

Information Acquisition and Use by Networked Players

David P. Myatt

London Business School

dmyatt@london.edu

Chris Wallace

University of Manchester

christopher.wallace@manchester.ac.uk

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Abstract. In an asymmetric coordination (or anti-coordination) game, players acquire and use signals about a payoff-relevant fundamental from multiple costly information sources. Some sources have greater clarity than others, and generate signals that are more correlated and so more public. Players wish to take actions close to the fundamental but also close to (or far away from) others' actions. This paper studies how asymmetries in the game, represented as the weights that link players to neighbours on a network, affect how they use and acquire information. Relatively centrally located players (in the sense of Bonacich, when applied to the dependence of players' payoffs upon the actions of others) acquire fewer signals from relatively clear information sources; they acquire less information in total; and they place more emphasis on relatively public signals.

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Decision makers often seek to take actions close to some unknown state of the world and also close to (or sometimes far away from) the actions of others. Roughly speaking: they wish to do the right thing, and do it together. Such decision makers use any available information that resolves their uncertainty about the state of the world and that allows them better to coordinate (or anti-coordinate, in games of strategic substitutes) with others' actions. An established literature, surveyed below, has applied quadratic-payoff models with these features to understand information use and (more recently) costly information acquisition by the players of investment games, by price-setting and quantity-setting oligopolists, within financial markets, in a macroeconomic context, by the members of political parties with competing leaders, and in other important scenarios. A feature of most such studies is that players' payoffs are specified symmetrically. This paper studies situations in which players care asymmetrically about coordination. Two questions are answered. Firstly: how do the scale and pattern of asymmetric coordination motives influence how players use the information available to them? Secondly: if information sources are costly, then how do the coordination asymmetries influence which sources receive attention and the total expenditure on information acquisition?

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The context for these questions is a quadratic-payoff game in which players are arranged on a network. They wish to take actions close to some (unknown) fundamental and close to (or far away from) the actions of others to whom they are linked. A link (a weighted edge of a directed graph) represents a player's desire to coordinate (or not, in an anti-coordination game) with the corresponding neighbour. Two illustrative applications are used: the supply of asymmetrically differentiated products by quantity-setting oligopolists; and the coordination of policy advocacy amongst the members of a political party. For the first application, the uncertain fundamental corresponds to the Cournot equilibrium output in a complete-information world, while the network describes the pattern of substitutability between products. For the second application, the uncertain fundamental corresponds to the ideal policy for the whole party, while the network characterizes the structure of sub-factions within the party's membership.

Each player can acquire and use information about the fundamental by paying costly attention to (multiple) information sources. Such a source is characterized by the precision of a noise component common to every player's observation—its underlying “accuracy”—and the precision of a player-specific noise component—its “clarity”. By paying more attention a player increases both the precision of the observed signal and its correlation with others' observations of it. The conditional (on the fundamental) correlation coefficient of signal realizations determines the publicity of an information source. In an applied political context (one of the two illustrative applications) a source might correspond to the speech of a leader heard by party members; the quality of the leader's judgement maps to the accuracy of the information from that source; the clarity of the leader's communication determines the precision of player-specific interpretations of what the leader has to say; and, finally, the attention of a player corresponds to the time spent listening to the leader's oratory. There are equivalent connections to the other application to the oligopolistic supply of differentiated products with uncertain demand.

A leading result is this: if players acquire and use the same set of information sources then they will (i) pay attention to a subset of information sources consisting of the clearest; (ii) acquire and use relatively clear signals more; and (iii) acquire more information the less central they are on the network (in the sense of Bonacich centrality). The weight placed on a signal deviates from its relative accuracy by the product of the player's centrality and a measure of the signal's relative clarity. Here, the centrality measure refers to the extent to which a player's payoff is influenced by others, and not the extent to which a player's own action changes others' payoffs.

Full characterizations are given for several commonly studied networks (e.g. symmetric networks, star networks, and core-periphery networks). In a (directed) hierarchy network in which players are linked only to those immediately above them, players further down the chain acquire a (weak) subset consisting of the clearest signals acquired by the player(s) above; they acquire less information in total; and players far enough down the chain behave exactly as they would in a symmetric network.

An important message is that relatively clear (and so, endogenously, relatively public) information has relatively great influence on players who are more central (in the sense of being more influenced, rather than more influential) to a network.

A fuller review of the literature is postponed to the concluding remarks. However, this paper links together a literature which uses network centrality measures in complete-information quadratic-payoff coordination games with another literature which considers the use (and more recently the costly acquisition) of information in incomplete-information but symmetric versions of such games.

