Information Acquisition and Use by Networked Players

David P. Myatt
London Business School
dmyatt@london.edu

Chris Wallace
University of Manchester
christopher.wallace@manchester.ac.uk

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Abstract. In an asymmetric coordination (or anti-coordination) game, players acquire and use signals about a payoff-relevant fundamental from multiple costly information sources. Some sources have greater clarity than others, and generate signals that are more correlated and so more public. Players wish to take actions close to the fundamental but also close to (or far away from) others' actions. This paper studies how asymmetries in players' coordination motives, represented as the weights that link players to neighbours on a network, affect how they use and acquire information. Relatively centrally located players (in the sense of Bonacich, when applied to the dependence of players' payoffs upon the actions of others) acquire fewer signals from relatively clear information sources; they acquire less information in total; and they place more emphasis on relatively public signals.

JEL Classifications. C72, D83, D85. **Keywords.** Networks, Bonacich Centrality, Information Acquisition and Use, Public and Private Information.

Decision makers often seek to take actions close to some unknown state of the world (a fundamental motive) and also close to (or sometimes far away from) the actions of others (a coordination motive). An established literature has applied quadratic-payoff games with these features to understand information use and (more recently) costly information acquisition in a variety of important economic environments.

This paper contributes a tractable model of situations in which players care asymmetrically about coordination. Links on a network represent players' desires to coordinate (or not) with their neighbours. Players also wish to match the fundamental state of the world. This structure allows for two different kinds of asymmetry. Firstly, two different players may balance differently the payoff component from coordination with the payoff from matching the state of the world. Secondly, even if two players agree on this balance they may care differently about the identities of those with whom they coordinate. For example, in a hierarchical environment a player might care about coordinating with higher members of that hierarchy but not with lower members.

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Players learn about the unknown state of the world (and so about likely action choices of others) via multiple information sources. Each source generates a signal realization for a player equal to the true state plus a noise term which can be correlated across players. Such an information source is characterized by two things: the precision of a common noise component—its underlying "accuracy"—and the precision of a player-specific component—its "clarity". Paying more (costly) attention allows a player to reduce the player-specific noise. This increases both the informativeness of the signal and its correlation with others' observations. In this context, there is distinction between the "use" and "acquisition" of information. Specifically, information acquisition refers to the expenditure of costly attention across information sources, where that attention reduces player-specific noise. Conditional on such acquisition decisions, or indeed in situations where the information structure is entirely exogenous, information use refers to how different signal realizations influence players' final action choices.

Two questions are answered. Firstly: how do the scale and pattern of asymmetric coordination motives influence how players use the information available to them? Secondly: if information sources are costly, then how do the coordination asymmetries influence which sources receive attention and the total expenditure on information acquisition?

The answer to the first question is that information use (the response of actions to signals) is jointly determined by a player's centrality and the correlation of signal realizations. Centrality refers to how a player's payoff is influenced by others' actions, measured à la Bonacich with a decay parameter equal to the conditional (on the state) correlation coefficient of signals across players. A player's (Bonacich) centrality is influenced by both forms of coordination asymmetry: differences in the desire to coordinate, and differences in the identities of those with whom a player wishes to coordinate. The key finding is that a bias toward public signals is stronger for more central players.

A clean answer to the second question emerges when players (endogenously) pay to acquire the same subset of the available information sources. Those sources are the easiest to interpret, even if they have very poor underlying accuracy; equivalently, these are the sources that are cheapest in the sense that the marginal cost of increasing the precision of the player-specific noise is lowest. Players then use clearer (or cheaper) signals relatively more: the influence of a signal deviates from its relative accuracy by the product of the player's (unweighted) Bonacich centrality and a measure of the signal's relative clarity. Strikingly, more central players spend less in total on information acquisition.

Fuller characterizations, including for corner-solution cases in which different players ignore different information sources, are developed for two commonly studied network classes. In a two-type core-perhiphery network, central players acquire fewer information sources, and make greater use of clearer (rather than more accurate) signals. Secondly, in a hierarchy in which players seek to coordinate only with those immediately above them, players further down the chain acquire a subset comprising the clearest signals acquired by the player(s) above; they acquire less information in total.

An important message is that relatively clear (and so, endogenously, relatively public) information has more influence on the players who are more centrally dependent upon the actions of others in a network; but those players pay less to acquire information.

This paper links a literature which uses network centrality measures in asymmetric complete-information quadratic-payoff games (Ballester, Calvó-Armengol, and Zenou, 2006) to one which studies the use of dispersed information in symmetric games (Morris and Shin, 2002; Angeletos and Pavan, 2007), while incorporating the signal technology of Dewan and Myatt (2008) and Myatt and Wallace (2012). Almost all existing analyses of quadratic-payoff games with dispersed information specify symmetric players. The distinction of this paper is that it admits the tractable analysis of an arbitrary pattern of coordination motives with two kinds of asymmetry: players have different aggregate coordination motives and also care differently about with whom they coordinate.[2]

The model and its full-information benchmark solution are described in Section 1. The equilibrium is characterized in Section 2, with sharp results reported when players acquire and use the same set of signals. A benchmark result in Section 3 reports that asymmetric players act symmetrically when they share a common aggregate coordination motive. Two particular formulations are then discussed: "two-type" (for example, core-periphery) networks in Section 4 and a hierarchy structure in Section 5. Both cases admit the possibility that an information source is used by some players but not others. Concluding remarks and a discussion of related literature are contained in Section 6.

1. A Quadratic-Payoff Coordination Game on a Network

1.1. **Players and Payoffs.** Each player $m \in \{1, \ldots, M\}$ simultaneously chooses a real-valued action $a_m \in \mathbb{R}$. For a pair of players m and m', $\gamma_{mm'}$ is the (relative) influence of the action of player m' upon the payoff of player m, which is

$$u_m \equiv \text{constant} - \left[(1-\beta_m)(a_m-\theta)^2 + \beta_m \sum\nolimits_{m' \neq m} \gamma_{mm'}(a_m - a_{m'})^2\right]. \tag{1}$$

θ is a common real-valued "fundamental" target, $\gamma_{mm'} \geq 0$, $\sum_{m' \neq m} \gamma_{mm'} = 1$, and β_m (which can be positive or negative) is the aggregate influence of others on player m.[3] This is a quadratic-payoff game in which players wish be close to the fundamental θ and close to (or far away from) the actions of others. Assume $|\beta_m| < 1$ for all m, so that coordination (or anti-coordination) motives are not overly strong.[4]

[2] Two existing papers incorporate player asymmetry: Myatt and Wallace (2018) allows for the first kind of asymmetry, but not for the second, in a specific price-setting model; Leister (2017) allows for general player asymmetry, but restricts to a single perfectly private signal. See Section 6 for a fuller discussion.

[3] A player wishes either to coordinate with ($\beta_m > 0$) or against ($\beta_m < 0$) all others. This is straightforward to relax. Indeed, $\gamma_{mm'} \geq 0$ is assumed for expositional purposes only: it plays no role in any of the proofs.

[4] The specification of (1) is not particularly restrictive. See Section 1.3 for a fuller discussion.

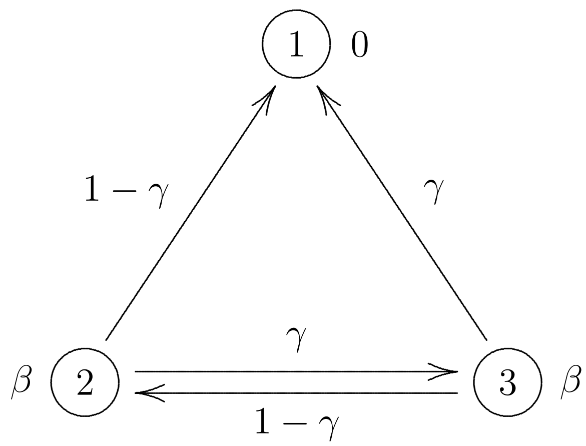

FIGURE 1. A Three-Player Network

Each directed link reflects the dependence of one player's payoff (at the root of the arrow) upon coordination with another player's action (at the head of the arrow). Here, the elements of Γ satisfy $\gamma_{23} = \gamma_{31} = \gamma \in [0, \frac{1}{2}]$ and $\gamma_{21} = \gamma_{32} = 1 - \gamma \in [\frac{1}{2}, 1]$. The players' concerns for coordination are $\beta_1 = 0$ and $\beta_2 = \beta_3 = \beta$ respectively. Player 1 does not care about coordination, and so γ_{12} and γ_{13} are omitted.

The parameters $\gamma_{mm'}$ represent the weights on the links in a directed graph in which each player is identified with a different node. The adjacency matrix for this network is

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1M} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{M1} & \gamma_{M2} & \cdots & \gamma_{MM} \end{bmatrix},$$

where $\gamma_{mm} = 0$ for all m. The mth row captures the relative influence of others' actions on the payoff of m. The absolute influence also includes m's desire to coordinate. Writing $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_M)'$ and $\text{diag}[\boldsymbol{\beta}]$ for the diagonal matrix with mth diagonal element β_m, the adjusted (for the strengths of the coordination motive) adjacency matrix is

$$\bar{\Gamma} \equiv \text{diag}[\boldsymbol{\beta}]\Gamma = \begin{bmatrix} \beta_1\gamma_{11} & \beta_1\gamma_{12} & \cdots & \beta_1\gamma_{1M} \\ \beta_2\gamma_{21} & \beta_2\gamma_{22} & \cdots & \beta_2\gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_M\gamma_{M1} & \beta_M\gamma_{M2} & \cdots & \beta_M\gamma_{MM} \end{bmatrix}.$$

$\bar{\Gamma}$ incorporates two sources of player asymmetry. Firstly, players may be asymmetrically connected: players m and m' may care relatively differently about some third player m'', so $\gamma_{mm''} \neq \gamma_{m'm''}$. Secondly, even if connections are symmetric and equal ($\gamma_{mm'} = 1/(M-1)$ for all m and m') then players may care differently about coordination: $\beta_m \neq \beta_{m'}$.

A three-player example is displayed in Figure 1. The nodes are the three players. Adjacent to a node is the player's aggregate coordination concern: $\beta_1 = 0$ (Player 1 cares only about the fundamental) while $\beta_2 = \beta_3 = \beta > 0$ (Players 2 and 3 care equally, in aggregate, about coordination). There is an asymmetry between those who care about coordination and those who do not. A further form of asymmetry concerns the network of interdependencies. Setting $0 \le \gamma \le \frac{1}{2}$, Player 3 cares mostly about coordinating with Player 2 (because $\gamma_{32} = 1-\gamma \ge \gamma = \gamma_{31}$) whereas Player 2 cares mostly about coordination with Player 1 (because $\gamma_{23} = \gamma \le 1-\gamma = \gamma_{21}$). Player 3 cares more (relative to Player 2) about coordinating with another coordinator. The adjusted adjacency matrix is

$$\bar{\Gamma} = \begin{bmatrix} 0 & 0 & 0 \\ \beta(1-\gamma) & 0 & \beta\gamma \\ \beta\gamma & \beta(1-\gamma) & 0 \end{bmatrix}.$$

1.2. **Centrality.** β_m (the sum of the mth row of $\bar{\Gamma}$) measures m's direct concern for coordination. However, the effective overall concern for coordination depends on the players with whom m wishes to coordinate. If m wishes to match those who also have strong coordination motives then this amplifies m's incentive to take a coordinated action. The final strength of the desire to coordinate depends on the entire network and on the centrality of a player within it. This paper uses notions of "Bonacich" centrality that follow those employed by Ballester, Calvó-Armengol, and Zenou (2006) and others.[5]

Definition (Bonacich Centrality)**.** *Given a network described by $\bar{\Gamma}$, player m's Bonacich centrality with decay parameter ρ is the mth entry of*

$$\mathbf{b} = [\mathrm{I} - \rho\bar{\Gamma}]^{-1}\mathbf{1} = \sum\nolimits_{k=0}^{\infty} \rho^k \bar{\Gamma}^k \mathbf{1}, \quad \textit{where} \quad \mathbf{1} = (1,\ldots,1)'.$$

Briefly, if $\bar{\Gamma}$ is a symmetric matrix containing only 1s and 0s then the mth element of $\rho^k\bar{\Gamma}^k\mathbf{1}$ counts the paths of length k that begin (or end) at player m from every other player. Such paths "decay" (are discounted) by ρ^k. If the matrix contains numbers other than 1 and 0, each path is further discounted by the weight of each component link.

This definition easily extends to allow different players (or nodes on a network) to have different exogenous influences, summarized in a vector $\boldsymbol{\alpha}$. Doing so, the vector of $\boldsymbol{\alpha}$-weighted Bonacich centralities (with decay parameter ρ) is $\mathbf{b}_{\boldsymbol{\alpha}} = [\mathrm{I} - \rho\bar{\Gamma}]^{-1}\boldsymbol{\alpha}$.

Bonacich centrality is readily illustrated using the example of Figure 1, for which

$$\mathbf{b} = \frac{1}{1-\beta^2\gamma(1-\gamma)} \begin{pmatrix} 1-\beta^2\gamma(1-\gamma) \\ 1+\beta+\beta^2\gamma^2 \\ 1+\beta+\beta^2(1-\gamma)^2 \end{pmatrix}. \tag{2}$$

Given that $\gamma \le \frac{1}{2}$, it is straightforward to confirm that $b_3 \ge b_2 > b_1$. Hence, Player 3 is the most central in terms of coordination dependency upon other players' actions.

1.3. **A Full Information Benchmark.** A natural benchmark case is when θ is known.

Each player m chooses a_m to maximize (1). The maintained assumption $|\beta_m| < 1$ is sufficient for concavity, and first-order conditions yield unique best-replies:

$$\text{Best reply of } m = a_m = (1-\beta_m)\theta + \beta_m \sum\nolimits_{m' \ne m} \gamma_{mm'} a_{m'}. \tag{3}$$

[5]Bonacich (1987) developed many notions of centrality. Nevertheless, the definition here has become popularly associated with Bonacich. It is an affine transformation of one appearing in Bonacich (1987).

$a_m = \theta$ for all m uniquely satisfies this system: everyone perfectly coordinates, and so play is completely symmetric.[6] This is by design: the objective is to study the impact of asymmetries on information use and acquisition, and so it is instructive to abstract away from asymmetries in actions that would occur in a full-information world.[7]

The symmetry of play holds because players care about matching the same fundamental θ and the actions of others, rather than scaled versions of these target variables.[8] The asymmetry in the payoff specification arises because players balance the fundamental and various coordination motives (one for each other player) differently. This allows for two sources of asymmetry. Firstly, Γ represents a directed network with weighted links (its elements are not restricted to take values in $\{0, 1\}$) and it need not be symmetric (the influence of player m' on m need not match that of m on m'). Secondly, the aggregate influence of others' actions is not identical across players: $\beta_m \neq \beta_{m'}$, in general. These parameters (representing the aggregate coordination motive) can be either positive or negative (and so actions can be either strategic substitutes or complements).

A feature of the specification (1) is that the coefficients applied to the quadratic-loss terms sum to one: $1 - \beta_m + \beta_m \sum_{m' \neq m} \gamma_{mm'} = 1$. This is not restrictive: a player's payoff can be scaled up or down (so scaling the coefficients) until this equality holds. Indeed, all results on information use apply even if this sum-to-one equality is dropped. It does matter, however, for the analysis of costly information acquisition, which is described just below: scaling players' payoffs would also scale up or down information-aquisition costs, and such costs are assumed (again just below) to be symmetric across players.

1.4. **Information.** The information structure follows closely that introduced (to political science) by Dewan and Myatt (2008) and (to economics) by Myatt and Wallace (2012).

Players do not know θ, but share a common prior that $\theta \sim \mathrm{N}(x_0, \kappa_0^2)$. Many of the results reported here focus, without loss of generality and for expositional simplicity, on the improper prior limit where $\kappa_0^2 \to \infty$.[9] Players have access to n sources of information about θ. Each player receives a signal of θ from information source $i \in \{1, \ldots, n\}$, where

$$\text{Signal } i \text{ received by player } m = x_{im} = \theta + \eta_i + \varepsilon_{im} \tag{4}$$

[6](3) may be rewritten in matrix notation: $\mathbf{a} = (\mathrm{I} - \bar{\Gamma})\theta\mathbf{1} + \bar{\Gamma}\mathbf{a}$, where $\bar{\Gamma} = \mathrm{diag}[\boldsymbol{\beta}]\Gamma$ and where $\mathbf{a} = (a_1, \ldots, a_M)'$ is the vector of players' actions, $\mathbf{1}$ is the $M \times 1$ vector of 1s, and I is the $M \times M$ identity matrix. $\mathbf{a} = \theta\mathbf{1}$ if $(\mathrm{I} - \bar{\Gamma})$ is invertible. $|\beta_m| < 1$ is sufficient; it ensures that the strategic complementarity (or substitutability) of actions does not overwhelm the incentive to take an action close to the fundamental.

[7]The specification (1) is a variant of the payoffs found in Ballester, Calvó-Armengol, and Zenou (2006). In that paper, for the setting described here, equilibrium actions are proportional to weighted Bonacich centralities (see their Remark 1, p. 1409). The formulation of (1) exactly counteracts the centrality of the player, so that all players choose the same action. The purpose of this paper is to understand how such variations in network position affect the use and acquisition of information. Section B.1 in Appendix B explores more fully the relationship between the specification here and that of their paper.

[8]This contrasts with a recent paper by Myatt and Wallace (2018) which studies price competition between suppliers with asymmetrically sized portfolios of differentiated products. In that paper, the actions are prices and the fundamental state of the world θ is a common demand shifter. This applied environment generates a payoff structure equivalent to the one here, but where different players apply different scaling factors to θ (for example: larger suppliers seek to set prices that respond more strongly to demand conditions) and so the equilibrium is no longer symmetric in a full-information world.

[9]A prior $\theta \sim \mathrm{N}(x_0, \kappa_0^2)$ is equivalent to adding an $(n+1)$st signal $i = 0$ with parameters κ_0^2 and $\xi_0^2 = 0$.

and where the noise terms are all independent. η_i is a source of noise common across players with $\eta_i \sim \mathrm{N}(0, \kappa_i^2)$. The associated precision $1/\kappa_i^2$ is the "accuracy" of the information source. It represents noise inherent in the source itself, perhaps attributable to errors made when the signal is "sent". ε_{im}, on the other hand, is an idiosyncratic noise component, attributable to errors made by the "receiver" of the signal. The associated precision may be (to some extent at least) under the control of the player. Assume that

$$\varepsilon_{im} \sim \mathrm{N}(0, \xi_{im}^2) \quad \text{where} \quad \xi_{im}^2 = \frac{\xi_i^2}{z_{im}}.$$

The precision $1/\xi_i^2$ is the underlying "clarity" of information source i. z_{im} measures the (costly) attention player m pays to signal i. Two different specifications are considered.

