \bar{w}^\star\left[\sum_{j=0}^{n}\hat{\psi}_j\right]^{-1}\left\{\hat{\rho}_i\sum_{j=0}^{n}\hat{\psi}_j - \sum_{j=0}^{n}\hat{\psi}_j\hat{\rho}_j\right\} = \bar{w}^\star(\hat{\rho}_i-\hat{\rho}),\end{aligned}$$

where $\hat{\rho}$ is also defined in the proposition. Substituting back into the derivative found above,

$$\frac{\partial w_i^\star}{\partial M} = -w_i^\star\bar{w}^\star\big\{(1+\beta+c)+(1-\beta)(2+c)\bar{w}^\star\pi(\hat{\rho}_i-\hat{\rho})\big\}.$$

Now $(1+\beta+c) > (1-\beta)$, $(\hat{\rho}_i-\hat{\rho}) \le 1$, and $\bar{w}^\star\pi \le (2+c)^{-1}$ since

$$\bar{w}^\star\pi = \frac{1}{2\beta M+2(1-\beta)+cM} \le \frac{1}{2+c}$$

(this value is attained only if $M=1$). Therefore $\partial w_i^\star/\partial M < 0$ as required. Using exactly the same method to evaluate the differential of $w_i^\star$ with respect to c,

$$\frac{\partial w_i^\star}{\partial c} = -w_i^\star\bar{w}^\star M\big\{1-(\pi-1)(\hat{\rho}_i-\hat{\rho})\big\}.$$

Recall that $\pi\in[1,2)$ and again $\hat{\rho}_i-\hat{\rho}\le 1$. Once again $\partial w_i^\star/\partial c<0$ as required. For the differential with respect to β, the very same method is used one last time. Now

$$\frac{\partial w_i^\star}{\partial \beta} = -w_i^\star \bar{w}^\star (M-1)\{1 - M(2+c)\bar{w}^\star \pi(\hat{\rho}_i - \hat{\rho})\}.$$

Now from the definitions of π and $\bar{w}^\star$,

$$M(2+c)\bar{w}^\star \pi = \frac{M(2+c)}{2\beta M + 2(1-\beta) + cM} \equiv \pi^\dagger,$$

from (14). Thus $\partial w_i^\star / \partial \beta > 0$ if and only if $\pi^\dagger(\hat{\rho}_i - \hat{\rho}) > 1$ as required. □

Proof of Proposition 3. The expected profit of supplier m is reported in (20). The first three terms depend only on w_m. A change in $w_{m'}$ has no first-order effect on the final interaction term when evaluated at a symmetric strategy profile. Hence, any externality imposed by supplier m' on supplier m operates via the externality term. Specifically,

$$\left.\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im'}}\right|_{w_{m'}=w_m} = -\frac{1-\beta}{2}\frac{\partial\, \mathrm{E}[\bar{Q}_{m'}^2]}{\partial w_{im'}}.$$

Given that $\mathrm{E}[\bar{Q}_{m'}^2] = \bar{w}_{m'}^2 x_0^2 + \kappa_0^2(\bar{w}_{m'} - w_{0m'})^2 + \sum_{i=1}^n w_{im'}^2 \kappa_i^2$,

$$\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im'}} = -(1-\beta)\left[\bar{w}_m\left(x_0^2 + \kappa_0^2\right) - w_{0m}\kappa_0^2 + w_{im}\kappa_i^2\right] \quad \text{for } i \neq 0, \quad \text{and}$$

$$\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{0m'}} = -(1-\beta)\bar{w}_m x_0^2.$$

Now consider supplier m' shifting weight from signal j to signal i. Thus m's profits change by

$$\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im'}} - \frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{jm'}} = (1-\beta)\left[w_{jm}\kappa_j^2 - w_{im}\kappa_i^2\right],$$

which is positive if and only if $w_{jm}\kappa_j^2 > w_{im}\kappa_i^2$. Employing the equilibrium weights in (7),

$$w_j^\star \kappa_j^2 > w_i^\star \kappa_i^2 \quad \Leftrightarrow \quad \frac{\kappa_j^2}{\pi\kappa_j^2 + \xi_j^2} > \frac{\kappa_i^2}{\pi\kappa_i^2 + \xi_i^2} \quad \Leftrightarrow \quad \frac{\kappa_j^2}{\kappa_j^2 + \xi_j^2} > \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2},$$

which is simply $\rho_j > \rho_i$. Thus the externality imposed by m' on m when moving weight from signal $j \neq 0$ to $i \neq 0$ is positive if and only if j is more public than i. As this is true for all m, and the first-order effect of such a shift on m' is small (the weights are being moved from their equilibrium values), industry profits are necessarily increased by such an exercise.

Now consider a shift in weight from the prior to signal i:

$$\frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{im'}} - \frac{\partial\, \mathrm{E}[\mathrm{Profit}_m]}{\partial w_{0m'}} = -(1-\beta)\left[(\bar{w}_m - w_{0m})\kappa_0^2 + w_{im}\kappa_i^2\right]. \tag{25}$$

$\bar{w}^\star - w_0^\star = \sum_{i=1}^n w_i^\star > 0$ and so this is negative: it is optimal to move weight toward the prior. At equilibrium, the expression in the square brackets in (25) can be written

$$\underbrace{\bar{w}^\star \kappa_0^2}_{\text{variance}} - \underbrace{\left(w_0^\star \kappa_0^2 - w_i^\star \kappa_i^2\right)}_{\text{public vs. private}}.$$

Moving weight towards the prior has two effects. First there is the effect that, from the perspective of another supplier, such a shift in weight results in a reduction in variance: this is good for the

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supplier. The first term measures this effect (the noise to which the supplier is exposed is reduced by κ_0^2, the variance on the prior, scaled by the total weight placed on all information $\bar{w}^\star$).

The term labeled "public vs. private" captures the negative (second) effect of moving weight toward the prior owing to the fact that the prior is necessarily a purely public signal: $w_0^\star\kappa_0^2 > w_i^\star\kappa_i^2$ since $\rho_0 = 1 > \rho_i$ for all $i \neq 0$. The cost of moving from relatively private to relatively public signals (argued above) is also present when shifting weight from relatively private signals to the prior, but, as (25) shows, this is always more than compensated for by the first effect.

The claims in the third part of the proposition, and in particular, the results listed in (14) require consideration of the industry profit. At a symmetric profile, the interaction term in supplier m's profit in (20) is identically zero. Moreover, $\mathrm{E}[\bar{Q}_{m'}^2] = \mathrm{E}[\bar{Q}_m^2]$ for all m'. Hence,

$$\begin{aligned}\mathrm{E}[\text{Profit}_m] = \mathrm{E}[\theta\bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\pi\,\mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\right)\\ - \frac{(1-\beta)(M-1)\,\mathrm{E}[\bar{Q}_m^2]}{2}.\end{aligned} \tag{26}$$

Noting the definition of π used earlier,

$$\pi\left(\beta M + (1-\beta) + \frac{cM}{2}\right) + \frac{(1-\beta)(M-1)}{2} = M\left(1 + \frac{c}{2}\right).$$

Hence, using the definition of $\pi^\dagger$ in the proposition,

$$\mathrm{E}[\text{Profit}_m] = \mathrm{E}[\theta\bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\left(\pi^\dagger\,\mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\right).$$