Specifically, Ballester, Calvó-Armengol, and Zenou (2006) identified a connection between network centrality measures and equilibrium actions in quadratic-payoff coordination games under complete information. Their approach is now standard (for textbook treatments, see Goyal, 2007; Jackson, 2008). This paper adapts their model to a setting with dispersed information in the tradition of Morris and Shin (2002) and Angeletos and Pavan (2007). It does so while allowing for information acquisition à la Dewan and Myatt (2008) and Myatt and Wallace (2012). Asymmetries in these kinds of games have received relatively little attention. One exception is Myatt and Wallace (2017) in which asymmetries arise from differences in supplier size in a price-setting industry. The network approach taken here admits the analysis of a much wider set of asymmetries.

Some interesting recent papers address connected themes. Golub and Morris (2017) use a network to investigate the extent to which players care about the higher-order expectations of others. In particular focus are situations in which players care (almost) entirely about coordination rather than the unknown fundamental. More closely related is work by Denti (2017) in which information is endogenously acquired by players of a coordination game. A prominent feature of his paper is the justification for and use of entropic costs for signal acquisition. Leister (2017) focuses on the welfare properties of equilibrium in a quadratic-payoff game played on a network where players may privately (or publicly) acquire a single, costly, but perfectly uncorrelated, signal (à la Colombo, Femminis, and Pavan, 2014). Herskovic and Ramos (2015) studied a network-formation game where the (again, purely private) source of information is identified with a given player (after Calvó-Armengol and de Martí, 2007, 2009).

The paper proceeds as follows. The game and network structure is described in Section 1, together with the equilibrium of a full-information benchmark model. That section also relates the game to two illustrative applications: Cournot competition, and a policy advocacy game. The information use and acquisition framework is presented in Section 2. The equilibrium is characterized in Section 3 for general networks, under the condition that players acquire and use the same set of signals. To gain a better understanding of networks in which players acquire different sets of signals it is necessary to go beyond the symmetric benchmark of Section 4. Two important formulations are discussed: “two-type” networks (encompassing, for example, core-periphery networks) in Section 5 and a hierarchy structure in Section 6. Finally, some concluding remarks and a fuller discussion of the related literature are contained in Section 7.

1. A SIMPLE COORDINATION GAME ON A NETWORK

This section describes the basic model. The equilibrium under full information is discussed as a benchmark case for the incomplete information structure introduced latterly.

1.1. Players and Payoffs. Each player $m \in \{1, \dots, M\}$ simultaneously chooses a real-valued action $a_m \in \mathbb{R}$. The M players are arranged on a network. $\gamma_{mm'}$ is the (relative) influence of the action of player m' upon the payoff of player m . Precisely,

$$\text{Payoff of } m \equiv u_m \equiv \text{constant} - \left[(1 - \beta_m)(a_m - \theta)^2 + \beta_m \sum_{m' \neq m} \gamma_{mm'}(a_m - a_{m'})^2 \right], \quad (1)$$

where θ is a real-valued ‘‘fundamental’’ target parameter for all players, and where $\gamma_{mm'} \geq 0$, $\sum_{m' \neq m} \gamma_{mm'} = 1$, and where β_m (which can be positive or negative) measures the extent to which player m cares (in aggregate) about the actions of others.²

This is a quadratic-payoff coordination (or anti-coordination, if $\beta_m < 0$) game in which players wish to take actions close to the fundamental θ and close to (or far away from) some aggregate measure of the actions of others. Assume $|\beta_m| < 1$ for all m throughout, so that coordination (or anti-coordination) motives are not overly strong.

The parameters $\gamma_{mm'}$ represent the weights on the links in a directed graph in which each player is identified with a different node. The adjacency matrix for this network is

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1M} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{M1} & \gamma_{M2} & \cdots & \gamma_{MM} \end{bmatrix},$$

where $\gamma_{mm} = 0$ for all m . Γ need not be symmetric. When it is, the network is undirected and player m is linked to m' if and only if m' is linked to m . This adjacency matrix captures, for each m , the relative influence of other players’ actions on the payoff of player m . The absolute influence of those actions also includes the extent to which each player m cares about coordination. Writing $\beta = (\beta_1, \dots, \beta_M)'$ and $\text{diag}[\beta]$ for the diagonal matrix with m th diagonal element β_m , the appropriate adjusted (for the M different strengths of the coordination motive) adjacency matrix is

$$\bar{\Gamma} \equiv \text{diag}[\beta]\Gamma = \begin{bmatrix} \beta_1\gamma_{11} & \beta_1\gamma_{12} & \cdots & \beta_1\gamma_{1M} \\ \beta_2\gamma_{21} & \beta_2\gamma_{22} & \cdots & \beta_2\gamma_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_M\gamma_{M1} & \beta_M\gamma_{M2} & \cdots & \beta_M\gamma_{MM} \end{bmatrix}.$$