Firstly, player i might simply receive each signal (free of charge). Setting $z_{im} \equiv 1$ for all i and m (more generally, fixing z_{im}) each signal is characterized by its accuracy and clarity or, equivalently, by its overall precision ψ_i and correlation across players ρ_i, where

$$\psi_i = \frac{1}{\kappa_i^2 + \xi_i^2} \quad \text{and} \quad \rho_i = \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2}.$$

More correlated signals are more public (if $\rho_i = 0$ observations are independent; if $\rho_i = 1$, then they are common) and so ρ_i indexes a signal's "publicity". The focus in this specification is on information use: how different signals (characterized by ψ_i and ρ_i, or equivalently κ_i^2 and ξ_i^2) influence the actions taken by differently positioned players.

Secondly, $z_{im} \geq 0$ might be a choice variable for player m. Prior to choosing an action (conditional on received information), player m chooses how much attention to pay to signal i. $z_{im} = 0$ is interpreted as ignoring the signal altogether, and results in a completely uninformative (infinite variance) realization of x_{im}. Attention is costly: let

$$C_m(z_{1m}, \ldots, z_{nm}) = \sum\nolimits_{i=1}^{n} z_{im} \quad \text{for all } m \tag{5}$$

be that linear cost which is deducted from u_m. This admits a sampling interpretation discussed by Myatt and Wallace (2015, p. 483) and justified formally by Han and Sangiorgi (2018). Pragmatically, the linearity admits explicit solutions for information acquisition.

Under this specification, the information sources are equally costly. However, the attention paid to a source enters into the signal structure only via the relationship $\xi_{im}^2 = \xi_i^2/z_{im}$. Scaling up or down the cost of acquiring any particular information source i is equivalent to scaling up or down the clarity parameter ξ_i^2. Hence, treating all information sources as equally costly is without of generality. In essence, z_{im} is to be interpreted as the expenditure on signal i by player m. With this interpretation, clearer signals (lower ξ_i^2) correspond to cheaper information sources.

Note that the parameters of information acquisition, such as ξ_i^2 or equivalently the marginal cost of attention, do not vary across the player set. By design, the information technology is exogenously symmetric; and so any asymmetries arise endogenously via players' different choices of attention z_{im}. This means that the exclusive sources of asymmetry arise from the specification of u_m in (1). This rules out situations in which some players find it particularly easy to observe specific information sources.[10]

The focus for this specification is information acquisition, particularly on which different signals are acquired by differently positioned players on the network. Note that the correlation (and precision) of each signal i is determined endogenously in this latter setting: each signal's publicity can differ across players and is an equilibrium phenomenon.

2. Information Use, Information Acquisition, and Centrality

This section characterizes equilibrium information use first by abstracting from the acquisition problem (so, setting $z_{im} \equiv 1$ for all i and m) and then by solving subsequently for equilibrium information acquisition and use when z_{im} is chosen optimally for each i.

2.1. **Information Use with Exogenous Signals.** The (Bayesian Nash) equilibrium considered is linear in signal realizations.[11] In particular, consider the affine strategies

$$a_m = w_{0m}x_0 + \sum\nolimits_{i=1}^{n} w_{im}x_{im},$$

where w_{im} is the weight that player m places on signal i, and w_{0m} is the weight on the prior.[12] Substituting these strategies into (1), and using x_{im} from (4), the expected payoff $\mathrm{E}[u_m] = \text{constant} - (1-\beta_m)\,\mathrm{E}[(a_m-\theta)^2] - \beta_m \sum_{m'\neq m}\gamma_{mm'}\,\mathrm{E}[(a_m - a'_m)^2]$ can be readily calculated in terms of the weights w_{im}; the proof of Lemma 1 reports the full expression. This generates a game in which each player m chooses $n+1$ weights to maximize $\mathrm{E}[u_m]$.

Concavity of $\mathrm{E}[u_m]$ is guaranteed by the assumption $|\beta_m| < 1$. Differentiating with respect to the weight placed on the prior mean yields M equations

$$(1-\beta_m)\left(\sum\nolimits_{i=0}^{n} w_{im} - 1\right) + \beta_m \sum\nolimits_{m'\neq m}\gamma_{mm'}\left(\sum\nolimits_{i=0}^{n}(w_{im} - w_{im'})\right) = 0,$$

which hold if and only if $\sum_{i=0}^{n} w_{im} = 1$ for all m: each player's action choice is a weighted average of the player's signal realizations and the prior mean.

Turning to the use of those signals, the $n \times M$ first-order conditions are

$$w_{jm}(\kappa_j^2 + \xi_{jm}^2) - \beta_m \sum\nolimits_{m'\neq m}\gamma_{mm'}w_{jm'}\kappa_j^2 = c_m,$$

[10]A more general cost function $C_m(z_m) = \sum_{i=1}^{n}\zeta_{im}z_{im}$ allows the marginal cost ζ_{im} of attention paid to source i to vary across players. Now consider a situation with three information sources in which players m and m' satisfy $\zeta_{im} = \zeta_{im'}$, $\zeta_{jm} < \zeta_{jm'}$, and $\zeta_{km} > \zeta_{km'}$. This can be interpreted as a situation in which source j is delivered in the native language of player m, source k is delivered in the native language of player m', and source i is some neutral lingua franca. In this situation, source i could play a role by giving a signal of common clarity for the players. Such a situation is ruled out by the specification here, and is left for future work. The authors thank a referee for suggesting such a situation as a possible interpretation of an information source with common clarity for different players.

[11]This is without much loss of generality. In the model of Dewan and Myatt (2008), any equilibrium involving strategies which are bounded above and below by linear strategies is itself linear.

[12]For $x_0 \neq 0$ any further additive constant term in this affine strategy can be captured via $w_{0m}x_0$. If $x_0 = 0$ then the constant can be re-instated, with no substantive changes to the results.

where c_m is a player-specific constant. Lemma 1 uses the precision and correlation notation (thereby assuming $z_{im} \equiv 1$ and hence $\xi_{im}^2 = \xi_i^2$ for all i and m).

Lemma 1 (Characterization)**.** *The unique linear equilibrium satisfies* $\sum_{i=0}^{n} w_{im} = 1$ *for each* m. *The weight each player* m *places on information source* i *satisfies*

$$w_{im} = \beta_m \sum\nolimits_{m' \neq m} \gamma_{mm'} w_{im'} \rho_i + c_m \psi_i, \tag{6}$$

where c_m *is an (equilibrium determined) player-specific constant. If the prior is diffuse then all weight is placed on the signals:* $\lim_{\kappa_0^2 \to \infty} w_{0m} = 0$ *and so* $\lim_{\kappa_0^2 \to \infty} \sum_{i=1}^{n} w_{im} = 1$.

Allowing the prior to become diffuse (so that $\kappa_0^2 \to \infty$) the weight on the prior mean must fall to zero (since otherwise a player's payoff would diverge) and so, with such a diffuse prior, a player's action is a weighted average of the n signals.

For expositional simplicity, the paper now proceeds with a diffuse prior, so $\sum_{i=1}^{n} w_{im} = 1$. This is without loss of generality: a proper prior is equivalent (from the perspective of players) to an additional $(n+1)$st signal $i = 0$ with $\xi_i^2 = 0$.

Rewriting the conditions of (6) in the vector notation of the previous section provides some general insight into how information is used by networked players. Define

$$\mathbf{w}_i \equiv (w_{i1}, \ldots, w_{iM})' \quad \text{and} \quad \mathbf{c} \equiv (c_1, \ldots, c_M)' \quad \Rightarrow \quad \mathbf{w}_i = \rho_i \bar{\Gamma} \mathbf{w}_i + \psi_i \mathbf{c}.$$

Since $\rho_i \leq 1$ for all i, $|\beta_m| < 1$ is sufficient for the inverse $(\mathrm{I} - \rho_i \bar{\Gamma})^{-1}$ to exist. Using the equality $\sum_{i=1}^{n} \mathbf{w}_i = \mathbf{1}$ to solve for c generates the following proposition's characterization of equilibrium weights (see Appendix A for all proofs).

Proposition 1 (Equilibrium Information Use)**.** *There is a unique linear equilibrium in which the vector of weights players place on their observations of signal* i *satisfies*

$$\mathbf{w}_i = \psi_i [\mathrm{I} - \rho_i \bar{\Gamma}]^{-1} \left[\sum\nolimits_{j=1}^{n} \psi_j [\mathrm{I} - \rho_j \bar{\Gamma}]^{-1} \right]^{-1} \mathbf{1}.$$

Looking across the player set, the use of a signal is proportional to a player's Bonacich centrality with decay parameter ρ_i. The use of a more public signal (with a higher correlation coefficient) decays more slowly through the network.[13] The influence of a signal is increasing in its publicity when the game is one of strategic complements (for instance, when the elements of $\bar{\Gamma}$ are all strictly positive). This effect is compounded for players who are the most centrally influenced by the actions of others.

As an illustration of the results so far, consider the example of Figure 1, and set $\gamma = \frac{1}{2}$ to make the algebra tractable. Applying Proposition 1, the weights are

$$w_{i1} = \frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \quad \text{and} \quad w_{i2} = w_{i3} = \frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \frac{1 + \rho_i \beta/2 - \beta \bar{\rho}}{1 - \rho_i \beta/2},$$

[13]Note that $\mathrm{A}^{-1} = \sum_{k=0}^{\infty} (\mathrm{I} - \mathrm{A})^k$ for any invertible matrix A, where $\mathrm{A}^0 \equiv \mathrm{I}$. Using $\mathrm{A} = \mathrm{I} - \rho_j \bar{\Gamma}$ and re-substituting for the constants c, the weights may be written $\mathbf{w}_i = \psi_i [\mathrm{I} - \rho_i \bar{\Gamma}]^{-1} \mathbf{c} = \psi_i \sum_{k=0}^{\infty} (\rho_i)^k \bar{\Gamma}^k \mathbf{c}$. Now $\bar{\Gamma}^k$ captures the influence of the weights chosen by all k-distant players on the network. Thus the influence of others' use of signal i decays through the network more slowly the higher is ρ_i.

where the constant $\bar{\rho}$ is a weighted average of the ρ_is:

$$\bar{\rho} = \sum_{j=1}^{n} \frac{\psi_j \rho_j}{1-\rho_j\beta/2} \Big/ \sum_{j=1}^{n} \frac{\psi_j}{1-\rho_j\beta/2}.$$

Player 1 uses precision weighting. Players 2 and 3 (these players are symmetric given that $\gamma = \frac{1}{2}$) care about the signal's publicity. In particular, $w_{i2} = w_{i3} > w_{i1}$ if and only if $\rho_i > \bar{\rho}$, so that the ith signal is more public than average.

2.2. **Endogenous Information Acquisition and Use.** With endogenous information acquisition, each player jointly chooses an acquisition policy $z_m \in \mathcal{R}_+^n$ and (focusing on strategies for which actions are linear in signal realizations) the weights to place on the signals. Setting $\xi_{im}^2 = \xi_i^2/z_{im}$, for $z_{im} > 0$, the first-order condition for w_{im} is

$$w_{im}\left(\kappa_i^2 + \frac{\xi_i^2}{z_{im}}\right) - \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{im'} \kappa_i^2 = c_m,$$

where c_m is again a player-specific constant.[14] The first-order condition for z_{im} is simply $w_{im}^2 \xi_i^2 / z_{im}^2 = 1$ (again, when it's positive). Rearranging yields an analogue to Lemma 1.

Lemma 2 (Equilibrium Properties)**.** *There is a unique linear equilibrium in which the weight player m places on information source i satisfies*

$$w_{im} = \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{im'} + \frac{c_m - \xi_i}{\kappa_i^2}, \quad \textit{and} \quad z_{im} = \xi_i w_{im} \tag{7}$$

is the attention paid to i, for all i such that $w_{im} > 0$ (equivalently $z_{im} > 0$); $w_{jm} = z_{jm} = 0$ otherwise. Here, c_m is an (equilibrium determined) player-specific constant.

The expressions in (7) may be applied directly, and are useful for the two settings discussed in Sections 4 and 5. In general, different players may listen to different sets of signals so that $z_{im} = 0$ but $z_{im'} > 0$ for some i and $m \neq m'$. Indeed, this will be the case for many interesting examples. However, a particularly clean result is available when all the players listen to the same (possibly strict) subset of the n signals.

To this end, suppose that $z_{im} > 0 \Leftrightarrow z_{im'} > 0$ for all i and $m \neq m'$, so that all players listen to precisely the same set of signals. Define $N_\star = \{i : z_{im} > 0 \text{ for all } m\}$: the non-empty subset of $\{1, \ldots, n\}$ containing all the signals that receive positive attention. For all $i \in N_\star$ the first-order conditions in (7) hold, and $\sum_{i \in N_\star} w_{im} = 1$ for all m.

A special case is when all players listen to all signals. A sufficient condition for this is that no player is too central, or that the least clear signal is not too unclear.[15] Given that clarity can be re-interpreted as the cost of an information source (as discussed

[14] This is a slight abuse of notation: c_m differs in general from the constant identified in Section 2.1. However, it is convenient for expositional purposes to use the same symbol for these constants.

[15] Precisely, $z_{im} > 0$ for all i and m (so that all sources receive positive attention) if

$$\max_m b_m < \left[\sum_{i=1}^{n} \frac{\xi_{\max} - \xi_i}{\kappa_i^2}\right]^{-1} \tag{8}$$

where b_m is the (unweighted) centrality of m; the mth element of the vector $[\mathrm{I} - \bar{\Gamma}]^{-1}\mathbf{1}$, and $\xi_{\max} = \max_i \xi_i$.

toward the end of Section 1.4) this holds if the most costly information source is not too expensive. Scaling down the costs of all sources is sufficient to achieve this.

Proposition 2 applies more broadly when information sources are partitioned into those that are universally acquired and those that are universally ignored.[16]

Proposition 2 (Equilibrium Information Acquisition)**.** *Suppose that, in equilibrium, any signal that is acquired by some player is acquired by everyone. The signals acquired are from the clearest (lowest ξ_i) sources. The weight placed on i by a player m is higher than its relative accuracy if and only if the information source is clearer than average:*

$$\mathbf{w}_i = \frac{1}{\kappa_i^2}\left\{\frac{1}{\sum_{j\in N_\star} 1/\kappa_j^2}\mathbf{1} - (\xi_i - \bar{\xi}_\star)[\mathrm{I} - \bar{\Gamma}]^{-1}\mathbf{1}\right\}, \quad \textit{where} \quad \bar{\xi}_\star = \frac{\sum_{j\in N_\star} \xi_j/\kappa_j^2}{\sum_{j\in N_\star} 1/\kappa_j^2}, \tag{9}$$

for all $i \in N_\star$. The weight's deviation from the signal's relative accuracy is proportional to the product of the difference between signal i's clarity and the average clarity of all the acquired signals and the player's unweighted Bonacich centrality.

To understand this proposition, consider $\xi_i = \bar{\xi}_\star$ for all i. Applying the solution in the proposition, $w_{im} \propto 1/\kappa_i^2$ so that play is symmetric and all players use their signals in proportion to the underlying accuracy of the information source. Given that they do so, the optimality of information acquisition from (7) implies

$$z_{im} = \xi_i w_{im} = \bar{\xi}_\star w_{im} \propto \frac{1}{\kappa_i^2} \quad \Rightarrow \quad \frac{1}{\kappa_i^2 + (\xi_i^2/z_{im})} \propto \frac{1}{\kappa_i^2},$$

and, moreover, all signals share the same correlation coefficient. With equally clear information sources, signal precisions are (endogenously) proportional to underlying information accuracies, and all signals are equally public.[17] This reinforces the use of information in proportion to the underlying accuracy of the corresponding source.

Now consider $\xi_i < \xi_j$. Beginning with a situation in which signals are accuracy-weighted, less attention is devoted to the clearer signal simply because it is easier to understand. Nevertheless, the overall (endogenous) clarity of the message from source i is now relatively greater than from source j. It is optimal to place more emphasis on source i. This explains the presence of the term $-(\xi_i - \bar{\xi}_\star)$ in the solution reported (9).

The term $-(\xi_i - \bar{\xi}_\star)$ is multiplied by the vector of Bonacich centralities $[\mathrm{I} - \bar{\Gamma}]^{-1}\mathbf{1}$, which says that the effect of greater relative clarity is amplified for more central players. The

[16]The symmetric case (see Section 3) satisfies the conditions of Proposition 2; $\max_m b_m$ can be replaced with $1/(1-\beta)$ in the condition (8). Equivalently: the coordination motive is not too large.

[17]Clearer signals are endogenously more public in the sense of having higher correlation coefficients in equilibrium. Note that the correlation between the observation of source i by m and m' is

$$\rho_{imm'} = \kappa_i^2\left[\left(\kappa_i^2 + \frac{\xi_i^2}{z_{im}}\right)\left(\kappa_i^2 + \frac{\xi_i^2}{z_{im'}}\right)\right]^{-\frac{1}{2}}.$$

In the equilibrium described in this section by (7), and in those to follow, $z_{im} = \xi_i w_{im}$ when positive. But, from (9), $w_{im} = f_m(\xi_i)/\kappa_i^2$ when positive, where f_m is a decreasing (player-specific) function of ξ_i. It is straightforward to check that $\rho_{imm'} > \rho_{jmm'} \Leftrightarrow \xi_i < \xi_j$ if both m and m' acquire i and j. If either m or m' does not acquire some i, then $\rho_{imm'} = 0$. As will be seen throughout, players acquire a subset consisting of the most clear signals. So, in equilibrium, the clearer the signal the more endogenously public it is.

reason is that clearer sources are more public, and an increase in centrality shifts weight toward more public (and so clearer) information sources. Naturally, there are equilibrium considerations too: if others shift toward the clearer source then those who wish to coordinate with them (a desire which is captured by $\bar{\Gamma}$) face an enhanced incentive to place more weight on and devote more attention to the clearer information. This logic underpins the solution reported in Proposition 2. That solution applies whether the action choices of players are strategic complements ($\beta_m > 0$, in which case the centralities are larger and the emphasis on clearer sources is stronger) or are strategic substitutes ($\beta_m < 0$, in which case the effect is weaker). An illustration is provided for players with symmetric coordination motives ($\beta_m = \beta$ for all m) in Proposition 5 below.