The role of π is taken by $\pi^\dagger$ when considering the effect of changes in w on each supplier's profit. In fact, after appropriate substitution and algebraic simplification, total industry profit is

$$\begin{aligned}\mathrm{E}[\text{Profit}] = M \times \Bigg\{\bar{w}\big(x_0^2 + \kappa_0^2\big) - w_0\kappa_0^2 - M\left[1 + \frac{c}{2}\right]\\ \times\left(\bar{w}^2\big(x_0^2+\kappa_0^2\big) - 2\bar{w}w_0\kappa_0^2 + \sum_{i=0}^n w_i^2\kappa_i^2\right)\\ - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\sum_{i=0}^n w_i^2\xi_i^2\Bigg\}.\end{aligned}$$

Differentiating firstly with respect to w_0 and then with respect to w_i, $i \neq 0$,

$$\begin{aligned}\frac{\partial\,\mathrm{E}[\text{Profit}]}{\partial w_0} &= M\big\{x_0^2 - M(2+c)\bar{w}x_0^2\big\}, \quad \text{and}\\ \frac{\partial\,\mathrm{E}[\text{Profit}]}{\partial w_i} &= M\big\{x_0^2 + \kappa_0^2 - M(2+c)\big[\bar{w}\big(x_0^2+\kappa_0^2\big) - w_0\kappa_0^2 + w_i\kappa_i^2\big]\\ &\quad - \big(2\beta M + 2(1-\beta) + cM\big)w_i\xi_i^2\big\}.\end{aligned}$$

Setting the first expression to zero gives $\bar{w}^\dagger \equiv \sum_{i=0}^n w_i^\dagger = 1/M(2+c)$ as required. Using this fact, setting the second expression to zero yields the condition

$$M(2+c)\big(w_0^\dagger\kappa_0^2 - w_i^\dagger\kappa_i^2\big) = \big(2\beta M + 2(1-\beta) + cM\big)w_i^\dagger\xi_i^2.$$

Substituting with $\pi^\dagger$ this becomes $w_0^\dagger \pi^\dagger \kappa_0^2 = w_i^\dagger(\pi^\dagger\kappa_i^2 + \xi_i^2)$. The proportionality result stated in the proposition follows immediately. Furthermore, recalling that $\xi_0^2 = 0$, the weights that (collusively) maximize joint profits for $i = \{0, \ldots, n\}$ are

$$w_i^\dagger = \bar{w}^\dagger \left[\sum_{j=0}^{n} \frac{\pi^\dagger\kappa_i^2 + \xi_i^2}{\pi^\dagger\kappa_j^2 + \xi_j^2} \right]^{-1} \quad \text{where } \bar{w}^\dagger = \frac{1}{M(2+c)}.$$

Note from the definitions of π and $\pi^\dagger$ that these fractions have the same denominator. Since $M > 1$ the numerator for $\pi^\dagger$ exceeds that of π and so $\pi^\dagger > \pi > 1$. To obtain the final statement of the proposition the relative equilibrium weights for any two signals i and j may be compared to the relative weights they would attract at the collusive optimum just described:

$$\frac{w_i^\star}{w_j^\star} > \frac{w_i^\dagger}{w_j^\dagger} \quad \Leftrightarrow \quad \frac{\pi\kappa_j^2 + \xi_j^2}{\pi\kappa_i^2 + \xi_i^2} > \frac{\pi^\dagger\kappa_j^2 + \xi_j^2}{\pi^\dagger\kappa_i^2 + \xi_i^2} \quad \Leftrightarrow \quad \pi^\dagger\left(\kappa_i^2\xi_j^2 - \kappa_j^2\xi_i^2\right) > \pi\left(\kappa_i^2\xi_j^2 - \kappa_j^2\xi_i^2\right).$$

Since $\pi^\dagger > \pi$, this holds if and only if $\kappa_i^2\xi_j^2 > \kappa_j^2\xi_i^2$, which in turn is true if and only if $\rho_i > \rho_j$. □

Proof of Proposition 4. Using the equations from the main text, the consumer surplus associated with the quantity produced by supplier m alone is $U_m - p_m Q_m$, or

$$\mathrm{CS}_m = \frac{1}{2}\left\{ \left[\beta M + (1-\beta)\right] Q_m^2 + (1-\beta) \sum_{m' \neq m} Q_{m'} Q_m \right\}.$$

Summing over all $m = \{1, \ldots, M\}$, taking the expectation, and (as in the proof of Proposition 1) substituting in for $\mathrm{E}[Q_m^2]$ and $\mathrm{E}[Q_m Q_{m'}]$, expected consumer surplus is

$$\mathrm{E}[\mathrm{CS}] = \frac{\beta M + (1-\beta)}{2} \sum_{m=1}^{M} \left(\mathrm{E}\left[\bar{Q}_m^2\right] + \mathrm{E}\left[(Q_m - \bar{Q}_m)^2\right] \right) + \frac{1-\beta}{2} \sum_{m=1}^{M} \sum_{m' \neq m} \mathrm{E}[\bar{Q}_m \bar{Q}_{m'}].$$

For a symmetric strategy profile (so that $w_{im} = w_i$ for all m) note that $\mathrm{E}[\bar{Q}_m\bar{Q}_{m'}] = \mathrm{E}[\bar{Q}_m^2]$. Hence:

$$\mathrm{E}[\mathrm{CS}] = \frac{M[\beta M + (1-\beta)]}{2} \left(\mathrm{E}\left[(Q_m - \bar{Q}_m)^2\right] + \pi^\ddagger \mathrm{E}\left[\bar{Q}_m^2\right] \right)$$
$$\text{where } \pi^\ddagger \equiv \frac{M}{\beta M + (1-\beta)}.$$

Substituting in for $\mathrm{E}[(Q_m - \bar{Q}_m)^2]$ and $\mathrm{E}[\bar{Q}_m^2]$,

$$\mathrm{E}[\mathrm{CS}] = \frac{M(\beta M + (1-\beta))}{2} \left(\pi^\ddagger\left(\bar{w}^2\left(x_0^2 + \kappa_0^2\right) - 2\bar{w}w_0\kappa_0^2\right) + \sum_{i=0}^{n} w_i^2\left(\pi^\ddagger\kappa_i^2 + \xi_i^2\right) \right). \tag{27}$$

To prove the proposition, expected consumer surplus may be differentiated first with respect to w_0 and then with respect to w_i for $i \neq 0$:

$$\frac{\partial\, \mathrm{E}[\mathrm{CS}]}{\partial w_0} = \frac{M(\beta M + (1-\beta))}{2} \left\{ 2\pi^\ddagger\left[\bar{w}\left(x_0^2 + \kappa_0^2\right) - \bar{w}\kappa_0^2 - w_0\kappa_0^2 + w_0\kappa_0^2\right] \right\}$$
$$= M^2\bar{w}x_0^2 > 0, \tag{28}$$
$$\frac{\partial\, \mathrm{E}[\mathrm{CS}]}{\partial w_i} = M\left(\beta M + (1-\beta)\right)\left\{ \pi^\ddagger\bar{w}\left(x_0^2 + \kappa_0^2\right) - \pi^\ddagger w_0\kappa_0^2 + w_i\left(\pi^\ddagger\kappa_i^2 + \xi_i^2\right) \right\} > 0. \tag{29}$$

Consumer surplus is strictly increasing in all of the weights used by the suppliers. However, fixing $\bar{w}$ and shifting weight from signal j to signal i:

$$\frac{\partial\, \mathrm{E[CS]}}{\partial w_i} - \frac{\partial\, \mathrm{E[CS]}}{\partial w_j} = M\big(\beta M + (1-\beta)\big)\big\{w_i\big(\pi^{\ddagger}\kappa_i^2 + \xi_i^2\big) - w_j\big(\pi^{\ddagger}\kappa_j^2 + \xi_j^2\big)\big\},$$

which is greater than zero if and only if $w_i(\pi^{\ddagger}\kappa_i^2+\xi_i^2) > w_j(\pi^{\ddagger}\kappa_j^2+\xi_j^2)$. Once again, evaluating at the equilibrium weights described in (7), this is true if and only if

$$\frac{\pi^{\ddagger}\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2} > \frac{\pi^{\ddagger}\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2} \quad \Leftrightarrow \quad \pi^{\ddagger}\big(\kappa_i^2\xi_j^2 - \kappa_j^2\xi_i^2\big) > \pi\big(\kappa_i^2\xi_j^2 - \kappa_j^2\xi_i^2\big).$$

Now $\pi^{\ddagger} > \pi^{\dagger} > \pi > 1$, so this latter inequality holds if and only if $\kappa_i^2\xi_j^2 > \kappa_j^2\xi_i^2$, which again reduces to $\rho_i > \rho_j$. In other words, moving weight from a signal j to another signal i increases consumer surplus if and only if i is more public than j.