$\bar{\Gamma}$ incorporates two sources of player asymmetry. Firstly, players may be asymmetrically connected: players m and m' may care relatively differently about some third player m'' , so $\gamma_{mm''} \neq \gamma_{m'm''}$. Secondly, even if connections are symmetric and equal ($\gamma_{mm'} = 1/(M-1)$ for all m and m') then players may care differently about coordination: $\beta_m \neq \beta_{m'}$.

²A player wishes either to coordinate with ($\beta_m > 0$) or against ($\beta_m < 0$) all others. This is straightforward to relax. Indeed, $\gamma_{mm'} \geq 0$ is assumed for expositional purposes only: it plays no role in any of the proofs.

1.2. A Full Information Benchmark. θ is the parameter over which players have uncertainty. Before specifying this uncertainty, however, it is instructive to examine the full-information benchmark case and to note how it connects to the literature.

Each player m chooses a_m to maximize (1). The maintained assumption $|\beta_m| < 1$ is sufficient for concavity, and first-order conditions yield unique best-replies:

$$\text{Best reply of } m = a_m = (1 - \beta_m)\theta + \beta_m \sum_{m' \neq m} \gamma_{mm'} a_{m'}. \quad (2)$$

Clearly, $a_m = \theta$ for all m satisfies this M -equation system. Thus, there is a symmetric equilibrium. The expression in (2) may be rewritten in matrix notation. That is,

$$\mathbf{a} = (\mathbf{I} - \bar{\Gamma})\theta\mathbf{1} + \bar{\Gamma}\mathbf{a} \quad \text{where} \quad \bar{\Gamma} = \text{diag}[\beta]\Gamma,$$

and where $\mathbf{a} = (a_1, \dots, a_M)'$ is the vector of players' actions, $\mathbf{1}$ is the $M \times 1$ vector of 1s, and \mathbf{I} is the $M \times M$ identity matrix. So long as $(\mathbf{I} - \bar{\Gamma})$ is invertible, $\mathbf{a} = \theta\mathbf{1}$ is the unique solution. $|\beta_m| < 1$ is sufficient for $(\mathbf{I} - \bar{\Gamma})$ to have full rank.³

1.3. Commentary. Under full information, the equilibrium is symmetric: every player m chooses $a_m = \theta$. This is by design: the objective is to study the impact of asymmetries on information use and acquisition, and so it is instructive to abstract away from asymmetries in actions that would follow from different payoff specifications. Plenty of asymmetry remains: $\bar{\Gamma}$ represents a directed network with weighted links (so its elements are not restricted to take values in $\{0, 1\}$), it need not be symmetric (so the influence of player m' on m need not match that of m on m'), and its elements can be positive or negative (actions can be strategic substitutes or complements). Moreover, the aggregate influence of others' actions is not identical across players: $\beta_m \neq \beta_{m'}$, in general.

The specification (1) is a special case of the quadratic payoffs found in Ballester, Calvó-Armengol, and Zenou (2006) and the subsequent literature. In that paper, for the setting described here, equilibrium actions are proportional to weighted Bonacich centralities.⁴ The weighting applied in (1) exactly counteracts the centrality of the player, so that all players choose the same action. Clearly, however, players have different centralities. The purpose of this paper is to understand how such variations in network position affect the use and acquisition of information.

1.4. Application: Asymmetric Cournot Competition. The specification (1) is (intentionally) abstract. Here this specification is linked to a specific applied model.

Consider a world in which a representative consumer enjoys quasi-linear quadratic utility from the consumption of M products. Specifically:

$$\text{Representative Consumer Utility} = \text{constant} + \sum_{m=1}^M q_m \left[\theta\alpha_m - \frac{1}{2} \sum_{m'=1}^M \delta_{mm'} q_{m'} \right],$$

³ $|\beta_m| < 1$ ensures that the strategic complementarity (or substitutability) of actions is not large enough to outweigh the incentive each player has to take an action close to the fundamental.