2.3. **Total Information Acquisition.** Beyond the weights attached to the various signals in use, total information acquisition (measured by $Z_m = \sum_{i=1}^{n} z_{im}$, and so corresponding to total cost paid for the information acquired) is amenable to analysis. Noting that $z_{im} > 0$ only if $i \in N_\star$, and using the first-order condition for such z_{im} in (7), premultiply (9) by ξ_i and sum over $i \in N_\star$. For every i, defining the M-dimensional vector

$$\mathbf{z}_i \equiv (z_{j1}, \ldots, z_{jM})', \quad \text{and hence} \quad \mathbf{Z} \equiv \sum\nolimits_{i=1}^{n} \mathbf{z}_i = (Z_1, \ldots, Z_M)',$$

yields immediately the last proposition of this section.

Proposition 3 (Total Information Acquisition)**.** *Suppose that, in equilibrium, any signal that is acquired by some player is acquired by everyone. Then, player m's total information acquisition is decreasing in the Bonacich centrality of that player. In fact,*

$$\mathbf{Z} = \bar{\xi}_\star \mathbf{1} - [\mathrm{I} - \bar{\Gamma}]^{-1} \mathbf{1} \sum\nolimits_{j \in N_\star} \frac{(\xi_j - \bar{\xi}_\star)^2}{\kappa_j^2}. \tag{10}$$

Consider a game with strategic complementarities (every element of $\bar{\Gamma}$ is positive). Referring to Proposition 2, and looking across the signals in positive use, the clearer a signal i (the lower ξ_i) the more weight is attached to it. Indeed, signals that are clearer than average (as measured by $\bar{\xi}_\star$) are acquired and used more than implied by their relative accuracy (as measured by $1/\kappa_i^2 / \sum_{j \in N_\star} 1/\kappa_j^2$). This effect is compounded by the player's position in the network: a player who is more central departs more from using signals according to their relative accuracy than one who is less central. More central players favour relatively clear (endogenously relatively public) information sources.

On the other hand, Proposition 3 says that more central players spend relatively little on information acquisition. They are influenced more by others and so they place more importance on coordination. This rebalances their use of information (meaning the influence of a signal on a player's action choice) toward more public information sources. Such public information sources are those that are clearer. A player faces a stronger incentive to improve the precision of signals that are used more. For a central player, the heavily used signals are clearer and so are (equivalently) cheaper to acquire; less costly attention is required to achieve any particular precision of observation. The focus on sources that are less costly reduces central players' total expenditure on costly information acquisition. Note that this does not say that they acquire less information, but instead simply says that they spend less by using the cheaper sources.

2.4. **Corner Solutions in a Three-Player Example.** A feature of the equilibrium characterized by (7) is that in general a signal i may be acquired (and used) by player m, but not by another player m'. The features of asymmetry in network position that would drive such behaviours are purposefully ignored in the above. To understand how and why different players might use different information (and what features of those sources determine which signals get acquired), two important network structures are explored in Sections 4 and 5. Before moving on to these cases, some insight may be gained from the three-player example of Figure 1.

For this three-player example, consider an environment with two information sources, and order those sources such that $\xi_1 < \xi_2$. For $\gamma < \frac{1}{2}$, the centralities of the players satisfy $b_3 > b_2 > b_1$ (see (2) and Figure 1). Under some parameter configurations (for example, if $\xi_2 - \xi_1$ is not too large) all three players use both information sources. Applying Proposition 2, the use of the less-clear signal by the moderately central player is

$$w_{22} = \hat{w}_{22} \quad \text{where} \quad \hat{w}_{22} = \frac{1}{\kappa_2^2}\left[\frac{1}{\sum_{i=1}^{2} 1/\kappa_i^2} - b_2(\xi_2 - \bar{\xi})\right] \quad \text{and} \quad \bar{\xi} = \frac{\sum_{j=1}^{2} \xi_j/\kappa_j^2}{\sum_{j=1}^{2} 1/\kappa_j^2}.$$

If this second information source becomes even less clear, so that ξ_2 rises, then eventually the most central player 3 stops acquiring and using it altogether. In fact, if

$$b_2 < \left[\frac{\xi_2 - \xi_1}{\kappa_1^2}\right]^{-1} < b_3$$

then $w_{23} = 0$ (player 3 uses only the first signal) but $w_{22} > 0$ and $w_{21} > 0$.[18] However, the explicit solution for w_{22} (other information-use coefficients are in Appendix B) becomes

$$w_{22} = \frac{1}{\kappa_2^2}\left[\frac{1}{\sum_{i=1}^{2} 1/\kappa_i^2} - \frac{1 - \beta\gamma + \beta}{1 - \beta\gamma}(\xi_2 - \bar{\xi})\right](1 - \beta\gamma) < \hat{w}_{22}.$$

For this illustrative corner-solution case, where one player ceases to acquire (and therefore use) a signal, other (less central) players reduce their use of that signal away from the centrality-driven solution of Proposition 2. Further analyses of corner solutions of this type (in which certain information is only acquired by a subset of players) are studied in Sections 4 and 5. Nevertheless, there is a class of asymmetric games for which such corner solutions do not apply. These are studied next.

3. A Symmetric Equilibrium in an Asymmetric Game

The general structure (captured by $\bar{\Gamma}$) admits a great deal of asymmetry. Typically, therefore, information use and acquisition differ across players. However, there is an important class of asymmetric networks for which the equilibrium is symmetric.

[18] If this holds then the sufficient condition of (8) reported in Footnote 15 fails.

3.1. **A Symmetric Benchmark.** Suppose that players care equally about coordination: $\beta_m = \beta$ for all m, and so $\bar{\Gamma} = \beta\Gamma$. No assumption is made on the connections $\gamma_{mm'}$ and so very asymmetric networks are permitted. Nonetheless, players share the same Bonacich centralities. Explicitly, an application of the centrality definition yields:

$$\mathbf{b} = \sum\nolimits_{k=0}^{\infty} \rho^k \bar{\Gamma}^k \mathbf{1} = \sum\nolimits_{k=0}^{\infty} (\beta\rho)^k \Gamma^k \mathbf{1} = \left(\sum\nolimits_{k=0}^{\infty} (\beta\rho)^k\right) \mathbf{1} = (1/(1-\rho\beta))\mathbf{1}.$$

A rough intuition here is this. Given that aggregate coordination motives do not vary, there is no direct reason for players to behave differently. If a player expects others to behave symmetrically, then the identities of others with whom a player wishes to coordinate (determined by Γ) does not matter. This suggests that the equilibrium is symmetric, which is tied to the property that players share the same Bonacich centralities.

To establish this symmetry property formally, and focusing on information use (acquisition is considered below), insert the symmetric weights into (6):

$$w_i = \beta \sum\nolimits_{m' \neq m} \gamma_{mm'} w_i \rho_i + c_m \psi_i = \beta \rho_i w_i + c_m \psi_i \quad \Rightarrow \quad c_m = c \quad \forall m \quad \Rightarrow \quad w_i = \frac{c\psi_i}{1-\beta\rho_i}.$$

c can be solved by summing these weights across i, and using the equality $\sum_{i=1}^n w_i = 1$. The following proposition summarizes these facts using the clarity-accuracy notation.

Proposition 4 (Information Use and Symmetric Coordination Motives)**.** *If players share the same aggregate coordination motive, so that $\beta_m = \beta$ for all m,*

$$w_{im} = w_i \quad \forall m \quad \textit{where} \quad w_i = \frac{1}{(1-\beta)\kappa_i^2 + \xi_i^2} \Big/ \sum\nolimits_{j=1}^{n} \frac{1}{(1-\beta)\kappa_j^2 + \xi_j^2}. \tag{11}$$

In this benchmark case, players use information in proportion to its precision-weighted publicity, a result familiar from Myatt and Wallace (2014, Proposition 1), for instance.

Now allow players to acquire information endogenously. Applying the first-order conditions in (7) from Lemma 2 and inserting $w_{im} = w_i$ and $z_{im} = z_i$ for all m and i, whenever $z_i > 0 \Leftrightarrow w_i > 0$, then

$$w_i = \beta w_i + \frac{c - \xi_i}{\kappa_i^2} \quad \text{and} \quad z_i = \xi_i w_i.$$

So, for $i \in N_\star \equiv \{j : z_j > 0\} \subseteq \{1, \dots, n\}$, equilibrium weights are given by

$$w_i = \frac{c - \xi_i}{(1-\beta)\kappa_i^2}, \quad \text{whereas for } j \notin N_\star \quad w_j = 0.$$

The constant $c_m = c$ (for all m) can be found by summing over $i \in N_\star$. Once again, using the average clarity notation from Section 2.2, $c = \bar{\xi}_\star + (1-\beta)/\sum_{i \in N_\star} 1/\kappa_i^2$.

Proposition 5 (Information Acquisition with Symmetric Coordination Motives)**.** *If players share the same aggregate coordination motive then $w_{im} = w_i$ and $z_{im} = z_i$, where*

$$w_i = \frac{1}{\kappa_i^2}\left\{\frac{1}{\sum_{j \in N_\star} 1/\kappa_j^2} - \frac{\xi_i - \bar{\xi}_\star}{1-\beta}\right\} \quad \textit{and} \quad Z = \bar{\xi}_\star - \frac{1}{1-\beta}\sum_{j \in N_\star} \frac{(\xi_j - \bar{\xi}_\star)^2}{\kappa_j^2} \tag{12}$$

for $i \in N_\star$ and $Z_m = Z$ is the total information acquisition for player m. Moreover, $N_\star = \{i : \xi_i < \bar{\xi}_\star + (1-\beta)/\sum_{j \in N_\star} 1/\kappa_j^2\}$ is uniquely defined. The players use a (possibly strict) subset of the signals, consisting of the clearest. Signals $j \notin N_\star$ are ignored: $w_j = z_j = 0$.

The parallels between these results and those presented in Propositions 1–3 are plain. The earlier propositions may be applied directly to obtain (11) and (12). Using the expression for the inverse of $(\mathrm{I}-\bar{\Gamma})$ derived from the discussions in Footnote 13,

$$(\mathrm{I}-\bar{\Gamma})^{-1}\mathbf{1} = \sum\nolimits_{k=0}^{\infty}\bar{\Gamma}^k\mathbf{1} = \sum\nolimits_{k=0}^{\infty}\beta^k\Gamma^k\mathbf{1} = \sum\nolimits_{k=0}^{\infty}\beta^k\mathbf{1} = \frac{1}{1-\beta}\mathbf{1}.$$

The third equality holds because Γ is a row-stochastic matrix and so $\Gamma\mathbf{1}=\mathbf{1}$. Thus (9) and (10) directly imply (12). A similar exercise can be conducted for (11). As noted above, $\beta_m=\beta$ for all m gives every player the same Bonacich centrality.

A property of the symmetric equilibrium (this is also true in the presence of asymmetries) is that the clearest (lowest ξ_i) information sources are acquired. A clearer source is equivalent to one that is relatively cheap to acquire. (A lower marginal cost in the linear cost function is equivalent to a lower value of ξ_i.) Such cheap-to-acquire sources are used even if they do not accurately reflect the state of the world (that is, if κ_i^2 is high).

3.2. **Asymmetric Coordination Motives.** A necessary condition for asymmetric behaviour is that players differ in their desire to coordinate. This section describes briefly one situation in which such differences are present.

To proceed, suppose that $\gamma_{mm'}=1/(M-1)$ for all $m\neq m'$ so that there are no asymmetries in the connections between players. However, suppose that the aggregate coordination motives of players differ: $0<\beta_1<\beta_2<\ldots<\beta_M$.[19] Given that aggregate coordination motives are the only source of asymmetry, it is unsurprising that they determine the players' centralities, which satisfy $b_1<b_2<\cdots<b_M$ for any $1\geq\rho>0$.

With this in hand, earlier results apply immediately. Proposition 2 notes that the equilibrium weight placed on an endogenously acquired signal deviates from that signal's relative accuracy according to its relative clarity and according to the relevant player's Bonacich centrality. Similarly, Proposition 3 can also be applied directly.

Corollary (to Propositions 2 and 3)**.** *Suppose that players differ only in their aggregate desire to coordinate, and consider an equilibrium in which players' acquire and use the same set of information sources. A player with a stronger coordination motive makes more use of relatively clear information, and acquires less information overall.*

4. A Core-Periphery Network

That (at least two) players are differently influenced by others in aggregate is a necessary condition for asymmetry in the network structure to feed through into asymmetric information use and acquisition across players. Equivalently, players must have different Bonacich centralities. This section and the next present general formulations for two such networks, commonly found in the literature and analytically tractable, to explore how asymmetries in centrality affect asymmetries in information use and acquisition.[20]

[19] The restriction to coordination (rather than anti-coordination) shortens proofs and speeds exposition.

[20] The three-player example of Figure 1 is a special case of both: setting $\gamma=\frac{1}{2}$ yields a simple core-periphery network, while $\gamma=0$ (or $\gamma=1$) generates a simple three-player hierarchy.

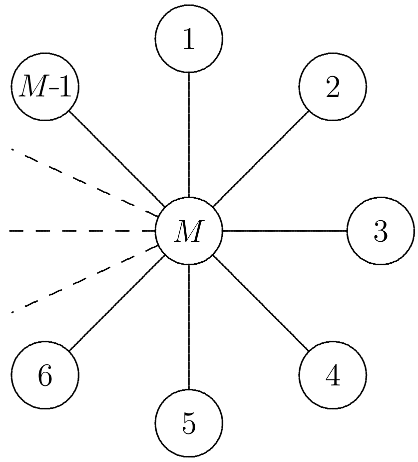

FIGURE 2. The Star Network with $M-1$ Spokes

Lines represent undirected links from m to m' with $\beta_m\gamma_{mm'} = \beta_{m'}\gamma_{m'm}$. If there is no line then $\gamma_{mm'} = 0$. Players $\{1,\ldots,M-1\}$ on the spokes care about the actions of other spoke players only through an indirect connection via the hub player $\{M\}$.

This section studies information use and acquisition across a two-type network, where the specification also allows for significant asymmetry within each group of types.

4.1. **A Two-Type Network of Players.** Partition the players into two subsets A and B of size M_A and $M_B = M - M_A$ respectively. Suppose that these subsets satisfy

$$\beta_m = \begin{cases} \beta_A & m \in A \\ \beta_B & m \in B \end{cases}, \quad \sum_{m'\in A} \gamma_{mm'} = \begin{cases} \omega_{AA} & m \in A \\ \omega_{BA} & m \in B \end{cases}, \quad \text{and} \quad \sum_{m'\in B} \gamma_{mm'} = \begin{cases} \omega_{AB} & m \in A \\ \omega_{BB} & m \in B \end{cases}.$$

These requirements say that two members of a group share the same aggregate concern for coordination with groups A and B respectively. For example, if players m and m' are both members of group A, then $\sum_{m''\in B}\beta_m\gamma_{mm''} = \sum_{m''\in B}\beta_{m'}\gamma_{m'm''}$. Similar claims apply when referring to intra-group coordination with other members of A. Furthermore, members of each group share the same aggregate coordination motive. Nevertheless, the opportunity for further substantial asymmetry remains.

This definition encompasses many important network structures. For instance, core-periphery networks fall under this specification, as do, therefore, star networks.

For a star network, suppose that player M is the hub player, connected to all other $M-1$ players, who in turn are connected only to player M. The usual specification has $\beta_m\gamma_{mm'} = \beta_{m'}\gamma_{m'm}$ if m and m' are connected, and $\gamma_{mm'} = 0$ otherwise. This fits the definition above with $A = \{1,\ldots,M-1\}$, $B = \{M\}$, and with aggregate coordination motives satisfying $\beta_B = (M-1)\beta_A$. Figure 2 illustrates such a specification.

However, the definition is broader than that: general core-periphery networks fit the construction here, as do many other network forms.[21] The key advantage of networks

[21]See Goyal (2007, Chapter 4, p. 80) for an example with $M_A = 8$ players on the periphery, $M_B = 4$ in the core, and $\beta_m\gamma_{mm'} = \beta_{m'}\gamma_{m'm}$ for connected players. The "windmill" networks of Dziubiński and Goyal (2017, p. 345) fall into this definition (at least, those with the same number of players in each clique do).

with this two-type structure is that solving for the weights (on signals) essentially boils down to inverting a 2×2 matrix, for which an explicit solution is available.

4.2. **Information Use.** Players' information use is symmetric within each group. To see why, consider a strategy profile that satisfies intra-group symmetry. Now consider (for example) a member of group A. This player notes that all members of group B act in the same away. Thus, the desire to coordinate with all of them according to the relevant row of the adjacency matrix $\bar{\Gamma}$ is equivalent to placing weight $\beta_A \omega_{AB}$ on one representative member of group B; the same is true when thinking about co-members of group A. From this, it follows that all members of A will choose best replies symmetrically.

Given that this is the case, the coordination motives of the M players within $\bar{\Gamma}$ can be summarized via the much simpler 2×2 adjacency matrix $\bar{\Omega}$ where

$$\bar{\Omega} \equiv \begin{bmatrix} \beta_A \omega_{AA} & \beta_A \omega_{AB} \\ \beta_B \omega_{BA} & \beta_B \omega_{BB} \end{bmatrix}.$$

Hence $(\mathrm{I} - \rho \bar{\Omega})^{-1}\mathbf{1}$ reports the Bonacich centralities of the two player groups.

The relative use of (exogenously provided) information by members of the two groups is determined by how public each signal is: whether one group rather than the other makes more use of a signal depends upon whether the correlation coefficient of that signal exceeds a critical value. The proof of Proposition 6 identifies this critical value $\hat{\rho}$.

Proposition 6 (Relative Information Use by the Player Types)**.** *If players in B care more about coordination than players in A, so that $\beta_B > \beta_A$, then players in B place more weight on a signal if and only if it is relatively public: $w_{iA} < w_{iB} \Leftrightarrow \rho_i > \hat{\rho}$ for some $\hat{\rho}$.*

The intuition is as before: relatively central players are those with stronger coordination motives, and they find that relatively public information is more useful for coordination because correlated signals reveal more about the actions of others.

4.3. **Information Acquisition.** The intuition above carries over to the case when players choose which signals (and how much of each) to acquire. Indeed, it is reinforced and compounded by the endogenous acquisition decisions made by the players.

A first observation is that either players in A acquire a subset of those signals acquired by players in B or vice versa. An examination of the weight given to each signal i which is acquired in (9) provides some general intuition. Take the most central player. For this player ξ_i is sufficiently small such that the term inside the brackets in (9) is positive. Thus, it must be positive for all other players. Essentially, if the most central player uses a signal, so does everyone else. Of course, this argument ignores the fact that the equilibrium conditions in (9) apply only when every player acquires the same set of signals. However, the broad intuition carries over to the two-type setting.