To establish the claim concerning the prior, consider shifting weight from the prior to a signal:

$$\frac{\partial\, \mathrm{E[CS]}}{\partial w_i} - \frac{\partial\, \mathrm{E[CS]}}{\partial w_0} = M\big(\beta M + (1-\beta)\big)\big\{\pi^{\ddagger}(\bar{w} - w_0)\kappa_0^2 + w_i\big(\pi^{\ddagger}\kappa_i^2 + \xi_i^2\big)\big\},$$

which, since $\bar{w} > w_0$ at equilibrium, is always positive. Thus consumer surplus always increases when weight is moved from the prior to any signal $i \neq 0$. Incorporating the notation $\xi_0^2 = 0$, the term within the brackets in the above expression may be written

$$\underbrace{\pi^{\ddagger}\bar{w}\kappa_0^2}_{\text{variance}} - \big(\underbrace{w_0\big(\pi^{\ddagger}\kappa_0^2 + \xi_0^2\big) - w_i\big(\pi^{\ddagger}\kappa_i^2 + \xi_i^2\big)}_{\text{public vs. private}}\big).$$

These two elements mirror those presented in the proof to Proposition 3. The first represents the increase in variance as a result of moving weight away from the prior: something that necessarily increases consumer surplus; the second represents the negative impact on consumer surplus associated with moving away from the (perfectly public) prior and towards a relatively private signal i. This latter term is always positive as $\rho_i < \rho_0 = 1$. Once again, the former effect always outweighs the latter. These observations complete the proof for Proposition 4. □

Proof of Proposition 5. Recall that for a symmetric strategy profile

$$\mathrm{E}[\mathrm{Profit}_m] = \mathrm{E}[\theta\bar{Q}_m] - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\big(\pi^{\dagger}\,\mathrm{E}\big[\bar{Q}_m^2\big] + \mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big]\big) \quad \text{and}$$

$$\mathrm{E[CS]} = \frac{M[\beta M + (1-\beta)]}{2}\big(\mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big] + \pi^{\ddagger}\,\mathrm{E}\big[\bar{Q}_m^2\big]\big),$$

where as before $\pi^{\dagger}$ and $\pi^{\ddagger}$ are defined as

$$\pi^{\dagger} \equiv \frac{(2+c)M}{2\beta M + 2(1-\beta) + cM} \quad \text{and} \quad \pi^{\ddagger} \equiv \frac{M}{\beta M + (1-\beta)}.$$

Social welfare W is the sum of consumer surplus and industry profits:

$$\begin{aligned}
\mathrm{E}[W] &= M\,\mathrm{E}[\mathrm{Profit}_m] + \mathrm{E[CS]} \\
&= M\,\mathrm{E}[\theta\bar{Q}_m] - M^2\left(\frac{1+c}{2}\right)\mathrm{E}\big[\bar{Q}_m^2\big] - M\left(\frac{\beta M + (1-\beta) + cM}{2}\right)\mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big] \\
&= M\left[\mathrm{E}[\theta\bar{Q}_m] - \left(\frac{\beta M + (1-\beta) + cM}{2}\right)\big(\mathrm{E}\big[(Q_m - \bar{Q}_m)^2\big] + \pi^{\diamond}\,\mathrm{E}\big[\bar{Q}_m^2\big]\big)\right],
\end{aligned}$$

where $\pi^\diamond$ is as stated in the proposition. Substituting in for the expectations and differentiating:

$$\frac{\partial\,\mathrm{E}[W]}{\partial w_0} = M x_0^2\big[1 - \bar{w}(1+c)M\big]. \tag{30}$$

A similar exercise for the derivative with respect to w_i (for all $i \neq 0$) yields

$$\begin{aligned}\frac{\partial\,\mathrm{E}[W]}{\partial w_i} = M\big\{&\big(x_0^2+\kappa_0^2\big) - (1+c)M\big[\bar{w}\big(x_0^2+\kappa_0^2\big) - w_0\kappa_0^2\big] \\ &- (1+c)M w_i\kappa_i^2 - \big[\beta M + (1-\beta) + cM\big]w_i\xi_i^2\big\}.\end{aligned} \tag{31}$$

Once again shifting weight from a signal $j \neq 0$ to another $i \neq 0$ has the effect:

$$\frac{\partial\,\mathrm{E}[W]}{\partial w_i} - \frac{\partial\,\mathrm{E}[W]}{\partial w_j} = M\big[\beta M + (1-\beta) + cM\big]\big\{w_j\big(\pi^\diamond\kappa_j^2+\xi_j^2\big) - w_i\big(\pi^\diamond\kappa_i^2+\xi_i^2\big)\big\},$$

where $\pi^\diamond = (1+c)M/(\beta M + (1-\beta) + cM)$. Now $\pi^\ddagger > \pi^\dagger > \pi^\diamond > \pi > 1$ whenever $M > 1$ and $\beta \in (0,1)$. Thus, at equilibrium, shifting weight from j to i is good for welfare whenever

$$\begin{aligned} w_j\big(\pi^\diamond\kappa_j^2+\xi_j^2\big) > w_i\big(\pi^\diamond\kappa_i^2+\xi_i^2\big) \quad &\Leftrightarrow \quad \frac{\pi^\diamond\kappa_j^2+\xi_j^2}{\pi\kappa_j^2+\xi_j^2} > \frac{\pi^\diamond\kappa_i^2+\xi_i^2}{\pi\kappa_i^2+\xi_i^2} \\ &\Leftrightarrow \quad \pi^\diamond\big(\kappa_j^2\xi_i^2 - \kappa_i^2\xi_j^2\big) > \pi\big(\kappa_j^2\xi_i^2 - \kappa_i^2\xi_j^2\big),\end{aligned}$$

where the equilibrium weights $w_i^\star$ are taken from (7). Now because $\pi^\diamond > \pi$ the final inequality holds if and only if $\rho_j > \rho_i$. Moving weight from j to i is beneficial for social welfare if and only if i is relatively private. Similarly for a move from the prior to a signal:

$$\begin{aligned}&\frac{\partial\,\mathrm{E}[W]}{\partial w_i} - \frac{\partial\,\mathrm{E}[W]}{\partial w_0} \\ &\quad = M\kappa_0^2 - M\big[\beta M + (1-\beta) + cM\big]\big\{\pi^\diamond(\bar{w} - w_0)\kappa_0^2 + w_i\big(\pi^\diamond\kappa_i^2+\xi_i^2\big)\big\}.\end{aligned}$$

Moving weight from the prior to a signal i is good for welfare if this is positive. This holds if

$$\begin{aligned}\frac{\kappa_0^2}{\beta M + (1-\beta) + cM} &> \pi^\diamond(\bar{w} - w_0)\kappa_0^2 + w_i\big(\pi^\diamond\kappa_i^2+\xi_i^2\big), \quad \text{or} \\ \frac{\kappa_0^2}{\beta M + (1-\beta) + cM} &> \pi^\diamond\bar{w}\kappa_0^2 - \big\{\underbrace{w_0\pi^\diamond\kappa_0^2 - w_i\big(\pi^\diamond\kappa_i^2+\xi_i^2\big)}_{\text{positive, since } \rho_0 > \rho_i}\big\}.\end{aligned}$$

The second term here is positive at equilibrium since the prior is a "perfectly public" signal (with $\xi_0^2 = 0$), and $\pi^\diamond > \pi$. Because $\bar{w}$ is given by the expression in (7), the ratio on the left-hand side of the previous inequality is strictly greater than the first term on the right-hand side. Thus the inequality holds, and welfare is always increased by moving weight from the prior towards any other (relatively private) signal. This proves the claims in the first two parts of the proposition.