⁴For a precise statement, see Ballester, Calvó-Armengol, and Zenou (2006, p. 1409, Remark 1).

where q_m is the supply of product m , and where the parameters satisfy $\delta_{mm'} = \delta_{m'm}$ and $\alpha_m > 0$. Differentiating with respect to each q_m yields prices:

$$p_m = \theta\alpha_m - \sum_{m'=1}^M \delta_{mm'} q_{m'}.$$

Now consider Cournot competition amongst M suppliers. Without loss of generality, assume that the constant marginal costs of production are zero. The linearity of the inverse-demand function generates the usual quadratic form for each supplier's profit, and so a best reply that is linear in competing outputs. Specifically,

$$\text{Profit of } m = q_m \left[\theta\alpha_m - \sum_{m'=1}^M \delta_{mm'} q_{m'} \right] \Rightarrow q_m = \frac{1}{2\delta_{mm}} \left[\theta\alpha_m - \sum_{m' \neq m} \delta_{mm'} q_{m'} \right].$$

This Cournot game has, in general, an asymmetric Nash equilibrium. Here the focus on information acquisition and use is maintained by choosing specifications which generate symmetric equilibria under complete information. This is obtained by ensuring that

$$\frac{\alpha_m - \sum_{m' \neq m} \delta_{mm'}}{2\delta_{mm}} = 1 \quad \text{for all } m \quad \Rightarrow \quad \alpha_m = \delta_{mm} + \sum_{m'=1}^M \delta_{mm'},$$

so that the complete-information Nash equilibrium satisfies $q_m^* = \theta$ for all m . This model now maps into the specification of equation (1) by setting, for all m and $m' \neq m$,

$$a_m = q_m, \quad \gamma_{mm'} = \frac{\delta_{mm'}}{\sum_{m'' \neq m} \delta_{mm''}} \quad \text{and} \quad \beta_m = -\frac{\sum_{m' \neq m} \delta_{mm'}}{2\delta_{mm}}.$$

A supplier's profit is equivalent (for behaviour) to the payoff of (1). The parameter β_m measures the overall extent to which supplier m 's product is a substitute for other products, while the parameters $\gamma_{mm'}$ identify the closest competitors to m .

1.5. Application: A Policy-Advocacy Game. Dewan and Myatt (2008, 2012) presented a theory of leadership in which the followers—the activist members of a political party—play a quadratic-payoff coordination game, and where the leaders correspond to providers of information. They specified payoffs of the form

$$\begin{aligned} \text{Payoff of Party Member } m &= \text{constant} - \underbrace{(1 - \beta)(a_m - \theta)^2}_{\text{concern for policy}} - \underbrace{\beta \left(a_m - \frac{\sum_{m' \neq m} a_{m'}}{M - 1} \right)^2}_{\text{desire for party unity}} \\ &= \text{other terms} - (1 - \beta)(a_m - \theta)^2 - \beta \sum_{m' \neq m} \frac{(a_m - a_{m'})^2}{M - 1}, \end{aligned}$$

where the other terms depend only the actions of other players. This fits directly into the specification (1) where $\gamma_{mm'} = 1/(M - 1)$ for all m and m' and where $\beta_m = \beta$ for all players; this is the completely symmetric case. However, this paper allows for intra-party variance in concerns for coordination ($\beta_m \neq \beta_{m'}$) and for groups (or factions) within the party to have relatively stronger dependencies within a faction (so that $\gamma_{mm'} > \gamma_{mm''}$ where m' and m'' are inside and outside, respectively, the faction of player m).

2. INFORMATION

The information structure follows closely that introduced (to political science) by Dewan and Myatt (2008) and (to economics) by Myatt and Wallace (2012).

2.1. Information Sources. Players do not know θ : they begin with an improper prior over it. They have access to n sources of information about θ . Each player receives an unbiased signal of θ from information source $i \in \{1, \dots, n\}$, where

$$\text{Signal } i \text{ Received by Player } m = x_{im} = \theta + \eta_i + \varepsilon_{im} \quad (3)$$

and where the noise terms are all independent. Players form beliefs about θ using these signals and their improper (diffuse) priors.⁵ η_i is a source of noise common across players with $\eta_i \sim N(0, \kappa_i^2)$. The associated precision $1/\kappa_i^2$ is the ‘‘accuracy’’ of the information source. It represents noise inherent in the source itself, perhaps attributable to errors made when the signal is ‘‘sent’’. ε_{im} , on the other hand, is an idiosyncratic noise component, attributable to errors made by the ‘‘receiver’’ of the signal. The associated precision may be (to some extent at least) under the control of the player. Assume that

$$\varepsilon_{im} \sim N(0, \xi_{im}^2) \quad \text{where} \quad \xi_{im}^2 = \frac{\xi_i^2}{z_{im}}.$$

The precision $1/\xi_i^2$ is the underlying ‘‘clarity’’ of information source i . z_{im} measures the (costly) attention player m pays to signal i . Two different specifications are considered.