Define the set of signals acquired in equilibrium by players in A and B respectively as $N_A = \{i : z_{iA} > 0\}$ and $N_B = \{i : z_{iB} > 0\}$, where $z_{iA} = z_{im}$ for $m \in A$ and similarly for z_{iB}. Similarly, define total acquisition as $Z_A = Z_m$ for $m \in A$ and $Z_B = Z_m$ for $m \in B$.

Proposition 7 (Information Acquisition in a Two-Type Network)**.** *Consider a two-type network with $\beta_B \geq \beta_A$: members of B care more about coordination than members of A.*

(i) Players in B acquire a (possibly weak) subset of the signals acquired by players in A: $N_B \subseteq N_A$. This subset consists of the clearest (lowest ξ_i) signals in N_A.

(ii) Players in B place more weight on relatively clear signals ($w_{iA} \leq w_{iB}$ if and only if ξ_i is sufficiently small). If $N_B = N_A$ then $w_{iA} \leq w_{iB} \Leftrightarrow \xi_i \leq \bar{\xi}$ where $\bar{\xi}$ is the accuracy-weighted average of ξ_i across the (common) set of acquired information sources.

(iii) Players in B acquire less information than players in A: $Z_A \geq Z_B$.

Not only do more central players place relatively high weight on relatively public signals, but they will also ignore entirely signals which are insufficiently clear. They do so even in circumstances when other players on the network pay attention to such (relatively private) information sources. In addition, the fact that more central players care more about the actions of others and less about the fundamental θ per se leads them not just to acquire fewer signals, but to acquire less information overall.

To see these propositions in action, recall the star network illustrated in Figure 2. Consider a simple example with just $n = 3$ information sources, with $\xi_1 < \xi_2 < \xi_3$, so that information sources are ordered by their clarity: 1 is the clearest. Suppose further that $\xi_3 > \xi_1 + (1 - \beta_A)\kappa_1^2$. This is sufficient for neither players on the spokes ($m \in A$) nor the hub player ($m \in B \equiv \{M\}$) to acquire a signal from source 3. Clarity determines whether a signal is acquired, and in this case source 3 is insufficiently clear for acquisition.

Suppose, on the other hand, that $\xi_2 < \xi_1 + (1 - \beta_A)\kappa_1^2$. Then certainly players in the spokes will acquire a signal from the second source. Proposition 7 can be applied: $\beta_B = (M-1) \times \beta_A$ and so $N_B \subseteq N_A$. Whether the signals acquired by the hub player constitute a strict subset of those acquired by the spoke players or not depends critically upon M. In particular, if M is sufficiently small, so that the hub player is "not too central" then $N_B = N_A$ and the players acquire the same set of signals. However, if

$$M > M^\star \equiv 1 + \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\beta_A[\beta_A \kappa_1^2 + (\xi_2 - \xi_1)]} > 2$$

then $w_{2B} = z_{2B} = 0$: the hub player M does not acquire a signal from information source 2. Instead, the hub player places all weight on the single clearest signal from source 1. In this case, the equilibrium values of the weights for spoke players in A are

$$w_{1A} = \frac{\beta_A \kappa_1^2 + \kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad w_{2A} = \frac{(1 - \beta_A)\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad \text{and} \quad w_{3A} = 0.$$

As mentioned, $w_{1B} = 1$ and $w_{2B} = w_{3B} = 0$. Thus $N_B = \{1\} \subset N_A = \{1, 2\}$. Now consider total information acquisition (or equivalently, the total cost paid for acquired information) by the different types of player. From (7), $Z_B = \xi_1$ trivially. $Z_A = \xi_1 w_{1A} + \xi_2 w_{2A}$ and so it is straightforward to verify that $Z_A > Z_B$ (so long as $w_{2A} > 0$). Spoke players acquire more information than the hub player does.

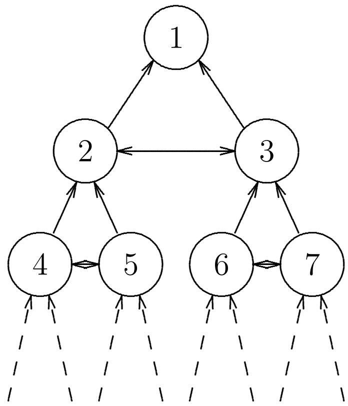

FIGURE 3. The Hierarchy Network with Two Members per Group

Arrows represent directed links from m to m' with $\gamma_{mm'} > 0$. If there is no arrow then $\gamma_{mm'} = 0$. Members of each (two player) group care about the actions of those within their group and those of a single player in the level directly above only.

5. A HIERARCHY NETWORK

This section turns attention to a hierarchy network in which most players share the same aggregate concern for coordination, but nevertheless acquire and use information differently owing to their positions within the hierarchy.

5.1. **The Hierarchy.** Suppose players are arranged in a linear hierarchy. Player 1 (at the top) does not care about coordination: $\beta_1 = 0$. Others share the same coordination motive: $\beta_m = \beta$ for all $m > 1$. Player 2 is connected to player 1 only, player 3 is connected to player 2 only, and so on: for $m > 1$, $\gamma_{mm'} = 1$ if $m' = m - 1$ and is zero otherwise. This is a directed and asymmetric network. Players "further down the chain" care more about coordination, not directly, but rather through their indirect connections to those above.

Although the results of this section will focus on the above story for simplicity, a more general network structure can be accommodated. In particular, suppose that each level in the hierarchy contains multiple players. Level $\ell \geq 2$ contains $(g+1)^{\ell-2}$ groups, each containing $g+1$ players whose payoffs depend upon the actions of all others within their group and exactly one player from the level above, $\ell - 1$. In level 1, there is a single player (player 1) who is unconnected to any other player.

A simple version is illustrated in Figure 3. Here, $g = 1$, $\gamma_{mm'} = \frac{1}{2}$ for all connections, and each group member is linked to exactly the same player in the level above. The payoff weighting attached to members of ones own group versus that attached to the player in the higher level may in general be different. For instance, set $\gamma_{mm'} = \gamma$ if m and m' are connected and in the same level and $\gamma_{mm'} = \gamma'$ if m' is the player from the level above m to whom m is linked (such that the normalization $\sum_{m' \neq m} \gamma_{mm'} = 1$ continues to hold). Else $\gamma_{mm'} = 0$. Note that it does not matter precisely to which player (or players) in level $\ell - 1$ the players from a single group in ℓ are connected: the equilibrium weights are the same for members of each level so long as the aggregate influence the actions of those above has upon them is the same. Nor will it matter precisely how many groups there are in any given level, nor their size: again, the aggregate influence the group's actions have upon each of its members is the only feature that matters.

This framework can be generalized even further with no important qualitative consequences for the results.[22] Here, then, the focus will be on a simple case where each level is identified with a single player: $g = 0$ and $\ell = m$. Note that other than player 1, who has $\beta_1 = 0$, each player $m > 1$ has coordination preference parameter $\beta_m = \beta$. In this sense, a hierarchy constitutes a minimal departure from symmetry.

5.2. **Information Use.** Applying the first-order conditions of (6) in Lemma 1,

$$w_{jm} = \beta w_{j(m-1)}\rho_j + c_m\psi_j \quad \text{for} \quad m > 1,$$

and $w_{j1} = c_1\psi_j$ for $m = 1$. Summing over j for $m = 1$ immediately yields

$$c_1 = \frac{1}{\sum_{i=1}^{n}\psi_i} \quad \text{and so} \quad w_{j1} = \frac{\psi_j}{\sum_{i=1}^{n}\psi_i} \equiv \hat{\psi}_j.$$

Player 1, entirely unaffected by those players below on the hierarchy, uses precision-weighted information. The objective is to explore information use for those players lower down the hierarchy. Players sufficiently far down the hierarchy behave as if the network was symmetric (see Proposition 4). Essentially, such players have the same centrality.

Proposition 8 (Information Use in a Hierarchy)**.** *Consider a hierarchy: (i) $\beta_1 = 0$ and (ii) for $m > 1$, $\beta_m = \beta$ and $\gamma_{mm'} = 1$ only if $m' = m - 1 \geq 1$ and is zero otherwise. Then*

$$w_{j1} = \frac{1}{\kappa_j^2 + \xi_j^2}\Big/\sum_{i=1}^{n}\frac{1}{\kappa_i^2 + \xi_i^2} \quad \textit{and} \quad \lim_{M\to\infty} w_{jM} = \frac{1}{(1-\beta)\kappa_j^2 + \xi_j^2}\Big/\sum_{i=1}^{n}\frac{1}{(1-\beta)\kappa_i^2 + \xi_i^2}$$

Player 1 uses each signal in proportion to its precision. Players far "down the chain" use weights approximately proportional to the precision-weighted publicity of each signal.

Moving down the chain of the hierarchy is equivalent to following a chain of iterative best replies, which naturally converges (further down the chain) to the equilibrium use of information in a game where all players share the same coordination motive.

5.3. **Information Acquisition.** Players within the hierarchy typically acquire different sets of signals. Without loss, order the information sources by clarity so that $\xi_1 < \xi_2 < \ldots < \xi_n$, and define $N_m = \{i : z_{im} > 0\} \subseteq \{1, \ldots, n\}$. Further, let $n_m = \max\{i \in N_m\}$. $n_m \leq n$ is the least clear signal that player m acquires and uses.

The objective is to show that $N_m = \{1, \ldots, n_m\} \subseteq N_{m-1}$ for all $m > 1$: that is, lower players in the hierarchy acquire (weakly) fewer signals than higher players, and that these consist of precisely the n_m clearest (lowest ξ_i) signals. Certainly player 1 acquires a subset consisting of the clearest signals. To see this, note that from (7), $w_{j1} = (c_1 - \xi_j)/\kappa_j^2$

[22]Appendix B.3 provides a recipe for doing so in the case where each level $\ell > 1$ contains several groups of $g+1$ players. Aside from a technicality or two, the proofs involve nothing more than a change of variables.

for any $j \in N_1$. Now c_1 is constant across i, so if for any $j > 1$, $\xi_j < c_1$ then $\xi_{j-1} < c_1$. But then $w_{j-1,1} > 0$ and hence $z_{j-1,1} > 0$. Indeed, c_1 can be directly calculated from (7) and

$$\sum\nolimits_{i \in N_1} w_{i1} = 1, \quad \text{so} \quad c_1 = \frac{1 + \sum_{i \in N_1} \xi_i / \kappa_i^2}{\sum_{i \in N_1} 1/\kappa_i^2}.$$

The proof to the next proposition, which characterizes the acquired sets of signals for each player, confirms N_1 is uniquely determined. Thus $N_1 = \{1, \ldots, n_1\}$ as required. Players "further down" the hierarchy acquire fewer signals. In fact, they acquire subsets of signals acquired by players above them, consisting of the most clear: player m chooses to acquire the clearest n_m signals only, and $n_{m+1} \leq n_m$ for all m.

Proposition 9 (Information Acquisition in a Hierarchy)**.** *For a hierarchy network:*

(i) Each player acquires a (weak) subset of the signals acquired by the player above. These are the most clear: for all $m \geq 1$, there is a unique n_m such that $w_{jm} > 0 \Leftrightarrow z_{jm} > 0$ for all $j \leq n_m$ and $w_{jm} = z_{jm} = 0$ for all $j > n_m$ with $n_{m+1} \leq n_m$.

(ii) Players lower in the hierarchy acquire (weakly) less information: $Z_{m+1} \leq Z_m$.

(iii) Player $m+1$ places more weight on signal j than player m does, so that $w_{jm+1} > w_{jm}$, and acquires more of j, so that $z_{jm+1} > z_{jm}$, if and only if j is clear enough.

(iv) If players m and $m+1$ use the same signals then player $m+1$ places more weight on signal j than player m (and acquires more of it) if and only if j is clearer than average.

Claim (i) states that players further down the chain use a weak subset of the signals used by those above. This subset can be strict, as a simple example suffices to show.

Suppose that there are $n = 3$ sources ordered in terms of their clarity $\xi_1 < \xi_2 < \xi_3$. Let

$$\xi_1 < \xi_1 + \frac{\kappa_1^2}{1+\beta} < \xi_2 < \xi_1 + \kappa_1^2 < \xi_3.$$

(Note $\beta < 1$, so this chain of inequalities is feasible.) With this example, $n_1 = 2$ and $n_m = 1$ for all $m \geq 2$. The weights on the signals are

$$w_{11} = \frac{\kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad w_{21} = \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad \text{and} \quad w_{31} = 0.$$

Furthermore, $w_{1m} = 1$ and $w_{2m} = w_{3m} = 0$ for all $m \geq 2$. The top player in the hierarchy ignores information source 3, but acquires signals from sources 1 and 2. Lower players acquire a signal from source 1 only (and trivially must place weight 1 upon it, therefore). Total acquisition, equivalently the total cost paid for information, is given by

$$Z_1 = \xi_1 \frac{\kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2} + \xi_2 \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2} \quad \text{and} \quad Z_m = \xi_1,$$

for all $m \geq 2$. A quick calculation confirms that $Z_1 > Z_m \Leftrightarrow \xi_1 < \xi_2$, as assumed. Thus, as claim (ii) of the proposition guarantees, the player at the top of the hierarchy acquires more information (equivalently, pays more for information) than those below.

This reemphasizes the main message: more central players (in this case, those further down the hierarchy) acquire less information, acquire relatively clear (or public) information, and use relatively clear (or public) information more intensively. The set of sources they acquire is a subset of those acquired by less central (higher) players.

6. Related Literature and Concluding Remarks

This paper links two strands of literature concerning quadratic-payoff games: studies of information use, after Morris and Shin (2002); and studies of coordination with networked player dependencies, following Ballester, Calvó-Armengol, and Zenou (2006).

The contribution of Morris and Shin (2002) generated a large literature investigating information use in quadratic-payoff coordination games and its welfare consequences. Such models have been applied to investment games, business cycles, oligopolies, political leadership, and financial markets (Angeletos and Pavan, 2004, 2007; Myatt and Wallace, 2014, 2015, 2018; Dewan and Myatt, 2008, 2012; Allen, Morris, and Shin, 2006). The typical setting is one in which all players receive public (perfectly correlated) and private (completely uncorrelated) signals about the fundamental.

Relative to a public-and-private specification, this paper allows for the acquisition and use of multiple information sources. The structure was introduced by Dewan and Myatt (2008, 2012), extended by Myatt and Wallace (2012), and has been applied extensively (Myatt and Wallace, 2014, 2015, 2018; Pavan, 2016; Galperti and Trevino, 2018). It allows players (at some cost) to alter both the precision and correlation properties of their information sources. Other work has restricted to binary acquire-or-not decisions (Hellwig and Veldkamp, 2009) or to choosing the precision of a single private signal (Llosa and Venkateswaran, 2013; Colombo, Femminis, and Pavan, 2014; Leister, 2017).

Almost all related papers specify symmetric players. A novelty here is that the model admits a very general class of asymmetries in players' preferences for coordination, represented by the links in a network. Ballester, Calvó-Armengol, and Zenou (2006) studied a general class of quadratic-payoff games where players' coordination preferences are described in this way.[23] In a complete information setting, players' (weighted Bonacich) centralities determine their equilibrium actions.[24] Only a very small selection of recent papers allow for some form of asymmetry when information is dispersed. Three are identified here: Myatt and Wallace (2018), Leister (2017), and Denti (2017).

Myatt and Wallace (2018) studies a price-setting oligopoly with differentiated products, uncertain linear demand conditions, asymmetrically sized firms, and the information structure used here. Their applied setting allows for players who care differently about

[23]For textbook treatments see Goyal (2007) and Jackson (2008). The structure has been extended by Belhaj, Bramoullé, and Deroïan (2014), and used widely. For example, König, Tessone, and Zenou (2014) studied the stochastic stability of equilibria in a network-formation game in which payoffs take this form. For applications to pricing in which the network reflects consumption externalities, see Fainmesser and Galeotti (2016a,b). None of these papers studies the acquisition and use of dispersed information.

[24]This connects an older literature (Katz, 1953; Bonacich, 1987) on indices of network centrality to equilibrium play in a broad class of games. This connection extends to situations with dispersed information.

coordination (specifically: larger firms control more products, and care less about the prices of others) but do not care differentially about others. In essence, their model is one in which $\beta_m \neq \beta_{m'}$ for two players m and m', but where $\gamma_{mm'} = 1/(M-1)$.

Leister (2017) gives each player costly control of the precision of only a single "perfectly private" signal and assumes enough structure to ensure an interior equilibrium. On the other hand, Leister (2017) admits a wider range of cost functions; considers precision choices that are publicly observed prior to play; and evaluates welfare. His messages are complementary, and it remains an open (and welcome) question as to whether his conclusions carry over to a more general information structure.

Each networked player in Denti (2017) designs a signal's correlation structure with the signals received by others and the state. The cost of acquiring such information takes the entropy form of the rational inattention literature (Sims, 2003, 2006). As here, Bonacich centrality combines with—now optimally chosen under entropic costs—signal correlation to determine the network effects of information acquisition. As a result of the non-convexities generated by entropic costs, however, and unlike here, multiplicity may arise (as in the symmetric example of Myatt and Wallace, 2012, Section 9).

In this paper, information acquisition is separate from the network itself. This contrasts with work in which the network describes the communication links between players (Calvó-Armengol and de Martí, 2007, 2009). For example, Herskovic and Ramos (2015) studied a network-formation game with players who each have access to uncorrelated signals. Rather than (as here) investigating the impact of a network structure on information use, that paper (and much of the literature from which it proceeds) explores the impact that information use has on network structure. Interestingly, publicity is key here also: players with particularly "good" information attract others who link with them. The more who link, the more public the signal becomes, the more useful it is to others trying to coordinate.[25] Thinking of such players as "opinion makers", Herskovic and Ramos (2015) relate their result to the origins of leadership. In an asymmetric setting, Calvó-Armengol, de Martí, and Prat (2015) study communication in which players can control the precision with which they send and receive signals to and from others (at some cost).[26] Again, players exogenously receive only a single, private, signal; although the aggregated information is endogenously public via the communication process.[27,28]

[25]So, in Herskovic and Ramos (2015), the better informed become the more influential. This contrasts with the "tyranny of the uninformed" result of Golub and Morris (2017) who provide an extended discussion of the distinction between these two results. It is interesting to compare these results with the observation of the current paper, that relatively centrally located players tend to focus on fewer, relatively public, signals. In other words, those who are more influenced by others acquire less information.

[26]Interestingly, players are unable to ignore entirely another player's signal by assumption. The current paper, the paper by Herskovic and Ramos (2015) discussed above, and the work of Currarini and Feri (2015), who study bilateral information sharing on networks, suggest this may not be entirely innocuous.

[27]Similar questions relating to information transmission in networks are addressed by Hagenbach and Koessler (2010) and Galeotti, Ghiglino, and Squintani (2013). In those papers, communication is modelled as "cheap talk" and payoff asymmetries enter through biases on the fundamental motive (rather than via the coordination motive). The focus is, therefore, upon the potential for credible communication.