The third part of the proposition is obtained by maximizing welfare with respect to each of the weights w_i for $i = \{0, \ldots, n\}$. This can be achieved by setting the derivatives in (30) and (31) to zero: the first of which yields a value for the aggregate weights at the social optimum,

$$\bar{w}^\diamond = \frac{1}{(1+c)M}.$$

Substituting this value into the expression in (31) and equating to zero immediately gives

$$w_i^\diamond = \bar{w}^\diamond \left[\sum_{j=0}^{n} \frac{\pi^\diamond \kappa_i^2 + \xi_i^2}{\pi^\diamond \kappa_j^2 + \xi_j^2} \right]^{-1} \quad \text{for all } i = \{0, \ldots, n\}.$$

Thus each weight is proportional to the appropriate quantity as stated in (17). For the very final statement of the proposition, compare the ratios

$$\frac{w_i^\star}{w_j^\star} > \frac{w_i^\diamond}{w_j^\diamond} \quad \Leftrightarrow \quad \frac{\pi \kappa_j^2 + \xi_j^2}{\pi \kappa_i^2 + \xi_i^2} > \frac{\pi^\diamond \kappa_j^2 + \xi_j^2}{\pi^\diamond \kappa_i^2 + \xi_i^2} \quad \Leftrightarrow \quad \pi^\diamond \left(\kappa_i^2 \xi_j^2 - \kappa_j^2 \xi_i^2\right) > \pi \left(\kappa_i^2 \xi_j^2 - \kappa_j^2 \xi_i^2\right).$$

Now $\pi^\diamond > \pi$: this inequality holds if and only if $\kappa_i^2 \xi_j^2 > \kappa_j^2 \xi_i^2$, or $\rho_i > \rho_j$, completing the proof. □

Lemma *(Restatement of Lemma 2). $(\pi^\dagger/\pi)$ is increasing M, and decreasing in β and c. For $(\pi^\diamond/\pi)$, there exists an industry size $\bar{M} > 1$ such that if $c \leq 2\beta(\beta + c)$ then*

$$\frac{\partial[\pi^\diamond/\pi]}{\partial M} > 0 \quad \textit{and} \quad \frac{\partial[\pi^\diamond/\pi]}{\partial \beta} < 0 \quad \textit{for all } M,$$

and if $c > 2\beta(\beta + c)$ then

$$\frac{\partial[\pi^\diamond/\pi]}{\partial M} > 0 \quad \textit{and} \quad \frac{\partial[\pi^\diamond/\pi]}{\partial \beta} < 0 \quad \textit{for } M < \bar{M},$$
$$\frac{\partial[\pi^\diamond/\pi]}{\partial M} < 0 \quad \textit{and} \quad \frac{\partial[\pi^\diamond/\pi]}{\partial \beta} > 0 \quad \textit{for } M > \bar{M}.$$

Finally, $\partial[\pi^\diamond/\pi]/\partial c < 0$ for all (M, β, c).

Proof. It can be confirmed that $\pi^\dagger = 2\pi - 1$ and so $\pi^\dagger/\pi = 2 - (1/\pi)$. This is increasing in π. Applying Lemma 1 yields the comparative-static properties of $\pi^\dagger/\pi$.

For $\pi^\diamond/\pi$, the proof is more involved. For $\phi \in \{M, \beta, c\}$,

$$\frac{\partial[\pi^\diamond/\pi]}{\partial \phi} > 0 \quad \Leftrightarrow \quad \frac{1}{\pi^2}\left[\frac{\partial \pi^\diamond}{\partial \phi}\pi - \frac{\partial \pi}{\partial \phi}\pi^\diamond\right] > 0. \tag{32}$$

The expressions for $\partial\pi/\partial\phi$ are in the proof of Proposition 2. It is straightforward to show

$$\frac{\partial \pi^\diamond}{\partial M} = (1 + c)(1 - \beta)\left[\bar{w}^\diamond \pi^\diamond\right]^2, \qquad \frac{\partial \pi^\diamond}{\partial \beta} = -M(M - 1)(1 + c)\left[\bar{w}^\diamond \pi^\diamond\right]^2, \quad \text{and}$$
$$\frac{\partial \pi^\diamond}{\partial c} = -M\left(\pi^\diamond - 1\right)\bar{w}^\diamond \pi^\diamond = -M(M - 1)(1 - \beta)\left[\bar{w}^\diamond \pi^\diamond\right]^2.$$

Starting with the effect of M, the equivalence in (32) is true if and only if

$$(1 + c)(1 - \beta)\left[\bar{w}^\diamond \pi^\diamond\right]^2 \pi > (2 + c)(1 - \beta)\left[\bar{w}^\star \pi\right]^2 \pi^\diamond \quad \Leftrightarrow$$
$$(1 + c)\left[\bar{w}^\diamond\right]^2 \pi^\diamond > (2 + c)\left[\bar{w}^\star\right]^2 \pi \quad \Leftrightarrow$$
$$\frac{1}{(\beta M + (1 - \beta) + cM)M} > \frac{2 + c}{(2\beta M + 2(1 - \beta) + cM)(\beta M + (1 - \beta)(M + 1) + cM)},$$

which, after some algebraic manipulation, reduces to $2(\beta + \frac{1}{M}(1 - \beta))(\beta + \frac{1}{M}(1 - \beta) + c) > c$. So $[\pi^\diamond/\pi]$ is increasing in M if and only if this inequality holds. For $M \to \infty$ this reduces to $2\beta(\beta + c) > c$. So, since the left-hand side is decreasing in M, and since the inequality surely holds at $M = 1$, there exists an $\bar{M}$ such that for all $M < \bar{M}$, $[\pi^\diamond/\pi]$ is increasing in M and for all $M > \bar{M}$, $[\pi^\diamond/\pi]$ is decreasing in M if and only if $2\beta(\beta + c) < c$. If not, then $[\pi^\diamond/\pi]$ is increasing in M for all M as required. Consider now the derivative with respect to β. (32) becomes

$$-M(M-1)(1+c)\big[\bar{w}^\diamond\pi^\diamond\big]^2\pi > -M(M-1)(2+c)\big[\bar{w}^\star\pi\big]^2\pi^\diamond$$
$$\Leftrightarrow \quad (2+c)\big[\bar{w}^\star\big]^2\pi > (1+c)\big[\bar{w}^\diamond\big]^2\pi^\diamond,$$

reversing the inequality found for M, yielding the result. Finally, consider c. (32) becomes

$$-M(M-1)(1-\beta)\big[\bar{w}^\diamond\pi^\diamond\big]^2\pi > -M(M-1)(1-\beta)\big[\bar{w}^\star\pi\big]^2\pi^\diamond$$
$$\Leftrightarrow \quad \big[\bar{w}^\star\pi\big]^2\pi^\diamond > \big[\bar{w}^\diamond\pi^\diamond\big]^2\pi \quad \Leftrightarrow \quad \big[\bar{w}^\star\big]^2\pi > \big[\bar{w}^\diamond\big]^2\pi^\diamond.$$