Firstly, player i might simply receive each signal (free of charge). Setting $z_{im} \equiv 1$ for all i and m , each signal is characterized by its accuracy and clarity or, equivalently, by its overall precision ψ_i and correlation across players ρ_i , where

$$\psi_i = \frac{1}{\kappa_i^2 + \xi_i^2} \quad \text{and} \quad \rho_i = \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2}.$$

More correlated signals are more public in nature (at the extremes, if $\rho_i = 0$ observations of i are independent across players; if $\rho_i = 1$, i is commonly observed), so ρ_i indexes a signals ‘‘publicity’’. The focus in this specification is simply on information use, particularly on how different signals (characterized by ψ_i and ρ_i , or equivalently κ_i^2 and ξ_i^2) are used by differently positioned players on the network.

Secondly, $z_{im} \geq 0$ might be a choice variable for player m . Prior to choosing an action (conditional on received information), player m chooses how much attention to pay to signal i . $z_{im} = 0$ is interpreted as ignoring the signal altogether, and results in a completely uninformative (infinite variance) realization of x_{im} . Attention is costly: let

$$C_m(z_{1m}, \dots, z_{nm}) = \sum_{i=1}^n z_{im} \quad \text{for all } m$$

be that linear cost. This admits a sampling interpretation along the lines discussed in Myatt and Wallace (2015, p. 483) and justified formally by Han and Sangiorgi (2015).⁶

⁵A prior $\theta \sim N(x_0, \kappa_0^2)$ is equivalent to adding an $(n+1)$ st signal $i=0$ with parameters κ_0^2 and $\xi_0^2 = 0$.

⁶More general cost functions yield little extra insight at the expense of much expositional inconvenience.

The focus for this specification is information acquisition, particularly on which different signals are acquired by differently positioned players on the network. Note that the correlation (and precision) of each signal i is determined endogenously in this latter setting: each signal's publicity can differ across players and is an equilibrium phenomenon.

2.2. Interpretation: Listening to Leaders. Section 1.5 observed that the quadratic-payoff coordination game studied here is (equivalent to) an asymmetric version of the policy advocacy game played by party-member followers in the theory of political leadership proposed by Dewan and Myatt (2008, 2012).

In the Dewan-Myatt theory, a leader helps followers to learn about the world (the party's ideal policy, in their interpretation) and helps followers to coordinate. The precision $1/\kappa_i^2$ is the underlying judgement of the i th leader: it is that leader's ability to understand the world. The precision terms $1/\xi_i^2$ and z_{im}/ξ_i^2 are associated with the clarity of communication with the leader: the former term is the clarity of the leader's speech, as it determines the ease with which a message can be understood; the latter term is the (endogenous) precision with which that message is received. When allowing for endogenous attention, Dewan and Myatt (2008) specified an attention span constraint: each follower chooses an information-acquisition policy (essentially: choosing to whom to listen) subject to the constraint $\sum_{i=1}^n z_{im} \leq 1$. The Lagrange multiplier on that constraint corresponds to the marginal cost of attention in the linear-cost specification studied here.

3. INFORMATION USE, INFORMATION ACQUISITION, AND CENTRALITY

This section characterizes equilibrium information use first by abstracting from the acquisition problem (so, setting $z_{im} \equiv 1$ for all i and m) and then by solving subsequently for equilibrium information acquisition and use when z_{im} is chosen optimally for each source i . Information acquisition and use are functions of the signal's clarity and accuracy (or publicity and precision) and the player's Bonacich centrality.

3.1. Information Use. The (Bayesian Nash) equilibrium considered is linear in signal realizations.⁷ In particular, consider strategies of the form

$$a_m = \sum_{i=1}^n w_{im} x_{im}, \quad \text{where} \quad \sum_{i=1}^n w_{im} = 1$$

where w_{im} is the weight that player m places on signal i . The property that $\sum_{i=1}^n w_{im} = 1$ follows from the players' incentives to play θ in expectation; if not, then the expectation of (1) would diverge. More generally, if a common prior $\theta \sim N(x_0, \kappa_0^2)$ were specified then a general property of a linear equilibrium is that $\sum_{i=1}^m w_{im} = 1$ where w_{0m} is the weight placed on the prior mean. It is without loss of generality to treat the prior as an extra (perfectly public) signal, so that the weights on all signals sum to one. The next step is to find the set of such weights which maximize each player's expected utility.