[28]In related work, Galeotti and Goyal (2010) present a model in which the players' payoffs depend on information acquired from their neighbours. Their focus is on the outcome of a network formation process

Information use by networked players has also been studied in different strategic settings. A recent paper by Leister, Zenou, and Zhou (2017) studies an exchange-rate attack environment (so, a binary-action coordination game) with a common unknown payoff-relevant variable (θ). Players receive a single costless (uncorrelated, conditional on θ) signal about θ before choosing their action. The focus is on the cut-off equilibria typical of exchange-rate attack problems and not on information acquisition and use, but players' payoff-dependencies are also represented by a network structure.

Relative to the social value of information literature, this paper studies a general class of preference asymmetries via the network on which players are arranged. Relative to the literature on games and communication in networks, the model here focuses on the acquisition and use of information sources independent of the network structure itself. Instead, the model incorporates a rich correlation structure over multiple different sources whose publicity and precision are affected by the acquisition decisions of the players themselves. A key contribution is to identify a connection between information acquisition (and use), the signal's publicity, and the players' centrality in the network.

Appendix A. Proofs of Lemmas and Propositions

Proof of Lemma 1. A first step is to show that the expected payoff of player m is

$$\mathrm{E}[u_m] = \text{constant} - (1-\beta_m)\,\mathrm{E}[(a_m-\theta)^2] - \beta_m \sum\nolimits_{m'\neq m} \gamma_{mm'}\,\mathrm{E}[(a_m - a'_m)^2]$$

$$\text{where}\quad \mathrm{E}[(a_m-\theta)^2] = \Big(\sum\nolimits_{i=0}^{n} w_{im} - 1\Big)^2 x_0^2 + \Big(\sum\nolimits_{i=1}^{n} w_{im} - 1\Big)^2 \kappa_0^2 + \sum\nolimits_{i=1}^{n} w_{im}^2(\kappa_i^2 + \xi_{im}^2)$$

$$\text{and}\quad \mathrm{E}[(a_m-a_{m'})^2] = \Big(\sum\nolimits_{i=0}^{n} (w_{im} - w_{im'})\Big)^2 x_0^2 + \Big(\sum\nolimits_{i=1}^{n} (w_{im} - w_{im'})\Big)^2 \kappa_0^2 + \sum\nolimits_{i=1}^{n} (w_{im}-w_{im'})^2\kappa_i^2 + \sum\nolimits_{i=1}^{n} w_{im}^2 \xi_{im}^2 + \sum\nolimits_{i=1}^{n} w_{im'}^2 \xi_{im'}^2.$$

With linear strategies of the form stated in the main text,

$$a_m - \theta = \Big(\sum\nolimits_{i=1}^{m} w_{im} - 1\Big)(\theta - x_0) + \sum\nolimits_{i=1}^{m} w_{im}(\eta_i + \varepsilon_{im}) + \Big(\sum\nolimits_{i=0}^{m} w_{im} - 1\Big) x_0.$$

All but the final term are zero in expectation, and all terms are uncorrelated. Hence, squaring and taking the expectation yields $\mathrm{E}[(a_m-\theta)^2]$. A similar procedure yields $\mathrm{E}[(a_m - a'_m)^2]$.

Differentiating with respect to w_{0m},

$$\frac{\partial\,\mathrm{E}[(a_m-\theta)^2]}{\partial w_{0m}} = 2\Big(\sum\nolimits_{i=0}^{m} w_{im} - 1\Big) x_0^2 = 2(\bar{w}_m - 1)x_0^2$$

$$\frac{\partial\,\mathrm{E}[(a_m-a_{m'})^2]}{\partial w_{0m}} = 2\Big(\sum\nolimits_{i=0}^{n} (w_{im} - w_{im'})\Big) x_0^2 = 2(\bar{w}_m - \bar{w}_{m'})x_0^2.$$

where $\bar{w}_m \equiv \sum_{i=0}^{n} w_{im}$ and similarly for $\bar{w}_{m'}$. Hence:

$$-\frac{\partial\,\mathrm{E}[u_m]}{\partial w_{0m}} = 2x_0^2\Big[(1-\beta_m)(\bar{w}_m - 1) + \beta_m \sum\nolimits_{m'\neq m} \gamma_{mm'}(\bar{w}_m - \bar{w}_{m'})\Big] = 0.$$

These M equations solve to yield $\bar{w}_m = 1$ for all players.

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when equilibrium play of the game is itself network-dependent (using the networked public-good provision game of Bramoullé and Kranton 2007, later generalized in Bramoullé, Kranton, and D'Amours 2014). A recent experimental treatment of these network formation issues can be found in Goyal, Rosenkranz, Weitzel, and Buskens (2017), while Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2010) investigate a variant in which players have incomplete information about the structure of the network.

Next, differentiate with respect to w_{jm} and evaluate at $\bar{w}_m = 1$ to obtain

$$\frac{\partial\,\mathrm{E}[(a_m-\theta)^2]}{\partial w_{jm}} = -2w_{0m}\kappa_0^2 + 2w_{jm}(\kappa_j^2+\xi_{jm}^2)$$

$$\begin{aligned}
\text{and } \frac{\partial\,\mathrm{E}[(a_m-a_{m'})^2]}{\partial w_{im}} &= -2\left(w_{0m}-w_{0m'}\right)\right)\kappa_0^2 + 2(w_{jm}-w_{jm'})\kappa_j^2 + 2w_{jm}\xi_{jm}^2 \\
&= \frac{\partial\,\mathrm{E}[(a_m-\theta)^2]}{\partial w_{jm}} + 2w_{0m'}\kappa_0^2 - 2w_{jm'}\kappa_j^2 \\
\Rightarrow -\frac{1}{2}\frac{\partial\,\mathrm{E}[u_m]}{\partial w_{jm}} &= -w_{0m}\kappa_0^2 + w_{jm}(\kappa_j^2+\xi_{jm}^2) + \beta_m\sum\nolimits_{m'\neq m}\gamma_{mm'}[w_{0m'}\kappa_0^2 - w_{jm'}\kappa_j^2].
\end{aligned}$$

Setting this equal to zero yields the condition stated in the lemma, where

$$c_m = w_{0m}\kappa_0^2 - \beta_m\sum\nolimits_{m'\neq m}\gamma_{mm'}w_{0m'}\kappa_0^2.$$

Uniqueness follows from the invertibility of the matrix described in the next proof. □

Proof of Proposition 1. $|\rho_i\beta_m| < 1$ for all m and so $[\mathrm{I}-\rho_i\bar{\Gamma}]$ has full rank for all i. Using vector notation, the first-order condition of (6) in Lemma 1 can be written as $\mathbf{w}_i = \psi_i[\mathrm{I}-\rho_i\bar{\Gamma}]^{-1}\mathbf{c}$. Summing over signals gives $\mathbf{1} = \sum_{j=1}^n \psi_j[\mathrm{I}-\rho_j\bar{\Gamma}]^{-1}\mathbf{c}$. There is a unique solution to the optimization problem if and only if $\sum_{j=1}^n \psi_j[\mathrm{I}-\rho_j\bar{\Gamma}]^{-1}$ is invertible: $\mathbf{c} = [\sum_{j=1}^n \psi_i[\mathrm{I}-\rho_j\bar{\Gamma}]^{-1}]^{-1}\mathbf{1}$.[29] Then, as required, the equilibrium weights may be written in vector notation as

$$\mathbf{w}_i = \psi_i[\mathrm{I}-\rho_i\bar{\Gamma}]^{-1}\left[\sum\nolimits_{j=1}^n\psi_j[\mathrm{I}-\rho_j\bar{\Gamma}]^{-1}\right]^{-1}\mathbf{1} = \psi_i\sum\nolimits_{k=0}^\infty(\rho_i)^k\bar{\Gamma}^k\left[\sum\nolimits_{j=1}^n\psi_j\sum\nolimits_{l=0}^\infty(\rho_j)^l\bar{\Gamma}^l\right]^{-1}\mathbf{1},$$

where the second equality follows the discussion in Footnote 13, and which further justifies the discussion immediately following the proposition. □

Proof of Lemma 2. Differentiating $\mathrm{E}[u_m]$ with respect to w_{im}, and setting to zero, gives exactly the expression in the main text above Lemma 1. Differentiating with respect to z_{im} gives $w_{im}^2\xi_i^2/z_{im}^2 = 1$. Noting $\xi_{im}^2 = \xi_i^2/z_{im}$ and substituting using the first-order condition for z_{im} gives the expression reported in Lemma 2 for an interior solution. When $z_{im} = 0$, payoffs would diverge if $w_{im} \neq 0$, yielding the second part of the lemma. □

Proof of Proposition 2. The first-order conditions in (7) may be written $\mathbf{w}_i = \bar{\Gamma}\mathbf{w}_i + (1/\kappa_i^2)[\mathbf{c}-\xi_i\mathbf{1}]$ so long as $w_{im} > 0$ for every m, that is $i \in N_\star$. Restricting to the case where if i is acquired by any player m then i is acquired by all others too, so that $w_{im} > 0$ for all m,

$$\begin{aligned}
&[\mathrm{I}-\bar{\Gamma}]\mathbf{w}_i = \frac{1}{\kappa_i^2}[\mathbf{c}-\xi_i\mathbf{1}] \quad\Rightarrow\quad \mathbf{w}_i = \frac{1}{\kappa_i^2}[\mathrm{I}-\bar{\Gamma}]^{-1}[\mathbf{c}-\xi_i\mathbf{1}] \quad\Rightarrow \\
&\qquad \mathbf{1} = \sum_{i\in N^\star}\mathbf{w}_i = \sum_{i\in N_\star}\frac{1}{\kappa_i^2}[\mathrm{I}-\bar{\Gamma}]^{-1}[\mathbf{c}-\xi_i\mathbf{1}] = [\mathrm{I}-\bar{\Gamma}]^{-1}\mathbf{c}\sum_{i\in N_\star}\frac{1}{\kappa_i^2} - [\mathrm{I}-\bar{\Gamma}]^{-1}\mathbf{1}\sum_{i\in N_\star}\frac{\xi_i}{\kappa_i^2},
\end{aligned}$$

which, using the definition of $\bar{\xi}_\star$ given in (9), can be solved explicitly for $\mathbf{c}$:

$$\mathbf{c} = \frac{1}{\sum_{i\in N_\star}1/\kappa_i^2}[\mathrm{I}-\bar{\Gamma}]\mathbf{1} + \bar{\xi}_\star\mathbf{1}.$$

[29] This is not immediate: the sum of many invertible matrices is not necessarily itself invertible. It can be guaranteed by restricting the β_m parameters (that they be small enough: the inverse exists if $\beta_m = 0$ for all m, so continuity of the matrix inverse function guarantees such positive values can be found).

Now $\mathbf{c}$ can be substituted back into the expression for w_i, giving

$$\mathbf{w}_i = \frac{1}{\kappa_i^2}\left\{\frac{1}{\sum_{j\in N_\star} 1/\kappa_j^2}\mathbf{1} - (\xi_i - \bar{\xi}_\star)[\mathrm{I}-\bar{\Gamma}]^{-1}\mathbf{1}\right\},$$

The fact that players acquire the clearest signals follows from inspection. □

Proof of Proposition 3. Multiply (9) through by ξ_i to obtain

$$\mathbf{z}_i = \frac{\xi_i}{\kappa_i^2}\left\{\frac{1}{\sum_{j\in N_\star} 1/\kappa_j^2}\mathbf{1} - (\xi_i - \bar{\xi}_\star)[\mathrm{I}-\bar{\Gamma}]^{-1}\mathbf{1}\right\},$$

then sum over $i \in N_\star$, and note the identity $\sum_{i\in N_\star}\xi_i(\xi_i-\bar{\xi}_\star)/\kappa_i^2 \equiv \sum_{i\in N_\star}(\xi_i-\bar{\xi}_\star)^2/\kappa_i^2$, by the definition of $\bar{\xi}_\star$ in (9), yielding the expression in (10). □

Proof of Proposition 4. Follows directly from arguments in the main text. □

Proof of Proposition 5. The first part and (12) follow directly from arguments in the main text. The formulation of $N_\star$ follows immediately from inspection of the first equation in (12). For $N_\star$ unique, see the proof in Myatt and Wallace (2012, Proposition 2). □

Section 3.2 observes that if there are no asymmetries in the connections between players (so that $\gamma_{mm'} = 1/(M-1)$ for all m and $m' \neq m$) then players centralities are determined by their aggregate concerns for coordination. This is recorded here formally as a lemma.

Lemma 3 (Coordination and Centrality)**.** *If there are no asymmetries in the connections between players and if $\beta_1 < \cdots < \beta_N$, then players' centralities satisfy $b_1 < b_2 < \cdots < b_M$ for any $1 \geq \rho > 0$.*

Proof of Lemma 3. For expositional simplicity (and without loss of generality) set $\rho = 1$. The vector of Bonacich centralities is $\mathbf{b} = [I-\bar{\Gamma}]^{-1}\mathbf{1} = \sum_{k=0}^{\infty}\mathbf{b}^k$ where $\mathbf{b}^k \equiv \bar{\Gamma}^k\mathbf{1}$. $\mathbf{b}^1$ satisfies $b_m^1 = \beta_m$ and so (i) $b_1^1 < \cdots < b_M^1$ and (ii) $(b_1^1/\beta_1) \geq \cdots \geq (b_M^1/\beta_M)$. This is an induction basis. As an induction hypothesis suppose that, for $k \geq 1$, both (i) and (ii) hold. Now, for any $m < M$,

$$b_m^{k+1} = \frac{\beta_m}{M-1}\sum_{m'\neq m} b_{m'}^k \quad \text{and so} \quad b_m^{k+1} < b_{m+1}^{k+1} \quad \Leftrightarrow \quad \beta_m \sum_{m'\neq m} b_{m'}^k < \beta_{m+1}\sum_{m'\neq m+1} b_{m'}^k$$
$$\Leftrightarrow \quad \beta_m b_{m+1}^k - \beta_{m+1} b_m^k < (\beta_{m+1}-\beta_m)\sum_{m'\neq m,m+1} b_{m'}^k.$$

The right-hand side is positive, and so a sufficient condition for this to hold is $\beta_m b_{m+1}^k \leq \beta_{m+1} b_m^k$ or equivalently $(b_{m+1}^k/\beta_{m+1}) \leq (b_m^k/\beta_m)$, which holds from the induction hypothesis. Further,

$$\frac{b_m^{k+1}}{\beta_m} \geq \frac{b_{m+1}^{k+1}}{\beta_{m+1}} \quad \Leftrightarrow \quad \sum_{m'\neq m} b_{m'}^k \geq \sum_{m'\neq m+1} b_{m'}^k \quad \Leftrightarrow \quad b_{m+1}^k \geq b_m^k,$$

which also holds owing to the induction basis. By the principle of induction, (i) and (ii) hold for all k. This in turn implies that $b_m = \sum_{k=0}^{\infty} b_m^k$ is strictly increasing in m. □

Proof of Proposition 6. Define

$$\phi(\rho) = (1-\beta_A\omega_{AA}\rho)(1-\beta_B\omega_{BB}\rho) - \beta_A\beta_B\omega_{AB}\omega_{BA}\rho^2 \tag{13}$$

which is the determinant of $(\mathrm{I}-\rho\bar{\Omega})$. Next, define the weighted averages ψ_+ and ρ_+ as

$$\psi_+ = \sum\nolimits_{i=1}^{n}\frac{\psi_i}{\phi(\rho_i)} \quad \text{and} \quad \rho_+ = \sum\nolimits_{i=1}^{n}\frac{\psi_i\rho_i}{\phi(\rho_i)}. \quad \text{Finally, let} \quad \hat{\rho} \equiv \frac{\rho_+}{\psi_+}. \tag{14}$$

The maintained assumption $|\beta_m| < 1$ guarantees $\phi(\rho) > 0$ for all $\rho \in [0, 1]$. Moreover, $\phi(\rho) \leq 1$ for all $\rho \in [0, 1]$. The "average publicity" term $\hat{\rho}$ is also between zero and one. Whether a signal is more heavily used by members of A rather than B turns on whether or not ρ_i exceeds $\hat{\rho}$.

A solution for the equilibrium will be found by assuming $w_{jm} = w_{jA}$ and $c_m = c_A$ for all $m \in A$; $w_{jw} = w_{jB}$ and $c_m = c_B$ for all $m \in B$. Then, using (6) in Lemma 1,

$$w_{jA} = \beta_A[\omega_{AA}w_{jA} + \omega_{AB}w_{jB}]\rho_j + c_A\psi_j;$$
$$w_{jB} = \beta_B[\omega_{BA}w_{jA} + \omega_{BB}w_{jB}]\rho_j + c_B\psi_j.$$

These equations solve readily to yield w_{jA} and w_{jB} in terms of c_A and c_B. For example,

$$w_{jA} = \psi_j \frac{\beta_A\omega_{AB}\rho_j c_B + (1 - \beta_B\omega_{BB}\rho_j)c_A}{(1 - \beta_A\omega_{AA}\rho_j)(1 - \beta_B\omega_{BB}\rho_j) - \beta_A\beta_B\omega_{AB}\omega_{BA}\rho_j^2.} \tag{15}$$

Clearly, an equivalent is readily available for w_{jB} simply by swapping A and B in the above wherever they occur. Now, summing over j, and using $\sum_{i=1}^{n} w_{im} = 1$ for all m,

$$1 = \beta_A\omega_{AB}\rho_+ c_B + (\psi_+ - \beta_B\omega_{BB}\rho_+)c_A = \beta_B\omega_{BA}\rho_+ c_A + (\psi_+ - \beta_A\omega_{AA}\rho_+)c_B, \tag{16}$$

where ρ_+ and ψ_+ are given in (14). Equating the two expressions in (16), collecting terms, and noting that $\omega_{AA} + \omega_{AB} = \omega_{BA} + \omega_{BB} = 1$,

$$\frac{c_A}{c_B} = \frac{\psi_+ - \beta_A\rho_+}{\psi_+ - \beta_B\rho_+}, \tag{17}$$

from which $c_A > c_B \Leftrightarrow \beta_A < \beta_B$ is immediate. Now using (15), cancelling the common denominator and the ψ_j terms, $w_{jA} < w_{jB}$ if and only if

$$\beta_A\omega_{AB}\rho_j c_B + (1 - \beta_B\omega_{BB}\rho_j)c_A < \beta_B\omega_{BA}\rho_j c_A + (1 - \beta_A\omega_{AA}\rho_j)c_B.$$

Collecting terms and rewriting, this holds if and only if

$$\frac{1 - \beta_B\rho_j}{1 - \beta_A\rho_j} < \frac{c_B}{c_A} = \frac{1 - \beta_B\rho_+/\psi_+}{1 - \beta_A\rho_+/\psi_+},$$

where the last equality follows from (17). Assuming $\beta_A < \beta_B$ the first ratio is decreasing in ρ_j, so the inequality is equivalent to $\rho_j > \rho_+/\psi_+ \equiv \hat{\rho}$. □

An observation in the text is that either players in A acquire a subset of those signals acquired by players in B or vice versa. This is stated and proved here as a formal lemma.