Now $\pi^\diamond > \pi$ and $\bar{w}^\diamond > \bar{w}^\star$, a contradiction proving the final part of the lemma. □

Proof of Proposition 6. The proposition follows directly from Lemma 2. □

Lemma 4 *(The Social Value of Information). Let* $\alpha = \pi^\diamond(\bar{w}^\diamond/\bar{w}^\star - 1)$. *Then*

$$\frac{d\,\mathrm{E}[\text{Profit}]}{d[1/\xi_j^2]} > 0 \quad \Leftrightarrow \quad \frac{1}{\pi} - \frac{1}{2} + \Delta_j\big(\pi^\dagger\big) > 0 \quad \textit{and}$$
$$\frac{d\,\mathrm{E}[\text{Profit}]}{d[1/\kappa_j^2]} > 0 \quad \Leftrightarrow \quad \frac{1}{2\pi} + \Delta_j\big(\pi^\dagger\big) > 0,$$
$$\frac{d\,\mathrm{E}[\text{CS}]}{d[1/\xi_j^2]} > 0 \quad \Leftrightarrow \quad \frac{\pi^\ddagger}{\pi} - \frac{1}{2} - \Delta_j\big(\pi^\ddagger\big) > 0 \quad \textit{and}$$
$$\frac{d\,\mathrm{E}[\text{CS}]}{d[1/\kappa_j^2]} > 0 \quad \Leftrightarrow \quad \frac{\pi^\ddagger}{2\pi} - \Delta_j\big(\pi^\ddagger\big) > 0,$$
$$\frac{d\,\mathrm{E}[W]}{d[1/\xi_j^2]} > 0 \quad \Leftrightarrow \quad \frac{\alpha}{\pi} + \frac{1}{2} + \Delta_j\big(\pi^\diamond\big) > 0 \quad \textit{and}$$
$$\frac{d\,\mathrm{E}[W]}{d[1/\kappa_j^2]} > 0 \quad \Leftrightarrow \quad \frac{\alpha}{\pi} + \frac{\pi^\diamond}{2\pi} + \Delta_j\big(\pi^\diamond\big) > 0,$$

where the $\Delta_j(\cdot)$ *term is defined as*

$$\Delta_j(\varpi) = \sum_{i=0}^{n}\frac{w_i^\star}{\bar{w}^\star}\left[\frac{\varpi\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2} - \frac{\varpi\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2}\right].$$

Proof of Lemma 4 and Propositions 7–8. Consider first profits.[19] Profit per supplier is

$$\mathrm{E}[\text{Profit}] = \bar{w}\big(x_0^2 + \kappa_0^2\big) - w_0\kappa_0^2 - M\left[1 + \frac{c}{2}\right]\left(\bar{w}^2\big(x_0^2 + \kappa_0^2\big) - 2\bar{w}w_0\kappa_0^2 + \sum_{i=0}^{n} w_i^2\kappa_i^2\right)$$

[19] The $\star$ notation is suppressed throughout for simplicity.

$$- \left(\beta M + (1-\beta) + \frac{cM}{2}\right) \sum_{i=0}^{n} w_i^2 \xi_i^2$$

$$\text{where } \bar{w} = \frac{1}{2\beta M + (1-\beta)(M+1) + cM}.$$

Note that $\bar{w}$ does not depend on ξ_j^2 and κ_j^2. Hence the terms $\bar{w}(x_0^2 + \kappa_0^2)$ and $\bar{w}^2(x_0^2 + \kappa_0^2)$ are constant with respect to ξ_j^2 and κ_j^2 for $j \neq 0$. Hence:

$$\begin{aligned} \mathrm{E}[\text{Profit}] = \text{constant} + \left[(2+c)M - \frac{1}{\bar{w}}\right] w_0 \bar{w} \kappa_0^2 \\ - \frac{1}{2} \sum_{i=0}^{n} w_i^2 \big[(2+c)M\kappa_i^2 + \big(2\beta M + 2(1-\beta) + cM\big)\xi_i^2\big]. \end{aligned}$$

Using the definitions of π and $\pi^\dagger$, and dividing through E[Profit] by $2\beta M + 2(1-\beta) + cM$ obtains

$$\mathrm{E}[\text{Profit}] \propto \text{constant} + \big[\pi^\dagger - \pi\big] w_0 \bar{w} \kappa_0^2 - \frac{1}{2} \sum_{i=0}^{n} w_i^2 \big[\pi^\dagger \kappa_i^2 + \xi_i^2\big] \quad \Rightarrow$$

$$\frac{d\,\mathrm{E}[\text{Profit}]}{d\phi_j} \propto \frac{(\pi^\dagger - \pi)\kappa_0^2}{\bar{w}} \frac{dw_0}{d\phi_j} - \frac{w_j^2}{2\bar{w}^2} \frac{d[\pi^\dagger \kappa_j^2 + \xi_j^2]}{d\phi_j} - \sum_{i=0}^{n} \frac{w_i[\pi^\dagger \kappa_i^2 + \xi_i^2]}{\bar{w}^2} \frac{dw_i}{d\phi_j}.$$

Differentiating w_i with respect to $\phi_j \in \{\xi_j^2, \kappa_j^2\}$ for $j \neq i$ and $j = i$ respectively:

$$\frac{dw_i}{d\phi_j} = \frac{\bar{w}\hat{\psi}_i \hat{\psi}_j^2}{(\sum_{k=0}^{n} \hat{\psi}_k)^2} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j} \quad \text{and}$$

$$\frac{dw_j}{d\phi_j} = \left(\frac{\bar{w}\hat{\psi}_j^3}{(\sum_{k=0}^{n} \hat{\psi}_k)^2} - \frac{\bar{w}\hat{\psi}_j^2}{\sum_{k=0}^{n} \hat{\psi}_k}\right) \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j}.$$

Plugging these terms into the earlier derivative

$$\begin{aligned} \frac{d\,\mathrm{E}[\text{Profit}]}{d\phi_j} \propto \frac{(\pi^\dagger - \pi)\kappa_0^2 \hat{\psi}_0 \hat{\psi}_j^2}{(\sum_{k=0}^{n} \hat{\psi}_k)^2} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j} - \frac{w_j^2}{2\bar{w}^2} \frac{d[\pi^\dagger \kappa_j^2 + \xi_j^2]}{d\phi_j} \\ - \sum_{i=0}^{n} \frac{w_i[\pi^\dagger \kappa_i^2 + \xi_i^2]}{\bar{w}^2} \frac{\bar{w}\hat{\psi}_i \hat{\psi}_j^2}{(\sum_{k=0}^{n} \hat{\psi}_k)^2} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j} \\ + \frac{w_j[\pi^\dagger \kappa_j^2 + \xi_j^2]}{\bar{w}^2} \frac{\bar{w}\hat{\psi}_j^2}{\sum_{k=0}^{n} \hat{\psi}_k} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j}. \end{aligned}$$