⁷This is without much loss of generality. In a related model, Dewan and Myatt (2008) established that any equilibrium involving strategies which are bounded above and below by linear strategies is itself linear.

Substituting these linear-in-signal strategies actions into (1), using x_{im} from (3), and setting (without loss of generality) the constant term in (1) to zero,

$$-u_m = (1 - \beta_m) \left[\sum_{i=1}^n w_{im} (\eta_i + \varepsilon_{im}) \right]^2 + \beta_m \sum_{m' \neq m} \gamma_{mm'} \left[\sum_{i=1}^n (w_{im} - w_{im'}) \eta_i + \sum_{i=1}^n w_{im} \varepsilon_{im} - \sum_{i=1}^n w_{im'} \varepsilon_{im'} \right]^2.$$

Taking expectations, and using the independence of the various noise terms,

$$-E[u_m] = (1 - \beta_m) \sum_{i=1}^n w_{im}^2 (\kappa_i^2 + \xi_{im}^2) + \beta_m \sum_{m' \neq m} \gamma_{mm'} \left[\sum_{i=1}^n (w_{im} - w_{im'})^2 \kappa_i^2 + \sum_{i=1}^n w_{im}^2 \xi_{im}^2 + \sum_{i=1}^n w_{im'}^2 \xi_{im'}^2 \right].$$

Collecting terms and rewriting in a convenient form, $-E[u_m]$ is given by

$$\sum_{i=1}^n w_{im}^2 (\kappa_i^2 + \xi_{im}^2) - 2\beta_m \sum_{m' \neq m} \gamma_{mm'} \sum_{i=1}^n w_{im} w_{im'} \kappa_i^2 + \beta_m \sum_{m' \neq m} \gamma_{mm'} \sum_{i=1}^n w_{im'}^2 (\kappa_i^2 + \xi_{im'}^2). \quad (4)$$

The optimization programme is $\max_{\{w_{im}\}_{i=1}^n} E[u_m]$ subject to $\sum_{i=1}^n w_{im} = 1$, for every m . Equivalently, for every m minimize (4) with respect to the weights $\{w_{im}\}_{i=1}^n$, subject of course to the constraint $\sum_{i=1}^n w_{im} = 1$.⁸ Concavity is guaranteed by the assumption $|\beta_m| < 1$, and so the M constraints along with the $n \times M$ first-order conditions

$$w_{jm} (\kappa_j^2 + \xi_{jm}^2) - \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{jm'} \kappa_j^2 = c_m$$

are sufficient for a solution. c_m is a player-specific constant proportional to the m th constraint's Lagrange multiplier. Lemma 1 uses the precision and correlation notation (thereby assuming $z_{im} \equiv 1$ and hence $\xi_{im}^2 = \xi_i^2$ for all i and m).

Lemma 1 (Equilibrium First-Order Conditions). *There is a unique linear equilibrium in which the weight each player m places on information source i satisfies*

$$w_{im} = \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{im'} \rho_i + c_m \psi_i, \quad (5)$$

for all i , where c_m is an (equilibrium determined) player-specific constant.

This form for the first-order conditions proves useful for the examples of Sections 5 and 6. However, rewriting in the vector notation of the previous section provides some general insight into how information is used by networked players. Using the notation

$$\mathbf{w}_i = (w_{i1}, \dots, w_{iM})' \quad \text{and} \quad \mathbf{c} = (c_1, \dots, c_M)',$$

the first-order conditions in (5) may be rewritten as

$$\mathbf{w}_i = \rho_i \bar{\Gamma} \mathbf{w}_i + \psi_i \mathbf{c}.$$

Since $\rho_i \leq 1$ for all i , $|\beta_m| < 1$ is sufficient for the inverse $(I - \rho_i \bar{\Gamma})^{-1}$ to exist. Using the equality $\sum_{i=1}^n \mathbf{w}_i = \mathbf{1}$ to solve for \mathbf{c} generates the following proposition's characterization of equilibrium weights (see Appendix A for all proofs).

⁸Note again that $\sum_{i=1}^n w_{im} = 1$ is not imposed as a constraint, but is a property of a linear equilibrium.