Lemma 4 (Nested Attention). *Either $N_A \subseteq N_B$ or $N_B \subseteq N_A$ or both.*

Proof of Lemma 4. The first-order conditions for w_{iA} and w_{iB} (when positive) from (7) are

$$w_{iA} = \beta_A[\omega_{AA}w_{iA} + \omega_{AB}w_{iB}] + \frac{c_A - \xi_i}{\kappa_i^2} \quad \text{and} \quad w_{iB} = \beta_B[\omega_{BB}w_{iB} + \omega_{BA}w_{iA}] + \frac{c_B - \xi_i}{\kappa_i^2}. \tag{18}$$

The above first-order conditions apply if $i \in N_A \cap N_B \equiv N_{A\cap B}$. For $i \in N_A \cap \neg N_B \equiv N_{A/B}$,

$$w_{iA} = \beta_A\omega_{AA}w_{iA} + \frac{c_A - \xi_i}{\kappa_i^2} \quad \text{and} \quad w_{iB} = 0.$$

Clearly, for $i \in N_B \cap \neg N_A \equiv N_{B/A}$ the expressions are reversed. So, first, suppose that there exists $i \neq j$ such that $i \in N_{A/B}$ and $j \in N_{B/A}$. Then

$$w_{iA} = \frac{1}{1 - \beta_A\omega_{AA}} \frac{c_A - \xi_i}{\kappa_i^2} > 0 \quad \text{and} \quad w_{jB} = \frac{1}{1 - \beta_B\omega_{BB}} \frac{c_B - \xi_i}{\kappa_i^2} > 0, \tag{19}$$

whereas $w_{iB} = w_{jA} = 0$. For a player in B to not use signal i, the right-hand side of the first-order condition given above in (18) must be weakly negative. That is,

$$\beta_B[\omega_{BB}w_{iB} + \omega_{BA}w_{iA}] + \frac{c_B - \xi_i}{\kappa_i^2} \leq 0.$$

In equilibrium, then, $\xi_i \geq \beta_B\omega_{BA}w_{iA}\kappa_i^2 + c_B = \beta_B\omega_{BA}(c_A - \xi_i)/(1 - \beta_A\omega_{AA}) + c_B$. Equivalently,

$$\xi_i \geq \frac{\beta_B\omega_{BA}c_A + (1 - \beta_A\omega_{AA})c_B}{1 - \beta_A\omega_{AA} + \beta_B\omega_{BA}}.$$

Now $w_{iA} > 0$, so $c_A > \xi_i$. This in turn implies that, for signal i,

$$c_A > \xi_i \geq \frac{\beta_B\omega_{BA}c_A + (1 - \beta_A\omega_{AA})c_B}{1 - \beta_A\omega_{AA} + \beta_B\omega_{BA}}, \quad \text{and so} \quad c_A > c_B. \tag{20}$$

However, the very same exercise for signal $j \neq i$ can be conducted, yielding

$$c_B > \xi_j \geq \frac{\beta_A\omega_{AB}c_B + (1 - \beta_B\omega_{BB})c_A}{1 - \beta_B\omega_{BB} + \beta_A\omega_{AB}}, \quad \text{and so} \quad c_B > c_A.$$

Clearly, then, there cannot be both an $i \in N_{A/B}$ and a $j \in N_{B/A}$ in equilibrium. □

Proof of Proposition 7. (i) Suppose $N_B \subseteq N_A$. B-types use a (possibly weak) subset of the signals used by A-types. (18) and the first expression in (19) provide the first-order conditions for $i \in N_{A\cap B}$ and $i \in N_{A/B}$ respectively. $w_{iB} = 0$ for all $i \in N_{A/B}$ and all other weights are zero.

Consider the implication $N_B \subset N_A \Rightarrow \beta_B > \beta_A$ first. Suppose indeed that $N_B \subset N_A$. Then $N_{A\cap B} = N_B$. Substitute the first-order conditions for w_{iA} into those for w_{iB} in (18) for all $i \in N_B$ (recalling that $N_B \subset N_A$). This exercise yields

$$w_{iB}(1 - \beta_B\omega_{BB}) = \frac{\beta_B\omega_{BA}}{1 - \beta_A\omega_{AA}}\left[\beta_A\omega_{AB}w_{iB} + \frac{c_A - \xi_i}{\kappa_i^2}\right] + \frac{c_B - \xi_i}{\kappa_i^2}.$$

Rearranging to solve for w_{iB}, and using $\phi \equiv \phi(1)$ from (13),

$$\begin{aligned}
\phi w_{iB} &= \beta_B\omega_{BA}\frac{c_A - \xi_i}{\kappa_i^2} + (1 - \beta_A\omega_{AA})\frac{c_B - \xi_i}{\kappa_i^2} \\
w_{iB} &= \frac{1}{\phi\kappa_i^2}\Big[\beta_B\omega_{BA}c_A + (1 - \beta_A\omega_{AA})c_B - \xi_i(\beta_B\omega_{BA} + (1 - \beta_A\omega_{AA}))\Big].
\end{aligned}$$

Now, summing over N_B, and noting $\sum_{j\in N_B} w_{jB} = 1$,

$$\beta_B\omega_{BA}c_A + (1 - \beta_A\omega_{AA})c_B = \frac{\phi}{\sum_{j\in N_B} 1/\kappa_j^2} + \bar{\xi}_B(1 - \beta_A\omega_{AA} + \beta_B\omega_{BA}), \tag{21}$$

where $\bar{\xi}_B$ is the accuracy-weighted average clarity over signals used by type-B players (explicitly written in Proposition 7). Thus, for such $i \in N_B$, weights for B types are

$$w_{iB} = \frac{1/\kappa_i^2}{\sum_{j\in N_B} 1/\kappa_j^2} + \phi_B\left(\frac{\bar{\xi}_B - \xi_i}{\kappa_i^2}\right), \text{ with } \phi_B \equiv \frac{1 - \beta_A\omega_{AA} + \beta_B\omega_{BA}}{(1 - \beta_A\omega_{AA})(1 - \beta_B\omega_{BB}) - \beta_A\beta_B\omega_{AB}\omega_{BA}}. \tag{22}$$

The weights for type-A players depend on whether type-B players are using the signals or not. When they are, (18) applies for w_{iA}, when not, (19) applies. So

$$\begin{aligned}
i \in N_{A/B} \quad &\Rightarrow \quad w_{iA} = \frac{1}{1 - \beta_A\omega_{AA}}\frac{c_A - \xi_i}{\kappa_i^2} \\
i \in N_{A\cap B} \quad &\Rightarrow \quad w_{iA} = \frac{1}{1 - \beta_A\omega_{AA}}\frac{c_A - \xi_i}{\kappa_i^2} + \frac{\beta_A\omega_{AB}}{1 - \beta_A\omega_{AA}}w_{iB}.
\end{aligned} \tag{23}$$

Summing over all $i \in N_A = N_{A\cap B} \cup N_{A/B}$ and noting that $\sum_{j\in N_A} w_{jA} = \sum_{j\in N_B} w_{jB} = 1$,

$$1 = \frac{1}{1-\beta_A\omega_{AA}}\left[c_A \sum_{j\in N_A} \frac{1}{\kappa_j^2} - \sum_{j\in N_A} \frac{\xi_j}{\kappa_j^2}\right] + \frac{\beta_A\omega_{AB}}{1-\beta_A\omega_{AA}}.$$

Rearranging gives an expression for c_A. Now, by assumption, $N_B \subset N_A$ and so the chain of inequalities in the first expression of (20) holds for some i. Using (21) and

$$c_A = \frac{1-\beta_A}{\sum_{j\in N_A} 1/\kappa_j^2} + \bar{\xi}_A,$$

(20) can be true for this i if and only if

$$\bar{\xi}_A + \frac{1}{\phi_A}\frac{1}{\sum_{j\in N_A} 1/\kappa_j^2} > \xi_i \geq \bar{\xi}_B + \frac{1}{\phi_B}\frac{1}{\sum_{j\in N_B} 1/\kappa_j^2}, \quad \text{where} \quad \phi_A = \frac{1}{1-\beta_A}. \tag{24}$$

Assume the converse of the required result, so that $\gamma_B \leq \gamma_A$. Then

$$\begin{aligned}
\frac{1}{\phi_A} \leq \frac{1}{\phi_B} \quad &\Leftrightarrow \quad 1-\beta_A \leq \frac{(1-\beta_A\omega_{AA})(1-\beta_B\omega_{BB}) - \beta_A\beta_B\omega_{AB}\omega_{BA}}{1-\beta_A\omega_{AA}+\beta_B\omega_{BA}} \\
&\Leftrightarrow \quad (1-\beta_A\omega_{AA})^2 + (\beta_B\omega_{BA} - \beta_A\omega_{AB})(1-\beta_A\omega_{AA}) \leq (1-\beta_A\omega_{AA})(1-\beta_B\omega_{BB}) \\
&\Leftrightarrow \quad (1-\beta_A\omega_{AA}) + (\beta_B\omega_{BA} - \beta_A\omega_{AB}) \leq (1-\beta_B\omega_{BB}) \\
&\Leftrightarrow \quad \beta_B \leq \beta_A.
\end{aligned}$$

So, if $\beta_B \leq \beta_A$ then $1/\phi_A \leq 1/\phi_B$ and so

$$\bar{\xi}_A + \frac{1}{\phi_A}\frac{1}{\sum_{j\in N_A} 1/\kappa_j^2} \leq \bar{\xi}_A + \frac{1}{\phi_B}\frac{1}{\sum_{j\in N_A} 1/\kappa_j^2}. \tag{25}$$

But there must an exist an i such that (20) holds, and therefore an $i \notin N_B$ such that (24) holds.

$$\xi_i \geq \bar{\xi}_B + \frac{1}{\phi_B}\frac{1}{\sum_{j\in N_B} 1/\kappa_j^2} \quad \Leftrightarrow \quad \xi_i \geq \bar{\xi}_{B\cup\{i\}} + \frac{1}{\phi_B}\frac{1}{\sum_{j\in N_{B}\cup\{i\}} 1/\kappa_j^2},$$

defining $\bar{\xi}_{B\cup\{i\}}$ in an appropriate way and using the usual argument via cross multiplication and addition of ξ_i/κ_i^2 to both sides. If $N_A = N_B \cup \{i\}$ then this expression along with (25) contradicts (24). If N_A contains further signals not in N_B, then let $i = \{\arg\min_j \xi_j \,|\, j \in N_{A/B}\}$ and apply the above argument. Then repeat the last part of the argument for $i+1$, $i+2$, etc., until all signals in $N_{A/B}$ are included. A contradiction is reached again: if $N_B \subset N_A$ then $\beta_B > \beta_A$, as required.

Now suppose $\beta_B \leq \beta_A$. Then N_B is not a subset of N_A by modus tollens. Apply Lemma 4: $N_A \subseteq N_B$. Thus, swapping A for B, if $\beta_B \geq \beta_A$ then $N_B \subseteq N_A$, proving the proposition's first statement. Note that application of (24) immediately gives the final statement of claim (i), that this subset consists precisely of the clearest signals in N_A.

(ii) Recall the weights given in (22) and (23). Let $i \in N_{A \cap B}$ so that players in A and B acquire i,

$$\begin{aligned}
w_{iA} \leq w_{iB} \quad &\Leftrightarrow \quad \frac{1}{1-\beta_A\omega_{AA}}\frac{c_A-\xi_i}{\kappa_i^2} + \frac{\beta_A\omega_{AB}}{1-\beta_A\omega_{AA}} w_{iB} \leq w_{iB} \\
&\Leftrightarrow \quad \frac{c_A-\xi_i}{\kappa_i^2} \leq (1-\beta_A)w_{iB} \\
&\Leftrightarrow \quad \frac{c_A-\xi_i}{\kappa_i^2} \leq \frac{1}{\phi_A}\left[\frac{1/\kappa_i^2}{\sum_{j\in N_B} 1/\kappa_j^2} + \phi_B\left(\frac{\bar{\xi}_B-\xi_i}{\kappa_i^2}\right)\right] \\
&\Leftrightarrow \quad \phi_A\left[\frac{1}{\phi_A\sum_{j\in N_A} 1/\kappa_j^2} + (\bar{\xi}_A-\xi_i)\right] \leq \frac{1}{\sum_{j\in N_B} 1/\kappa_j^2} + \phi_B(\bar{\xi}_B-\xi_i) \\
&\Leftrightarrow \quad \xi_i \leq \frac{1}{\phi_B-\phi_A}\left[\frac{1}{\sum_{j\in N_B} 1/\kappa_j^2} - \frac{1}{\sum_{j\in N_A} 1/\kappa_j^2} + (\phi_B\bar{\xi}_B - \phi_A\bar{\xi}_A)\right],
\end{aligned}$$

where the fourth line follows from substitution for c_A and the final line from noting that $\phi_B > \phi_A$ if $\beta_B > \beta_A$. If $N_A = N_B$ then the summations are identical and cancel, and $\bar{\xi}_A = \bar{\xi}_B$, yielding the final result stated in claim (ii). If $N_B \subset N_A$ then the signals unused by B ($i \in N_{A/B}$) are the least clear used by A, trivially confirming the result for such i.

(iii) Again consider (22) and (23). Using the former, multiplying by ξ_i and summing over $i \in N_B$,

$$Z_B = \bar{\xi}_B - \phi_B \sum_{i\in N_B} \frac{(\xi_i-\bar{\xi}_B)^2}{\kappa_i^2}.$$

Similarly, multiply (23) through by ξ_i, and sum over $i \in N_A$,

$$Z_A = \frac{1}{1-\beta_A\omega_{AA}}\left[\sum_{i\in N_A}\frac{\xi_i}{\kappa_i^2}c_A - \sum_{i\in N_A}\frac{\xi_i^2}{\kappa_i^2}\right] + \frac{\beta_A\omega_{AB}}{1-\beta_A\omega_{AA}}Z_B,$$

since $w_{iB} = z_{iB} = 0$ for all $i \in N_{A/B}$. This is greater than or equal to Z_B if and only if

$$\begin{aligned}
&\sum_{i\in N_A}\frac{\xi_i}{\kappa_i^2}c_A - \sum_{i\in N_A}\frac{\xi_i^2}{\kappa_i^2} \geq Z_B(1-\beta_A) \\
\Leftrightarrow \quad &\bar{\xi}_A - \phi_A\sum_{i\in N_A}\frac{(\xi_i-\bar{\xi}_A)^2}{\kappa_i^2} \geq \bar{\xi}_B - \phi_B\sum_{i\in N_B}\frac{(\xi_i-\bar{\xi}_B)^2}{\kappa_i^2},
\end{aligned}$$

which follows by substituting for c_A and rearranging. Now $\beta_B > \beta_A \Rightarrow \phi_B > \phi_A$. Moreover $N_B \subseteq N_A$. By the proof method of the later Proposition 9, $Z_A \geq Z_B$ as required. □

Proof of Proposition 8. The objective is to examine the properties of w_{jM} as $M \to \infty$. First, w_{jm} is found for any j, m in terms of c_k with $k \in \{1, \ldots, m\}$. For $m > 1$, repeated substitution yields

$$\begin{aligned}
w_{jm} &= \beta\rho_j w_{jm-1} + \psi_j c_m \\
&= \beta\rho_j(\beta\rho_j w_{jm-2} + \psi_j c_{m-1}) + \psi_j c_m \\
&= \beta\rho_j(\beta\rho_j(\beta\rho_j w_{jm-3} + \psi_j c_{m-2}) + \psi_j c_{m-1}) + \psi_j c_m \\
&= \ldots \\
&= (\beta\rho_j)^{m-1} w_{j1} + \psi_j \sum_{k=0}^{m-2} (\beta\rho_j)^k c_{m-k} \\
&= \psi_j \sum_{k=0}^{m-1} (\beta\rho_j)^k c_{m-k},
\end{aligned}$$

where the last line uses $w_{j1} = \psi_j c_1$. Now, making a change of variable for k,

$$w_{jm} = \psi_j \sum\nolimits_{k=1}^{m} (\beta\rho_j)^{m-k} c_k. \tag{26}$$

Using the fact that $\sum_{i=1}^{n} w_{im} = 1$, and (26), the sequence $\{c_m\}_{m=1}^{M}$ can be deduced:

$$1 = \sum\nolimits_{i=1}^{n} \psi_i \sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m-k} c_k.$$

So, using this expression for m and $m+1$ yields

$$\begin{aligned}
\sum\nolimits_{i=1}^{n} \psi_i \sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m-k} c_k &= \sum\nolimits_{i=1}^{n} \psi_i \sum\nolimits_{k=1}^{m+1} (\beta\rho_i)^{m+1-k} c_k \\
&= \sum\nolimits_{i=1}^{n} \psi_i \left[\sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m+1-k} c_k + c_{m+1}\right] \\
&= \sum\nolimits_{i=1}^{n} \psi_i \beta\rho_i \sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m-k} c_k + \sum\nolimits_{i=1}^{n} \psi_i c_{m+1} \\
\Rightarrow \quad c_{m+1} \sum\nolimits_{i=1}^{n} \psi_i &= \sum\nolimits_{i=1}^{n} \psi_i (1-\beta\rho_i) \sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m-k} c_k \quad \text{or} \\
c_{m+1} &= \sum\nolimits_{i=1}^{n} \hat{\psi}_i (1-\beta\rho_i) \sum\nolimits_{k=1}^{m} (\beta\rho_i)^{m-k} c_k \\
&= \sum\nolimits_{k=1}^{m} c_k \left[\sum\nolimits_{i=1}^{n} \hat{\psi}_i (1-\beta\rho_i)(\beta\rho_i)^{m-k}\right] \\
&= \sum\nolimits_{k=1}^{m} c_k v_k^m, \quad \text{where} \quad v_k^m \equiv \sum\nolimits_{i=1}^{n} \hat{\psi}_i (1-\beta\rho_i)(\beta\rho_i)^{m-k}.
\end{aligned}$$