Simplifying a lot, and looking only at the sign:

$$\begin{aligned} \operatorname{sign}\left[\frac{d\,\mathrm{E}[\text{Profit}]}{d\phi_j}\right] = \operatorname{sign}\Bigg[\frac{(\pi^\dagger - \pi)}{\pi} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j} - \frac{1}{2} \frac{d[\pi^\dagger \kappa_j^2 + \xi_j^2]}{d\phi_j} \\ - \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j} \sum_{i=0}^{n} \frac{w_i[\pi^\dagger \kappa_i^2 + \xi_i^2]}{\bar{w}(\pi \kappa_i^2 + \xi_i^2)} + \frac{\pi^\dagger \kappa_j^2 + \xi_j^2}{\pi \kappa_j^2 + \xi_j^2} \frac{d[\pi \kappa_j^2 + \xi_j^2]}{d\phi_j}\Bigg]. \end{aligned}$$

Looking first at the effect of both idiosyncratic and common noise:

$$\operatorname{sign}\left[\frac{d\,\mathrm{E}[\text{Profit}]}{d\xi_j^2}\right] = \operatorname{sign}\left[\frac{\pi^\dagger}{\pi} - \frac{3}{2} + \sum_{i=0}^{n} \frac{w_i}{\bar{w}}\left[\frac{\pi^\dagger\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2} - \frac{\pi^\dagger\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2}\right]\right]$$

$$\operatorname{sign}\left[\frac{d\,\mathrm{E}[\text{Profit}]}{d\kappa_j^2}\right] = \operatorname{sign}\left[\frac{\pi^\dagger}{2\pi} - 1 + \sum_{i=0}^{n} \frac{w_i}{\bar{w}}\left[\frac{\pi^\dagger\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2} - \frac{\pi^\dagger\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2}\right]\right].$$

Noting that $\pi^\dagger = 2\pi - 1$ and reversing the sign to give the sign of the derivative with respect to precisions ($1/\xi_j^2$ and $1/\kappa_j^2$) gives the equivalences in the proposition.

Now $\pi > 1$ implies that $(1/2\pi) > (1/\pi) - (1/2)$. Hence if it is profitable (for the industry) to reduce the receiver noise in a signal then it is also profitable to reduce the sender noise.

Looking at a perfectly private signal:

$$\kappa_j^2 = 0 \quad \Rightarrow \quad \operatorname{sign}\left[\frac{d\,\mathrm{E}[\text{Profit}]}{d[1/\xi_j^2]}\right] \geq \operatorname{sign}\left[\frac{1}{\pi} - \frac{1}{2}\right] > 0,$$

as long as $\pi \leq 2$, as required. More generally, for any signal, the lower bound is:

$$\operatorname{sign}\left[\frac{d\,\mathrm{E}[\text{Profit}]}{d[1/\xi_j^2]}\right] \geq \operatorname{sign}\left[\frac{2}{\pi} - \frac{3}{2}\right] \geq 0 \quad \Leftrightarrow \quad \pi \geq \frac{4}{3},$$

which completes the proof of the statements in the proposition and corollaries concerning profits.

Using exactly the same method for consumer surplus, given in (27), it follows that

$$\frac{\partial\,\mathrm{E}[\mathrm{CS}]}{\partial\phi_j} \propto -\frac{\pi^\ddagger}{\pi}\frac{d(\pi\kappa_j^2 + \xi_j^2)}{d\phi_j} + \frac{1}{2}\frac{d(\pi^\ddagger\kappa_j^2 + \xi_j^2)}{d\phi_j} + \frac{d(\pi\kappa_j^2 + \xi_j^2)}{d\phi_j}\sum_{i=0}^{n}\frac{w_i}{\bar{w}}\left\{\frac{\pi^\ddagger\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2} - \frac{\pi^\ddagger\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2}\right\}.$$

So, after some cancellations and manipulation:

$$\operatorname{sign}\left[\frac{\partial\,\mathrm{E}[\mathrm{CS}]}{\partial[1/\xi_j^2]}\right] = \operatorname{sign}\left[\frac{\pi^\ddagger}{\pi} - \frac{1}{2} + \sum_{i=0}^{n}\frac{w_i}{\bar{w}}\left\{\frac{\pi^\ddagger\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2} - \frac{\pi^\ddagger\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2}\right\}\right], \quad \text{and}$$

$$\operatorname{sign}\left[\frac{\partial\,\mathrm{E}[\mathrm{CS}]}{\partial[1/\kappa_j^2]}\right] = \operatorname{sign}\left[\frac{\pi^\ddagger}{2\pi} + \sum_{i=0}^{n}\frac{w_i}{\bar{w}}\left\{\frac{\pi^\ddagger\kappa_j^2 + \xi_j^2}{\pi\kappa_j^2 + \xi_j^2} - \frac{\pi^\ddagger\kappa_i^2 + \xi_i^2}{\pi\kappa_i^2 + \xi_i^2}\right\}\right].$$

Now, $\pi^\ddagger > \pi$, so $\pi^\ddagger/\pi - \frac{1}{2} > \pi^\ddagger/2\pi$. So it would be sufficient to show the argument on the second line is positive. However, it can be negative. On the other hand, the argument on the first line is always positive. Since the ratio of $\pi^\ddagger\kappa_j^2 + \xi_j^2$ to $\pi\kappa_j^2 + \xi_j^2$ lies between 1 and $\pi^\ddagger/\pi$, the smallest the expression on the first line can be is

$$\frac{\pi^\ddagger}{\pi} - \frac{1}{2} + 1 - \frac{\pi^\ddagger}{\pi} \geq 0.$$

Thus, expected consumer surplus is always increasing in the precision of the receiver noise. On the other hand, it is increasing in the precision of the sender noise if $2\pi \geq \pi^\ddagger$, and otherwise may

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be decreasing. To obtain the inequality given in the corollary in the main text, note

$$\pi^\ddagger - 1 = \frac{(1-\beta)(M-1)}{\beta M + (1-\beta)} \quad \text{so} \quad \frac{\pi^\ddagger - 1}{\pi - 1} = \frac{2\beta M + 2(1-\beta) + cM}{\beta M + (1-\beta)} = 2 + c\pi^\ddagger,$$

where $\pi - 1$ comes from the definition of π in (6). Rearranging to obtain $\pi^\ddagger$ as an expression in π,

$$\pi^\ddagger = \frac{2\pi - 1}{1 - c(\pi - 1)} \quad \Rightarrow \quad 2\pi \geq \pi^\ddagger \quad \Leftrightarrow \quad \pi \leq \frac{1}{2}\left(1 + \sqrt{1 + \frac{2}{c}}\right) \quad \text{as required.}$$

Finally, turning to welfare: summing surplus from (27) and profits from above, collecting all of these terms together, and using the definition of $\pi^\diamond$:

$$\begin{aligned}\frac{\text{E[Welfare]}}{M} = {} & \left(x_0^2 + \kappa_0^2\right)\left[\bar{w} - M\left(1 + \frac{c}{2}\right)\bar{w}^2 + \frac{M}{2}\bar{w}^2\right] \\ & - w_0\kappa_0^2\left[1 - 2\bar{w}M\left(1 + \frac{c}{2}\right) + M\bar{w}\right] \\ & - \sum_{i=0}^{n} w_i^2\kappa_i^2\left[M\left(1 + \frac{c}{2}\right) - \frac{M}{2}\right] \\ & - \sum_{i=0}^{n} w_i^2\xi_i^2\left[\beta M + (1-\beta) + \frac{cM}{2} - \frac{1}{2}\left(\beta M + (1-\beta)\right)\right].\end{aligned}$$