Proposition 1 (Equilibrium Information Use). *There is a unique linear equilibrium in which the vector of weights players place on their observations of signal i satisfies*

$$\mathbf{w}_i = \psi_i [\mathbf{I} - \rho_i \bar{\Gamma}]^{-1} \left[\sum_{j=1}^n \psi_j [\mathbf{I} - \rho_j \bar{\Gamma}]^{-1} \right]^{-1} \mathbf{1}.$$

Looking across the player set, the use of a signal is proportional to a player's Bonacich centrality with parameter ρ_i . More public signals (with higher correlation coefficients) have lower "decay" factors.⁹ The influence of a signal is increasing in its publicity when the game is one of strategic complements (for instance, when the elements of $\bar{\Gamma}$ are all strictly positive). This effect is compounded for players who are the most centrally influenced (in the sense of Bonacich) by the actions of others.

3.2. Information Acquisition. If each player m chooses both the weights to place on each signal and an acquisition policy then the player's optimization problem is

$$\max_{w_m, z_m} \left\{ E[u_m] - C_m(z_m) \right\} \quad \text{subject to} \quad \sum_{i=1}^n w_{im} = 1.$$

Again focusing on the linear equilibrium, $E[u_m]$ may be calculated from (4). Note now that $\xi_{im}^2 = \xi_i^2 / z_{im}$, and so $E[u_m]$ is a function of the weights and the acquisition policy. When $z_{im} > 0$, the first-order condition for w_{im} is

$$w_{im} \left(\kappa_i^2 + \frac{\xi_i^2}{z_{im}} \right) - \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{im'} \kappa_i^2 = c_m,$$

where c_m is again a player-specific constant, proportional to the Lagrange multiplier associated with $\sum_{i=1}^n w_{im} = 1$.¹⁰ The first-order condition for z_{im} is simply $w_{im}^2 \xi_i^2 / z_{im}^2 = 1$ (again, when it's positive). Rearranging yields an analogue to Lemma 1.

Lemma 2 (Equilibrium Properties). *There is a unique linear equilibrium in which the weight player m places on information source i satisfies*

$$w_{im} = \beta_m \sum_{m' \neq m} \gamma_{mm'} w_{im'} + \frac{c_m - \xi_i}{\kappa_i^2}, \quad \text{and} \quad z_{im} = \xi_i w_{im} \quad (6)$$

is the attention paid to i , for all i such that $w_{im} > 0$ (equivalently $z_{im} > 0$); $w_{jm} = z_{jm} = 0$ otherwise. Here, c_m is an (equilibrium determined) player-specific constant.

The expressions in (6) may be applied directly, and are useful for the two settings discussed in Sections 5 and 6. In general, different players may listen to different sets of signals so that $z_{im} = 0$ but $z_{im'} > 0$ for some i and $m \neq m'$. Indeed, this will be the case for many interesting examples. However, a particularly clean result is available when all the players listen to the same (possibly strict) subset of the n signals.

⁹Note that $\mathbf{A}^{-1} = \sum_{k=0}^{\infty} (\mathbf{I} - \mathbf{A})^k$ for any invertible matrix \mathbf{A} , where $\mathbf{A}^0 \equiv \mathbf{I}$. Using $\mathbf{A} = \mathbf{I} - \rho_j \bar{\Gamma}$ and re-substituting for the constants \mathbf{c} , the weights may be written $\mathbf{w}_i = \psi_i [\mathbf{I} - \rho_i \bar{\Gamma}]^{-1} \mathbf{c} = \psi_i \sum_{k=0}^{\infty} (\rho_i)^k \bar{\Gamma}^k \mathbf{c}$. Now $\bar{\Gamma}^k$ captures the influence of the weights chosen by all k -distant players on the network. Thus the influence of others' use of signal i decays through the network more slowly the higher is ρ_i .

¹⁰This is a slight abuse of notation: c_m differs in general from the constant identified in Section 3.1. However, it is convenient for expositional purposes to use the same symbol for these constants.

To this end, suppose that $z_{im} > 0 \Leftrightarrow z_{im'} > 0$ for all i and $m \neq m'$, so that all players listen to precisely the same set of signals. Define $N_\star = \{i : z_{im} > 0 \text{ for all } m\}$: the non-empty subset of $\{1, \dots, n\}$ containing all the signals that receive positive attention. For all $i \in N_\star$ the first-order conditions in (6) hold, and $\sum_{i \in N_\star} w_{im} = 1$ for all m .

Proposition 2 (Equilibrium Information Acquisition). *Suppose that, in equilibrium, any signal that is acquired by some player is acquired by everyone. The signals acquired are from the clearest (lowest ξ_i) sources. The weight placed on i by a player m is higher than its relative accuracy if and only if the information source is clearer than average:*

$$\mathbf{w}_i = \frac{1}{\kappa_i^2} \left\{ \frac{1}{\sum_{j \in N_\star} 1/\kappa_j^2} \mathbf{1} - (\xi_i - \bar{\xi}_\star)[\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} \right\}, \quad \text{where} \quad \bar{\xi}_\star = \frac{\sum_{j \in N_\star} \xi_j / \kappa_j^2}{\sum_{j \in N_\star} 1/\kappa_j^2}, \quad (7)$$

for all $i \in N_\star$. The weight's deviation from the signal's relative accuracy is proportional to the product of the difference between signal i 's clarity and the average clarity of all the acquired signals and the player's Bonacich centrality.