Now, note that $v_k^m = v_{k-1}^{m-1}$ and $v_k^{m-1} > v_k^m$ for all $m \geq k > 1$. Define $\Delta c_m \equiv c_m - c_{m-1}$. Then

$$\begin{aligned}
\Delta c_{m+1} &= \sum\nolimits_{k=1}^{m} c_k v_k^m - \sum\nolimits_{k=1}^{m-1} c_k v_k^{m-1} = \sum\nolimits_{k=1}^{m} c_k v_k^m - \sum\nolimits_{k=2}^{m} c_{k-1} v_{k-1}^{m-1} \\
&= c_1 v_1^m + \sum\nolimits_{k=2}^{m} c_k v_k^m - \sum\nolimits_{k=2}^{m} c_{k-1} v_{k-1}^{m-1} \\
&= c_1 v_1^m + \sum\nolimits_{k=2}^{m} \Delta c_k v_k^m.
\end{aligned}$$

(The last line follows from $v_k^m = v_{k-1}^{m-1}$.) Now, by induction, it can be shown that $c_m < c_{m-1}$ for all $m > 1$, or equivalently that $\Delta c_m < 0$ for all $m > 1$. Suppose, first of all, that for some t, $\Delta c_t < 0$. Then, because $v_k^{t-1} > v_k^t$ for all $t \geq k > 1$,

$$\begin{aligned}
\Delta c_{t+1} &= c_1 v_1^t + \sum\nolimits_{k=2}^{t} \Delta c_k v_k^t = c_1 v_1^t + \sum\nolimits_{k=2}^{t-1} \Delta c_k v_k^t + \Delta c_t v_t^t \\
&< c_1 v_1^{t-1} + \sum\nolimits_{k=2}^{t-1} \Delta c_k v_k^{t-1} + \Delta c_t v_t^t = \Delta c_t + \Delta c_t v_t^t = (1 + v_t^t)\Delta c_t < 0,
\end{aligned}$$

by the induction hypothesis. So, if $\Delta c_t < 0$ then $\Delta c_{t+1} < 0$. Now consider $m = 2$,

$$c_2 = \sum\nolimits_{k=1}^{1} c_k v_k^1 = c_1 v_1^1 = c_1 \sum\nolimits_{i=1}^{n} \hat{\psi}_i (1-\beta\rho_i) < c_1,$$

since $v_k^m < 1$ for all $m \geq k \geq 1$. So indeed $c_2 < c_1$ or $\Delta c_2 < 0$. Therefore, by induction, $\Delta c_m < 0$ for all m. In other words, $\{c_m\}_{m=1}^{M}$ is a decreasing sequence. It is bounded below. In particular, again using $\sum_{i=1}^{n} w_{im} = 1$, for all $m > 1$

$$w_{jm} = \beta w_{jm-1}\rho_j + c_m \psi_j \quad \Rightarrow \quad c_m = \frac{1 - \beta \sum_{i=1}^{n} w_{im-1}\rho_i}{\sum_{i=1}^{n} \psi_i} \geq \frac{1-\beta}{\sum_{i=1}^{n} \psi_i} > 0.$$

Moreover, the value of c_1 is known, and so

$$c_m \in \left[\frac{1-\beta}{\sum_{i=1}^{n} \psi_i}, \frac{1}{\sum_{i=1}^{n} \psi_i}\right] \quad \text{for all } m.$$

Therefore $\{c_m\}_{m=1}^{M}$ converges as $M \to \infty$. It remains to establish that the sequence $\{w_{jm}\}_{m=1}^{M}$ converges as $M \to \infty$ for all j. In fact, subtracting w_{jm-1} from w_{jm} and using "Δ" notation

$\Delta w_{jm} \equiv w_{jm} - w_{jm-1} = \beta\rho_j \Delta w_{jm-1} + \psi_j \Delta c_m$ for $m > 2$. Evaluating at M and taking $M \to \infty$,

$$\lim_{M\to\infty} \Delta w_{jM} = \beta\rho_j \lim_{M\to\infty} \Delta w_{jM-1} + \psi_j \lim_{M\to\infty} \Delta c_M = \beta\rho_j \lim_{M\to\infty} \Delta w_{jM-1},$$

since $\lim_{M\to\infty} \Delta c_M = 0$. Hence $\lim_{M\to\infty} \Delta w_{jM} = 0$. Thus the sequence $\{w_{jm}\}_{m=1}^{M}$ converges as $M \to \infty$ for all j. Define $c_\infty \equiv \lim_{M\to\infty} c_M$. From the Mth first-order condition

$$w_{jM} = \beta w_{jM-1}\rho_j + c_M\psi_j \quad \Rightarrow \quad \lim_{M\to\infty} (w_{jM} - \beta\rho_j w_{jM-1}) = \psi_j c_\infty,$$

and so, defining $w_{j\infty} \equiv \lim_{M\to\infty} w_{jM}$, for all j,

$$w_{j\infty} = \frac{\psi_j}{1-\beta\rho_j} c_\infty \quad \Rightarrow \quad c_\infty = \left[\sum\nolimits_{i=1}^{n} \frac{\psi_i}{1-\beta\rho_i}\right]^{-1}.$$

Thus the weights converge to the familiar (from Section 3) expression

$$w_{j\infty} = \frac{\psi_j}{1-\beta\rho_j} \Big/ \sum\nolimits_{i=1}^{n} \frac{\psi_i}{1-\beta\rho_i} \quad \text{for all } j.$$

Summarizing in the accuracy/clarity notation and substituting for ρ_j and ψ_j gives the expression in the statement of the proposition. □

The next lemma is useful for the proof of Proposition 9, and is stated (in words) in the main text.

Lemma 5 (Shrinking Signal Acquisition). *If player m does not acquire signal j then nor does any later player $m' > m$ in the hierarchy. That is $z_{jm} = 0 \Rightarrow z_{jm'} = 0$ for all $m' > m$.*

Proof of Lemma 5. The first-order conditions for $m > 1$ when $w_{jm} > 0$ may be derived from (7):

$$w_{jm} = \beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2}. \tag{27}$$

The first task is to show that $c_m \le c_{m-1}$ for all $m > 1$. Consider (27). Sum over all $i \in N_m$, then

$$1 = \sum_{i\in N_m} w_{im} = \beta \sum_{i\in N_m} w_{im-1} + c_m \sum_{i\in N_m} \frac{1}{\kappa_i^2} - \sum_{i\in N_m} \frac{\xi_i}{\kappa_i^2}.$$

Note that $\sum_{i\in N_m} w_{im-1} \le 1$. Thus, for all $m > 1$, c_m can be bounded below:

$$c_m \ge \frac{(1-\beta) + \sum_{i\in N_m} \xi_i/\kappa_i^2}{\sum_{i\in N_m} 1/\kappa_i^2} \equiv \Xi(N_m; \beta).$$

For $j \notin N_m$,

$$\beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2} \le 0 \quad \Rightarrow \quad \beta \sum_{i\in N^-} w_{im-1} + c_m \sum_{i\in N^-} \frac{1}{\kappa_i^2} - \sum_{i\in N^-} \frac{\xi_i}{\kappa_i^2} \le 0,$$

where $N^- = \neg N_m \cap N_{m-1}$. For $j \in N^+ = N_m \cap N_{m-1}$ the condition in (27) applies, and

$$\sum_{i\in N^+} w_{im} = \beta \sum_{i\in N^+} w_{im-1} + c_m \sum_{i\in N^+} \frac{1}{\kappa_i^2} - \sum_{i\in N^+} \frac{\xi_i}{\kappa_i^2}.$$

For $j \in N^-$, $w_{jm} = 0$ and $N_{m-1} = N^+ \cup N^-$, so

$$1 \ge \sum_{i\in N_{m-1}} w_{im} \ge \beta \sum_{i\in N_{m-1}} w_{im-1} + c_m \sum_{i\in N_{m-1}} \frac{1}{\kappa_i^2} - \sum_{i\in N_{m-1}} \frac{\xi_i}{\kappa_i^2} = \beta + c_m \sum_{i\in N_{m-1}} \frac{1}{\kappa_i^2} - \sum_{i\in N_{m-1}} \frac{\xi_i}{\kappa_i^2}.$$

In other words, $c_m \le \Xi(N_{m-1}; \beta)$ for all $m > 1$. Thus, $c_m \le \Xi(N_{m-1}; \beta) \le c_{m-1}$ for all $m > 2$. Moreover, from the earlier fact that $c_1 = \Xi(N_1, 0)$, and noting that $\Xi(\cdot, \beta)$ is decreasing in β, $c_m \le \Xi(N_{m-1}; \beta) \le c_{m-1}$ for all $m > 1$: $\{c_m\}_{m=1}^{M}$ is a decreasing sequence, as required.

Now consider the statement of the lemma. If m does not use j, then $w_{jm} = 0$. Therefore $\beta w_{jm-1} + (c_m - \xi_j)/\kappa_j^2 \leq 0$. As a consequence, $\xi_j \geq c_m$. For $m+1$ to use j, w_{jm+1} must be strictly positive, and therefore (27) applies (evaluated for player $m+1$) and is strictly positive. But $w_{jm} = 0$, so it must be that $c_{m+1} > \xi_j$. But then $\xi_j \geq c_m \geq c_{m+1} > \xi_j$, a contradiction. Repeating this argument for all $m' > m+1$ yields the result. □

Proof of Proposition 9. From the discussion in the main text, $N_1 = \{1, \ldots, n_1\}$. To confirm that N_1 is unique, it is sufficient to confirm that $\Xi(N_1; 0)$ crosses the (rising) sequence of ξ_is only once (which will be after n_1 and before $n_1 + 1$, by definition). First, take $j = \max\{i \in N\}$ such that $\xi_{j+1} > \Xi(N; 0) > \xi_j$, if such exists. Then, for instance, for any $k > j$,

$$\xi_k \geq \xi_{j+1} > \Xi(N; 0) = \frac{1 + \sum_{i \in N} \xi_i/\kappa_i^2}{\sum_{i \in N} 1/\kappa_i^2} \quad \Leftrightarrow \quad \xi_k \sum_{i \in N} \frac{1}{\kappa_i^2} > 1 + \sum_{i \in N} \frac{\xi_i}{\kappa_i^2}$$
$$\Leftrightarrow \quad \xi_k \sum_{i \in N} \frac{1}{\kappa_i^2} + \frac{\xi_k}{\kappa_k^2} > 1 + \sum_{i \in N} \frac{\xi_i}{\kappa_i^2} + \frac{\xi_k}{\kappa_k^2} \quad \Leftrightarrow \quad \xi_k \sum_{i \in N \cup \{k\}} \frac{1}{\kappa_i^2} > 1 + \sum_{i \in N \cup \{k\}} \frac{\xi_i}{\kappa_i^2}$$
$$\Leftrightarrow \quad \xi_k > \frac{1 + \sum_{i \in N \cup \{k\}} \xi_i/\kappa_i^2}{\sum_{i \in N \cup \{k\}} 1/\kappa_i^2} = \Xi(N \cup \{k\}; 0).$$

A symmetrical argument applies for $k \leq j$, so that $\xi_k \leq \xi_j < \Xi(N; 0) \Leftrightarrow \xi_k < \Xi(N \backslash \{k\}; 0)$. Thus, by continued application of these facts, no superset or strict subset of N can satisfy this property. Therefore, there exists a unique $n_1 \geq 1$ such that $z_{j1} > 0 \Leftrightarrow w_{j1} > 0$ for all $j \leq n_1$ and $z_{j1} = w_{j1} = 0$ for all $j > n_1$. So $N_1 = \{1, \ldots n_1\}$ is unique as required.

Now, consider $m > 1$. In order to show $N_m = \{1, \ldots, n_m\}$ with $n_m \leq n_{m-1}$ for all $m > 1$, note that $N_m \subseteq N_{m-1}$ from Lemma 5. Next, $N_m = \{1, \ldots, n_m\}$ for all m is required. That is, each player uses a subset of signals consisting of the most clear (lowest ξ_j). This has been shown for $m = 1$. To see this for general m, consider the minimum m for which, for some j, $w_{jm} = 0$ but $w_{jm-1} > 0$. Now, again by Lemma 5, $w_{jm-1} > 0 \Rightarrow w_{jm'} > 0$ for all $m' < m - 1$. By way of a contradiction suppose that $j < n_m$ and there exists some $i > j$ for which $w_{im} > 0$. Then

$$\beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 \quad \Rightarrow \quad \beta \left(\beta w_{jm-2} + \frac{c_{m-1} - \xi_j}{\kappa_j^2} \right) + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0$$
$$\Rightarrow \quad \beta^2 w_{jm-2} + \beta \frac{c_{m-1} - \xi_j}{\kappa_j^2} + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 \quad \Rightarrow \quad \beta^{m-1} w_{j1} + \sum_{k=2}^{m} \beta^{m-k} \frac{c_k - \xi_j}{\kappa_j^2} \leq 0$$
$$\Rightarrow \quad \sum_{k=1}^{m} \beta^{m-k} \frac{c_k - \xi_j}{\kappa_j^2} \leq 0,$$

where the penultimate line follows from repeated substitution for w_{jm-2} and the final line from the value of w_{j1} established in the main text. Rearranging,

$$\xi_j \geq \sum\nolimits_{k=1}^{m} \beta^{m-k} c_k \Big/ \sum\nolimits_{k=1}^{m} \beta^{m-k}.$$

Signal i is used by m, and so is used by all $m' < m$. The very same calculation can be made for i, therefore; because $w_{im} > 0$, the first-order condition applies, and

$$w_{im} = \sum_{k=1}^{m} \beta^{m-k} \frac{c_k - \xi_i}{\kappa_i^2}. \tag{28}$$

But $i > j$, so $\xi_i > \xi_j \geq \sum_{k=1}^{m} \beta^{m-k} c_k / \sum_{k=1}^{m} \beta^{m-k}$ implying $w_{im} = 0$, a contradiction. No "gap" can open up for the first time at any $m > 1$. Since there are "no gaps" at $m = 1$, there are no gaps for any m. Finally, observe that n_m is uniquely determined for each $m > 1$ (applying precisely the method used above for n_1). These facts together prove the statements in claim (i).

Turning to total information acquisition, in claim (ii), define

$$\bar{\xi}_m = \frac{\sum_{i \in N_m} \xi_i / \kappa_i^2}{\sum_{i \in N_m} 1/\kappa_i^2}.$$

Now given the ordering of the ξ_is and the facts proven earlier that $N_m \subseteq N_{m-1}$ for all $m > 1$, and there are "no gaps" for any m so that $N_m = \{1, \ldots, n_m\}$, it is clear that this measure of "average clarity" declines: $\bar{\xi}_m \leq \bar{\xi}_{m-1}$ for all $m > 1$.[30] Using this notation, construct the positive weights for player m. In particular, since $w_{im} > 0$ implies that $w_{im'} > 0$ for all $m' < m$. (28) applies whenever $w_{im} > 0$. Summing over all such i for player m and rearranging,

$$\sum_{k=1}^{m} \beta^{m-k} c_k \sum_{i \in N_m} \frac{1}{\kappa_i^2} = 1 + \sum_{k=1}^{m} \beta^{m-k} \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} = 1 + \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2}.$$

Therefore, dividing through both sides by $\sum_{i \in N_m} 1/\kappa_i^2$ and using the $\bar{\xi}_m$ notation,

$$\sum_{k=1}^{m} \beta^{m-k} c_k = \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1-\beta^m}{1-\beta} \bar{\xi}_m. \tag{29}$$

From the first order conditions, $z_{im} = \xi_i w_{im}$. So, if $w_{im} > 0$ then, from (28), for all $m > 1$,

$$z_{im} = \sum_{k=1}^{m} \beta^{m-k} \left(\frac{c_k - \xi_i}{\kappa_i^2} \right) \xi_i. \tag{30}$$

Now, total information use (or total cost of information use) is $Z_m = \sum_{i \in N_m} z_{im}$,

$$\begin{aligned}
Z_m &= \sum_{k=1}^{m} \beta^{m-k} c_k \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} - \sum_{k=1}^{m} \beta^{m-k} \sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} \\
&= \left[\frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1-\beta^m}{1-\beta} \bar{\xi}_m \right] \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} \\
&= \bar{\xi}_m - \frac{1-\beta^m}{1-\beta} \left(\sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} - \bar{\xi}_m^2 \sum_{i \in N_m} \frac{1}{\kappa_i^2} \right) = \bar{\xi}_m - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)^2}{\kappa_i^2},
\end{aligned} \tag{31}$$

where the second equality follows from (29), the third from rearrangement and the definition of $\bar{\xi}_m$ and (31) from further rearrangement of the "variance-like" second term.

Now recall $i \in N_m$ if and only if $w_{im} > 0 \Leftrightarrow z_{im} > 0$. Using the recursive expression for w_{im} in (28), therefore, $i \in N_m$ if and only if

$$\begin{aligned}
w_{im} > 0 \quad &\Leftrightarrow \quad \sum_{k=1}^{m} \beta^{m-k} \frac{c_k - \xi_i}{\kappa_i^2} > 0 \quad \Leftrightarrow \quad \sum_{k=1}^{m} \beta^{m-k} c_k > \frac{1-\beta^m}{1-\beta} \xi_i \\
&\Leftrightarrow \quad \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1-\beta^m}{1-\beta} \bar{\xi}_m > \frac{1-\beta^m}{1-\beta} \xi_i \quad \Leftrightarrow \quad \xi_i < \bar{\xi}_m + \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2}.
\end{aligned} \tag{32}$$

Now $N_m \subseteq N_{m-1}$ for all $m > 1$, and $z_{im} > 0 \Rightarrow z_{i-1m} > 0$ for all $m \geq 1$. Using these facts, the statement of claim (ii), concerning total acquisition, may be proved.

[30] Of course, this measure is actually inversely related to average clarity (recall, $1/\xi_i^2$ is interpreted as information source i's clarity). Therefore, as expected, the signals acquired by players further down the hierarchy have higher clarity on average than those acquired by players above them.

If $N_m = N_{m-1}$ then inspection of (31) is sufficient. The last term is (weakly) positive, and does not change from $m-1$ to m, likewise the first term, but $\beta < 1$ so $\beta^m < \beta^{m-1}$, so $Z_m \leq Z_{m-1}$. The harder case is when $N_m \subset N_{m-1}$. Consider moving up the chain from player $m+1$ to player m. Assume, in the first instance, that $N_m = N_{m+1} \cup \{j\}$, so that j is the (sole) signal that m acquires, but $m+1$ does not.

First suppose that $\xi_j = \bar{\xi}_{m+1}$. Then $\bar{\xi}_m = \bar{\xi}_{m+1}$. Moreover, since $\bar{\xi}_m = \bar{\xi}_{m+1}$, from (31)

$$\begin{aligned}
Z_m &= \bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} \\
&= \bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_{m+1}} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \frac{(\xi_j - \bar{\xi}_{m+1})^2}{\kappa_j^2} \\
&= \bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_{m+1}} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} \geq Z_{m+1},
\end{aligned}$$

where the final equality follows from the supposition $\xi_j = \bar{\xi}_{m+1}$, and the inequality follows from $\beta^{m+1} < \beta^m$ and the (weak) positivity of the variance-like term.