Collecting constant terms (in ξ_j^2 and κ_j^2 for $j \neq 0$) into C and simplifying,

$$\begin{aligned}\frac{\text{E[Welfare]}}{M} = {} & C - w_0\kappa_0^2\left[1 - M\bar{w}(1+c)\right] - \frac{M(1+c)}{2}\sum_{i=0}^{n} w_i^2\kappa_i^2 \\ & - \frac{\beta M + (1-\beta) + cM}{2}\sum_{i=0}^{n} w_i^2\xi_i^2,\end{aligned}$$

which, after dividing through by the constant $(\beta M + (1-\beta) + cM)/2$, gives

$$\begin{aligned}\text{E[Welfare]} \propto {} & \text{constant} - w_0\kappa_0^2\left[\frac{2}{\beta M + (1-\beta) + cM} - 2\bar{w}\pi^\diamond\right] \\ & - \pi^\diamond\sum_{i=0}^{n} w_i^2\kappa_i^2 - \sum_{i=0}^{n} w_i^2\xi_i^2 \\ = {} & \text{constant} - 2w_0\pi^\diamond\kappa_0^2\left[w^\diamond - \bar{w}\right] - \sum_{i=0}^{n} w_i^2\left(\pi^\diamond\kappa_i^2 + \xi_i^2\right) \\ \propto {} & \text{constant} - \alpha\bar{w}w_0\kappa_0^2 - \frac{1}{2}\sum_{i=0}^{n} w_i^2\left(\pi^\diamond\kappa_i^2 + \xi_i^2\right),\end{aligned}$$

from the definition of $w^\diamond$, and where, from this and from the definition of $\bar{w}$,

$$\alpha \equiv \pi^\diamond w^\diamond\left[\frac{1}{\bar{w}} - \frac{1}{w^\diamond}\right] = \frac{\beta M + (1-\beta)}{\beta M + (1-\beta) + cM} \in [0, 1].$$

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Let $\phi_j \in \{\kappa_j^2, \xi_j^2\}$. Then, using the calculations for $dw_i/d\phi_j$ above,

$$\begin{aligned}\frac{d\,\mathrm{E}[\text{Welfare}]}{d\phi_j} &= -\alpha\bar{w}\kappa_0^2\frac{dw_0}{d\phi_j} - \sum_{i=0}^{n} w_i\frac{dw_i}{d\phi_j}\left(\pi^\diamond\kappa_i^2+\xi_i^2\right) - \frac{w_j^2}{2}\frac{d(\pi^\diamond\kappa_j^2+\xi_j^2)}{d\phi_j}\\ &= -\alpha\bar{w}\kappa_0^2\frac{\bar{w}\hat{\psi}_0\hat{\psi}_j^2}{(\sum_k\hat{\psi}_k)^2}\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j}\\ &\quad - \sum_{i=0}^{n} w_i\frac{\bar{w}\hat{\psi}_i\hat{\psi}_j^2}{(\sum_k\hat{\psi}_k)^2}\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j}\left(\pi^\diamond\kappa_i^2+\xi_i^2\right)\\ &\quad + w_j\frac{\bar{w}\hat{\psi}_j^2}{\sum_k\hat{\psi}_k}\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j}\left(\pi^\diamond\kappa_j^2+\xi_j^2\right) - \frac{w_j^2}{2}\frac{d(\pi^\diamond\kappa_j^2+\xi_j^2)}{d\phi_j}.\end{aligned}$$

Now using the definition of w_j in terms of $\hat{\psi}_j$ in the usual way,

$$\begin{aligned}\frac{d\,\mathrm{E}[\text{Welfare}]}{d\phi_j} &= -\frac{\alpha}{\pi}w_j^2\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j} - \frac{w_j^2}{2}\frac{d(\pi^\diamond\kappa_j^2+\xi_j^2)}{d\phi_j}\\ &\quad - w_j^2\sum_{i=0}^{n}\frac{w_i}{\bar{w}}\hat{\psi}_i\left(\pi^\diamond\kappa_j^2+\xi_j^2\right)\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j}\\ &\quad + w_j^2\hat{\psi}_j\left(\pi^\diamond\kappa_j^2+\xi_j^2\right)\frac{d(\pi\kappa_j^2+\xi_j^2)}{d\phi_j}.\end{aligned}$$

To sign this, the w_j^2 terms can be ignored, and (in terms of precisions),

$$\operatorname{sign}\left[\frac{\partial\,\mathrm{E}[\text{Welfare}]}{\partial[1/\xi_j^2]}\right] = \operatorname{sign}\left[\alpha+\frac{\pi}{2}-\pi\sum_{i=0}^{n}\frac{w_i}{\bar{w}}\left\{\frac{\pi^\diamond\kappa_j^2+\xi_j^2}{\pi\kappa_j^2+\xi_j^2}-\frac{\pi^\diamond\kappa_i^2+\xi_i^2}{\pi\kappa_i^2+\xi_i^2}\right\}\right], \quad \text{and}$$

$$\operatorname{sign}\left[\frac{\partial\,\mathrm{E}[\text{Welfare}]}{\partial[1/\kappa_j^2]}\right] = \operatorname{sign}\left[\alpha+\frac{\pi^\diamond}{2}-\pi\sum_{i=0}^{n}\frac{w_i}{\bar{w}}\left\{\frac{\pi^\diamond\kappa_j^2+\xi_j^2}{\pi\kappa_j^2+\xi_j^2}-\frac{\pi^\diamond\kappa_i^2+\xi_i^2}{\pi\kappa_i^2+\xi_i^2}\right\}\right].$$

For κ_j^2 then, this is always positive: the argument of the second expression is at least

$$\alpha+\frac{\pi^\diamond}{2}-\pi\frac{\pi^\diamond}{\pi}+\pi>0 \quad\Leftrightarrow\quad 2(\alpha+\pi)>\pi^\diamond,$$

but $2(\alpha+\pi)>2\pi>2\pi-1=\pi^\dagger>\pi^\diamond$. For ξ_j^2, in fact, it is also always positive under the maintained assumption $\pi\le 2$. To see this, note that $\pi^\diamond<\pi^\dagger$. It is sufficient to show that

$$\alpha+\frac{\pi}{2}-\pi^\diamond+\pi>0 \quad\Leftrightarrow\quad 2\alpha+3\pi>2\pi^\diamond.$$

Now $2\alpha+2>\pi \Rightarrow 2\alpha+3\pi+2>4\pi \Rightarrow 2\alpha+3\pi>4\pi-2=2\pi^\dagger$ since $2\pi-1=\pi^\dagger$. As mentioned above $\pi^\dagger>\pi^\diamond$, and so the inequality in the above displayed equation holds. So indeed welfare is increasing in the precision of information (both with reference to receiver and sender noise). □

Proof of Propositions 7 and 8. The propositions follow directly from the proof of Lemma 4. □

Proof of Lemma 3. Given the play of linear strategies,

$$\begin{aligned}
\mathrm{E}[\mathrm{Profit}_m] = {} & \bar{w}_m x_0^2 + (\bar{w}_m - w_{0m})\kappa_0^2 \\
& - \left(\beta M + \frac{(1-\beta)(M+1)}{2} + \frac{cM}{2}\right)\left(\bar{w}_m^2 x_0^2 + (\bar{w}_m - w_{0m})^2\kappa_0^2\right) \\
& - \left(\beta M + \frac{(1-\beta)(M+1)}{2} + \frac{cM}{2}\right)\sum_{i=1}^n w_{im}^2\kappa_i^2 \\
& - \left(\beta M + (1-\beta) + \frac{cM}{2}\right)\sum_{i=1}^n \frac{w_{im}^2\xi_i^2}{z_{im}} \\
& + (1-\beta)\sum_{m'\neq m}\left((\bar{w}_m - \bar{w}_{m'})^2\left(x_0^2 + \kappa_0^2\right) + \sum_{i=1}^n (w_{im} - w_{im'})^2\kappa_i^2\right) \\
& - \text{externality term} - \text{cost of information acquisition}.
\end{aligned} \tag{33}$$