Now treat Z_m as a function of ξ_j. Note that it is quadratic in ξ_j. Compute

$$\begin{aligned}
\frac{dZ_m}{d\xi_j} &= \frac{d\bar{\xi}_m}{d\xi_j} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{d}{d\xi_j} \frac{(\xi_i - \bar{\xi}_m)^2}{\kappa_i^2} \\
&= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \left[\frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2} - 2 \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)}{\kappa_i^2} \frac{d\bar{\xi}_m}{d\xi_j} \right] \\
&= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \left[\frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2} - \frac{2/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)}{\kappa_i^2} \right] \\
&= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2},
\end{aligned}$$

where the final line (and the quantity $d\bar{\xi}_m/d\xi_j$) follow from the definition of $\bar{\xi}_m$. So it follows that Z_m is increasing in ξ_j if and only if $\xi_j < \hat{\xi}_m$ where

$$\hat{\xi}_m \equiv \bar{\xi}_m + \frac{1}{2} \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2}.$$

Summarizing, Z_m is a quadratic in ξ_j with its maximum at $\hat{\xi}_m$ and it is greater than or equal to Z_{m+1} when evaluated at $\xi_j = \bar{\xi}_{m+1}$. It is therefore greater than or equal to Z_{m+1} (which does not depend on ξ_j by assumption) for all $\xi_j \in [\bar{\xi}_{m+1}, \hat{\xi}_m + (\hat{\xi}_m - \bar{\xi}_{m+1})]$. Now

$$\hat{\xi}_m + (\hat{\xi}_m - \bar{\xi}_{m+1}) = 2\hat{\xi}_m - \bar{\xi}_{m+1} = \bar{\xi}_m + \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + (\bar{\xi}_m - \bar{\xi}_{m+1}),$$

where the last term is strictly positive. But, for j to be acquired by m and not by $m+1$, it must be that (32) holds for m and fails for $m+1$. That is

$$\bar{\xi}_{m+1} < \bar{\xi}_{m+1} + \frac{1-\beta}{1-\beta^{m+1}} \frac{1}{\sum_{i \in N_{m+1}} 1/\kappa_i^2} \leq \xi_i < \bar{\xi}_m + \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2},$$

which implies ξ_j indeed lies (strictly) within the required range for Z_m to be larger than Z_{m+1}. This argument can be repeated for cases when N_{m+1} and N_m differ by more than one signal (in intermediate steps, starting with the highest ξ_j in N_m but not in N_{m+1}, and then the second highest, and so on). Therefore, $Z_m \geq Z_{m+1}$ for all m, as required.

For the third claim of the proposition, first note that by substitution of (29) into (28),

$$w_{jm} = \frac{1}{\kappa_j^2}\left[\frac{1}{\sum_{i\in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta}(\xi_j - \bar{\xi}_m)\right] \tag{33}$$

whenever $w_{jm} > 0$. Information acquisition of signal j is then simply $z_{jm} = \xi_j w_{jm}$. Consider (33) evaluated at m and $m+1$.

$$\begin{aligned} w_{jm+1} > w_{jm} \;\Leftrightarrow\;& \frac{1}{\sum_{i\in N_{m+1}} 1/\kappa_i^2} - \frac{1-\beta^{m+1}}{1-\beta}(\xi_j - \bar{\xi}_{m+1}) > \frac{1}{\sum_{i\in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta}(\xi_j - \bar{\xi}_m) \\ \Leftrightarrow\;& \frac{1}{\sum_{i\in N_{m+1}} 1/\kappa_i^2} - \frac{1}{\sum_{i\in N_m} 1/\kappa_i^2} + \frac{1-\beta^{m+1}}{1-\beta}\bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta}\bar{\xi}_m \\ & \qquad > \frac{1-\beta^{m+1}-(1-\beta^m)}{1-\beta}\xi_j = \frac{\beta^m(1-\beta)}{1-\beta}\xi_j = \beta^m\xi_j \\ \Leftrightarrow\;& \xi_j < \left\{\frac{1}{\sum_{i\in N_{m+1}} 1/\kappa_i^2} - \frac{1}{\sum_{i\in N_m} 1/\kappa_i^2} + \frac{1-\beta^{m+1}}{1-\beta}\bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta}\bar{\xi}_m\right\} \Big/ \beta^m. \end{aligned}$$

Noting $z_{jm} = \xi_j w_{jm}$ proves claim (iii) so long as j is used by both m and $m+1$. If j is not used by $m+1$, then the claim follows immediately ($m+1$ uses a subset consisting of the clearest signals used by m). For the final claim (iv), note that the last line in the above displayed inequality reduces to $\xi_j < \bar{\xi}_m = \bar{\xi}_{m+1}$ when $N_m = N_{m+1}$. □

Appendix B. Additional Material

B.1. **Quadratic Forms.** As noted in the text, $\sum_{m'\neq m}\gamma_{mm'} = 1$ is a normalization, which allows the aggregate coordination motive to be captured by β_m. The equality $1-\beta_m+\beta_m\sum_{m'\neq m}\gamma_{mm'} = 1$ holds if u_m is scaled appropriately. Dropping both normalizations and instead re-scaling the payoff u_m so that the weight on the match-the-fundamental component is equal to one leads to

$$u_m \equiv \text{constant} - (a_m - \theta)^2 - \sum\nolimits_{m'\neq m}\gamma_{mm'}(a_m - a_{m'})^2.$$

This allows for $\sum_{m'\neq m}\gamma_{mm'} \neq 1$ and $\gamma_{mm'} \gtreqless 0$ for any pair of players. In a full-information environment, the best reply of player m to an action profile of others is clearly

$$\text{BR}_m[a_{-m},\theta] = \frac{\theta + \sum_{m'\neq m}\gamma_{mm'}a_{m'}}{1+\sum_{m'\neq m}\gamma_{mm'}} \quad\Rightarrow\quad \frac{\partial \text{BR}_m[a_{-m},\theta]}{\partial\theta} + \sum\nolimits_{m'\neq m}\frac{\partial \text{BR}_m[a_{-m},\theta]}{\partial a_{m'}} = \text{constant}.$$

Thus a common change in θ and the actions of others changes the best reply of player m by the same amount. This "adding up" constraint implies that there is a (unique) symmetric equilibrium of the full-information game. Note, however, that there is no restriction on how the best reply of player m responds individually to θ and to the actions of others.

This is now compared with the model used by Ballester, Calvó-Armengol, and Zenou (2006). They set $u_i = \alpha_i x_i + \frac{1}{2}\sigma_{ii}x_i^2 + \sum_{j\neq i}\sigma_{ij}x_i x_j$, and immediately assumed $\alpha_i = \alpha > 0$ and $\sigma_{ii} = \sigma < 0$ for all i (although their Remarks 1–2 on page 1409 briefly revert to the more general setting). This uses their notation where i indexes players and x_i is the action choice of player i. Payoffs can be re-scaled so that $\sigma = -1$. Changing to the notation of this paper, this is equivalent to

$$u_m = \theta a_m - \tfrac{1}{2}a_m^2 + \sum\nolimits_{m'\neq m}\gamma_{mm'}a_m a_{m'}.$$

For this specification the best reply of m to the actions of others is

$$\text{BR}_m[a_{-m},\theta] = \theta + \sum\nolimits_{m'\neq m}\gamma_{mm'}a_{m'} \quad\Rightarrow\quad \frac{\partial \text{BR}_m[a_{-m},\theta]}{\partial\theta} = \text{constant}.$$

Thus every player's reaction to a change in the fundamental θ is identical. However, the "adding up" constraint does not have to hold, and so the equilibrium is not necessarily symmetric. In fact, symmetry of equilibrium with this payoff specification requires $\sum_{m' \neq m} \gamma_{mm'} =$ constant.

Summarizing, both the specification here and the one of Ballester, Calvó-Armengol, and Zenou (2006) allow for arbitrary weights on coordination with others. Ballester, Calvó-Armengol, and Zenou (2006) required the optimal reaction of players to a change in θ (or the expectation of θ, more generally) to be constant across the player set. Here, however, the optimal reaction of players to a common shift in both θ and others' actions is constant across the player set.

These specifications can be combined by allowing an extra parameter δ_m for each m and setting

$$u_m \equiv \text{constant} - (a_m - \delta_m \theta)^2 - \sum\nolimits_{m' \neq m} \gamma_{mm'} (a_m - a_{m'})^2,$$

so that each player m targets a differently scaled version of θ.[31] To obtain a specification equivalent to Ballester, Calvó-Armengol, and Zenou (2006) requires $\delta_m / \left(1 + \sum_{m' \neq m} \gamma_{mm'}\right)$ to be constant across the player set; to obtain the specification here requires δ_m to be constant.

B.2. **Weights for Three-Player Example.** In the example of Figure 1 with at most two signals in positive use by at least one player and $\xi_1 < \xi_2$, suppose there is an equilibrium in which

$$\begin{aligned}
w_{11} &= \frac{1}{\kappa_1^2} \left[\frac{1}{\sum_{i=1}^2 1/\kappa_i^2} - (\xi_1 - \bar{\xi}) \right], \\
w_{21} &= \frac{1}{\kappa_2^2} \left[\frac{1}{\sum_{i=1}^2 1/\kappa_i^2} - (\xi_2 - \bar{\xi}) \right], \\
w_{12} &= \frac{1}{\kappa_1^2} \left[\frac{1}{\sum_{i=1}^2 1/\kappa_i^2} - \frac{1 - \beta\gamma + \beta}{1 - \beta\gamma} (\xi_1 - \bar{\xi}) \right] (1 - \beta\gamma) + \beta\gamma, \\
w_{22} &= \frac{1}{\kappa_2^2} \left[\frac{1}{\sum_{i=1}^2 1/\kappa_i^2} - \frac{1 - \beta\gamma + \beta}{1 - \beta\gamma} (\xi_2 - \bar{\xi}) \right] (1 - \beta\gamma), \\
w_{13} &= 1 \quad \text{and} \quad w_{23} = 0.
\end{aligned}$$

It is straightforward to check that these weights solve the equations of Lemma 2, so long as $w_{23} = 0$ and $w_{22} > 0$. (Because $\xi_1 < \bar{\xi} < \xi_2$, with $\bar{\xi}$ as defined in the main text, if $w_{22} > 0$ then $w_{12} > 0$, and $w_{22} > 0 \Rightarrow w_{21} > 0 \Rightarrow w_{11} > 0$.) To confirm $w_{23} = 0$ and $w_{22} > 0$, using (7), and following some algebraic manipulations,

$$w_{23} = 0 \quad \Leftrightarrow \quad \frac{\xi_2 - \xi_1}{\kappa_1^2} \geq \frac{1 - \beta^2\gamma(1-\gamma)}{1 + \beta(1 + \beta(1-\gamma)^2)} = \frac{1}{b_3}. \tag{34}$$

On the other hand, straightforwardly, $w_{22} > 0$ if and only if

$$\frac{1 - \beta\gamma}{1 - \beta\gamma + \beta} > \frac{\xi_2 - \xi_1}{\kappa_1^2}. \tag{35}$$

The first expression in (35) can exceed the second expression in (34) only if $\gamma < \frac{1}{2}$ (as assumed in the main text). For player 2 to acquire the second signal and player 3 to ignore it, player 3 must be more central than player 2. Moreover, it is straightforward to check that, for $\gamma < \frac{1}{2}$, the inverse of the first expression in (35) is less than b_2, player 2's centrality (b_2 and b_3 are given in (2) explicitly). This justifies the sufficient condition for $w_{23} = 0$ and $w_{22} > 0$ given in the text.

[31] An additional coefficient $\nu_{mm'}$ can also be introduced to scale the target action of each other player, so that the coordination motive between player m and m' is captured by the loss $\gamma_{mm'}(a_m - \nu_{mm'} a_{m'})^2$.

B.3. **A Generalized Hierarchy Network.** Here, a more general version of the hierarchy network analysed in Section 5 is presented. The model introduced at the beginning of the section, for which Figure 3 illustrates an example, can be extended even further. Below, however, a recipe is provided for adapting Propositions 8–9 to the case where every level $\ell > 1$ contains several isolated groups, each containing $g + 1$ players. Any two players within a given group have $\gamma_{mm'} = \gamma_{m'm} = \gamma$. Each player m in level $\ell > 1$ is linked to precisely one player m' in layer $\ell - 1$ with $\gamma_{mm'} = 1 - g\gamma$. There is a single player in level 1 (player 1) who is linked to no-one. Further, suppose there are L levels in total.

First, an analogue to Proposition 8 is available. Define $w_{j\ell} \equiv w_{jm}$ and $c_\ell \equiv c_m$ for any player m residing in level ℓ. Applying Lemma 1, the optimal weight on signal j for a player in level ℓ is

$$w_{j\ell} = \beta\rho_j \left\{(1 - g\gamma)w_{j(\ell-1)} + g\gamma w_{j\ell}\right\} + c_\ell\psi_j,$$

for all $\ell > 1$. If $\ell = 1$ then $w_{j1} = c_1\psi_j$ as in the model described in Section 5. For $\ell > 1$,

$$\begin{aligned}(1 - \beta\rho_j g\gamma)w_{j\ell} &= \beta\rho_j(1 - g\gamma)w_{j(\ell-1)} + c_\ell\psi_j \\ w_{j\ell} &= \beta\rho_j^\star w_{j(\ell-1)} + c_\ell\psi_j^\star,\end{aligned} \tag{36}$$

where $\rho_j^\star \equiv \rho_j(1 - g\gamma)/(1 - \beta\rho_j g\gamma)$ and $\psi_j^\star \equiv \psi_j/(1 - \beta\rho_j g\gamma)$. This, however, is the very same expression as that of the opening statements in the proof to Proposition 8 in Appendix A, but with ρ_j replaced with $\rho_j^\star$ and ψ_j with $\psi_j^\star$. The only caveat is that, at $\ell = 1$, $w_{j1} = c_1\psi_j$.

Noting this difference at $\ell = 1$ is all that is required to show an analogue for Proposition 8 (replacing $1 - \beta$ with an appropriate constant). Repeated substitution in (36) yields (for $\ell > 1$)

$$w_{j\ell} = (\beta\rho_j^\star)^{\ell-1}w_{j1} + \psi_j^\star\sum\nolimits_{k=0}^{\ell-2}(\beta\rho_j^\star)^k c_{\ell-k}.$$

Now $w_{j1} = \psi_j c_1 = \psi_j^\star(1 - \beta\rho_j g\gamma)c_1 = \psi_j^\star c_1 - \psi_j^\star\beta\rho_j g\gamma c_1$. Therefore, (26) can be rewritten

$$w_{j\ell} = \psi_j^\star\sum\nolimits_{k=1}^{\ell}(\beta\rho_j^\star)^{\ell-k}c_k - \psi_j^\star(\beta\rho_j^\star)^\ell g\gamma c_1$$

for any player in level $\ell \geq 1$. Now, other than the second term, this is precisely the same as (26). Following exactly the method of the proof to Proposition 8,

$$c_{\ell+1} = \sum\nolimits_{i=1}^{n}\hat{\psi}_i^\star(1 - \beta\rho_i^\star)\sum\nolimits_{k=1}^{\ell}(\beta\rho_i^\star)^{\ell-k}c_k - g\gamma\sum\nolimits_{i=1}^{n}\hat{\psi}_i^\star(1 - \beta\rho_i^\star)(\beta\rho_i^\star)^\ell c_1,$$

where $\hat{\psi}_j^\star = \psi_j^\star/\sum_{i=1}^n\psi_i^\star$. Noting that this last term is the analogue of v_0^ℓ, but where ψ_j is replaced with $\psi_j^\star$, and ρ_j is replaced with $\rho_j^\star$ for all j, and abusing notation somewhat,

$$c_{\ell+1} = \sum\nolimits_{k=1}^{\ell}c_k v_k^\ell - g\gamma v_0^\ell c_1 \quad \text{where} \quad v_k^\ell \equiv \sum\nolimits_{i=1}^{n}\hat{\psi}_i^\star(1 - \beta\rho_i^\star)(\beta\rho_i^\star)^{\ell-k}. \tag{37}$$

Following step-by-step the approach in the proof to Proposition 8 yields

$$\Delta c_{\ell+1} = c_1 v_1^\ell + \sum\nolimits_{k=2}^{\ell}\Delta c_k v_k^\ell + g\gamma(v_0^{\ell-1} - v_0^\ell)c_1.$$

The last term is positive, given the definition of v_k^ℓ above. Showing that the sequence of c_ks is decreasing follows by induction. The only difficulty is the step at $\ell = 1$. But note, for all $\ell > 1$,

$$v_0^{\ell-1} - v_0^\ell = \sum\nolimits_{i=1}^{n}\hat{\psi}_i^\star(1 - \beta\rho_i^\star)^2(\beta\rho_i^\star)^{\ell-1} < \sum\nolimits_{i=1}^{n}\hat{\psi}_i^\star(1 - \beta\rho_i^\star)^2(\beta\rho_i^\star)^{\ell-2} = v_0^{\ell-2} - v_0^{\ell-1}.$$

Thus, the induction step follows even when adding this new term to Δc_{t+1}. Now, from (37), $c_2 = c_1 v_1^1 - g\gamma v_0^1 c_1 = c_1(v_1^1 - g\gamma v_0^1)$. The first component in the parentheses is smaller than one. Therefore, $c_2 < c_1$. Once again, $\{c_k\}_{k=1}^L$ is a declining sequence, bounded, and so converges. The remainder of the proof is exactly the same, replacing ψ_j with $\psi_j^\star$ and ρ_j with $\rho_j^\star$ in each expression. Then, as before,

$$w_{j\infty} = \frac{\psi_j^\star}{1-\beta\rho_j^\star} \Big/ \sum_{i=1}^{n} \frac{\psi_i^\star}{1-\beta\rho_i^\star} \quad \text{for all } j.$$

Replace $1-\beta$ with $1-\beta(1+g\gamma)$, and recall the maintained assumption that $|\beta(1+g\gamma)| < 1$. Therefore, the weight attached to each signal j is precisely as given in Proposition 8, but where $1-\beta$ is replaced with $1-\beta(1+g\gamma)$ and $M = L$ denotes the final level in the hierarchy.

Variants of Lemma 5 and Proposition 9 continue to hold, replacing the player subscript m with the associated level ℓ and using the notation described above. All that is required is to replace β appropriately, and to take care to adjust the κ_i^2 parameters in the proof.

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