Ignoring the cost of information acquisition, and the externality term which is independent of m's strategy, the linear terms may be separated out to yield

$$\begin{aligned}
\mathrm{E}[\mathrm{Profit}_m] = {} & \text{terms that are linear in } w_m - \underbrace{\left(\beta M + (1-\beta) + \frac{cM}{2}\right)}_{\text{(i)}}\sum_{i=1}^n \underbrace{\frac{w_{im}^2\xi_i^2}{z_{im}}}_{\text{(ii)}} \\
& - \underbrace{\left(\beta M - \frac{(1-\beta)(M-3)}{2} + \frac{cM}{2}\right)}_{\text{(iii)}}\underbrace{\left(\left(x_0^2 + \kappa_0^2\right)\bar{w}_m^2 + \sum_{i=1}^n w_{im}^2\kappa_i^2\right)}_{\text{(iv)}}.
\end{aligned}$$

The linear terms are concave in w_m. Turning to the remainder of the first line, the term (i) is positive, and multiplies the sum of convex functions: within each term (ii) w_{im}^2/z_{im} is (jointly) convex in w_{im} and z_{im}. Given that (i) is negated, this ensures that the first line is concave. Finally, consider the second line. By inspection, (iv) is a convex function of w_m. Hence, the second line is concave if the term (iii) is positive, which it is if and only if the condition $\pi < 2$ holds. Note that $\pi < 2$ is a critical sufficient condition: if it fails, then setting $\xi_i^2 = 0$ for each i (or choosing ξ_i^2 to be sufficiently small) would make $\mathrm{E}[\mathrm{Profit}_m]$ convex.

Given that $\pi < 2$, it is established that $\mathrm{E}[\mathrm{Profit}_m]$ is (jointly) concave in (w_m, z_m). This concavity remains if the interaction term is excluded: the interaction term is a convex function of w_m and so removing it ensures that the terms that remain in (19) are certainly concave. □

Proof of Proposition 9. The main text notes that $(w^\star, z^\star)$ must be a local maximizer of the expression in (19). If $\hat{C}(z_m)$ is convex then (from Lemma 3) this is concave, and there is a unique local maximizer. This also locally maximizes the expected profit of m, and from the concavity of this expected profit (Lemma 3 again) this ensures that $(w^\star, z^\star)$ is a symmetric equilibrium.

It remains to show that there are no asymmetric equilibria. To this, note that for the payoff of supplier m (expected profit) in (33) the externality term is irrelevant from the perspective of supplier m. The cost of information acquisition and the terms in the first three lines depend only on the decisions of m, and not on the choices of others. Hence m's objective is to maximize

$$\text{Payoff}_m = U(w_m, z_m) + (1-\beta)\sum_{m' \neq m}\left((\bar{w}_m - \bar{w}_{m'})^2\left(x_0^2 + \kappa_0^2\right) + \sum_{i=1}^{n}(w_{im} - w_{im'})^2\kappa_i^2\right),$$

where $U(w_m, z_m)$ collects together the terms from the first three lines of (33) and the cost of information acquisition. Given its concavity, the maximization of m's payoff is determined by first-order conditions. For example, the condition with respect to w_{im} for $i \in \{1, \ldots, m\}$ is

$$\begin{aligned}
0 = \frac{\partial \text{Payoff}_m}{\partial w_{im}} &= \frac{\partial U(w_m, z_m)}{\partial w_{im}} + 2(1-\beta)\sum_{m' \neq m}\left((\bar{w}_m - \bar{w}_{m'})\left(x_0^2 + \kappa_0^2\right) + (w_{im} - w_{im'})\kappa_i^2\right) \\
&= \frac{\partial U(w_m, z_m)}{\partial w_{im}} + 2(1-\beta)\sum_{m'=1}^{M}\left((\bar{w}_m - \bar{w}_{m'})\left(x_0^2 + \kappa_0^2\right) + (w_{im} - w_{im'})\kappa_i^2\right) \\
&= \frac{\partial U(w_m, z_m)}{\partial w_{im}} + 2(1-\beta)M\left((\bar{w}_m - \bar{w})\left(x_0^2 + \kappa_0^2\right) + (w_{im} - w_i)\kappa_i^2\right) \\
&\text{where } w_i \equiv \frac{\sum_{m'=1}^{M} w_{im'}}{M} \quad \text{and} \quad \bar{w} \equiv \frac{\sum_{m'=1}^{M} \bar{w}_{m'}}{M}.
\end{aligned}$$

An equivalent expression holds for $i = 0$. This implies that the first-order conditions for player m depend only on the choices of other players via the averages w_i and $\bar{w}$.

Consider an equilibrium strategy profile. Fix $\bar{w}$ and w_i for each i (the average weights across the player set) and replace the payoff of player m with the payoff

$$\text{Payoff}_m^\star = U(w_m, z_m) + (1-\beta)M\left((\bar{w}_m - \bar{w})^2\left(x_0^2 + \kappa_0^2\right) + \sum_{i=1}^{n}(w_{im} - w_i)^2\kappa_i^2\right),$$

where $\bar{w}$ and w_i for each i are treated as fixed parameters by player m. The first-order conditions for the maximization of $\text{Payoff}_m^\star$ are satisfied at the original equilibrium strategy profile. Moreover, so long as $\text{Payoff}_m^\star$ is concave in m's choices (which is checked below) then these choices are the unique maximizers of $\text{Payoff}_m^\star$. Note, however, that if players are symmetric then $\text{Payoff}_m^\star$ is the same for every player. Hence, each player's optimizing choice must be the same. This implies that $w_m = w_{m'}$ for each $m \neq m'$. That is, the equilibrium must be symmetric.

The missing element is the confirmation that $\text{Payoff}_m^\star$ is concave in m's choices. Note that:

$$\begin{aligned}
\text{Payoff}_m^\star = \text{terms that are linear in } w_m &- \underbrace{\left(\beta M + (1-\beta) + \frac{cM}{2}\right)}_{\text{(i)}}\sum_{i=1}^{n}\underbrace{\frac{w_{im}^2\xi_i^2}{z_{im}}}_{\text{(ii)}} \\
&- \underbrace{\left(\beta M - \frac{(1-\beta)(M-1)}{2} + \frac{cM}{2}\right)}_{\text{(iii)}}\underbrace{\left(\left(x_0^2 + \kappa_0^2\right)\bar{w}_m^2 + \sum_{i=1}^{n} w_{im}^2\kappa_i^2\right)}_{\text{(iv)}}.
\end{aligned}$$

A sufficient condition for concavity is that (iii) is positive, which it is if $3\beta + c > 1$.

Turning to the second set of claims, attention is strictly costly and so $z_i^\star > 0$ only if $w_i^\star > 0$. The solution for the equilibrium influence weights implies that $w_i^\star = 0$ if $z_i^\star = 0$. The displayed equation is the re-arranged first-order condition for $z_i^\star$. If $z_i^\star = 0$ and $\xi_i < K_i$ then it can be confirmed that there is a local profitable deviation to shift both w_{im} and z_{im} away from zero. □

Proof of Proposition 10. This proposition follows directly from arguments in the main text. □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2014.07.011.

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