To unpack the claims of this proposition, it is useful to begin with the case where all information sources share equal clarity. This means that $\xi_i = \bar{\xi}_\star$ for all i . Applying the solution in the proposition, this means that $w_{im} \propto 1/\kappa_i^2$ so that play is symmetric and all players use their signals in proportion to the underlying accuracy of the information source. Given that they do so, the optimality of information acquisition, from (6) within the statement of Lemma 2, means that

$$z_{im} = \xi_i w_{im} = \bar{\xi}_\star w_{im} \propto \frac{1}{\kappa_i^2} \quad \Rightarrow \quad \frac{1}{\kappa_i^2 + (\xi_i^2/z_{im})} \propto \frac{1}{\kappa_i^2},$$

and, moreover, all signals share the same correlation coefficient. With equal clarity information sources, signal precisions are (endogenously) proportional to underlying information accuracies, and all signals are equally public. This reinforces the use of information in proportion to the underlying accuracy of the corresponding source.

Now consider a situation in which $\xi_i < \xi_j$. Beginning with a situation in which signals are accuracy-weighted, there is less attention devoted to the clearer signal simply because it is easier to understand. Nevertheless, the overall (endogenous) clarity of the message from source i is now relatively greater than from source j , and so source i becomes more public. If play is in strategic complements, this shifts weight toward the clearer information source. Naturally, there are equilibrium considerations too: if others shift toward the clearer source then those who wish to coordinate with them (a desire which is captured by the adjusted adjacency matrix $\bar{\Gamma}$) will face an enhanced incentive to place more weight and devote more attention to the clearer information. Tracing this logic through leads, in equilibrium, to the solution reported in Proposition 2.

3.3. Total Information Acquisition. Beyond the weights attached to the various signals in use, total information acquisition (measured by $Z_m = \sum_{i=1}^n z_{im}$, and so corresponding to total cost paid for the information acquired) is amenable to analysis. Noting

that $z_{im} > 0$ only if $i \in N_*$, and using the first-order condition for such z_{im} in (6), pre-multiply (7) by ξ_i and sum over $i \in N_*$. For every i , defining the M -dimensional vector

$$\mathbf{z}_i \equiv (z_{j1}, \dots, z_{jM})', \quad \text{and hence} \quad \mathbf{Z} \equiv \sum_{i=1}^n \mathbf{z}_i = (Z_1, \dots, Z_M)',$$

yields immediately the last proposition of this section.

Proposition 3 (Total Information Acquisition). *Suppose that, in equilibrium, any signal that is acquired by some player is acquired by everyone. Then, player m 's total information acquisition is decreasing in the Bonacich centrality of that player. In fact,*

$$\mathbf{Z} = \bar{\xi}_* \mathbf{1} - [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} \sum_{j \in N_*} \frac{(\xi_j - \bar{\xi}_*)^2}{\kappa_j^2}. \quad (8)$$

Consider a game with strategic complementarities (so that every element of $\bar{\Gamma}$ is positive). Referring to Proposition 2, and looking across the set of signals in positive use, the clearer a signal i (the lower ξ_i) the more weight is attached to it in equilibrium. Indeed, signals that are clearer than average (as measured by $\bar{\xi}_*$) are acquired and used more than implied by their relative accuracy (as measured by $1/\kappa_i^2 / \sum_{j \in N_*} 1/\kappa_j^2$). This effect is compounded by the player's position in the network: a player who is more central departs more from using signals according to their relative accuracy than one who is less central (as measured by their Bonacich centrality). More central players favour relatively clear (endogenously relatively public) information sources.¹¹

On the other hand, Proposition 3 demonstrates that more central players acquire relatively little information. Players who are central are those whose payoffs are influenced more by others on the network: they place more importance on coordination (and hence their desire for relatively public information). The value of θ itself is of comparatively little importance to them and so acquired information has a lower marginal benefit.

A feature of the equilibrium characterized by (6), however, is that in general a signal i may be acquired (and used) by player m , but not by another player m' . The features of asymmetry in network position that would drive such behaviours are purposefully ignored in the above. To understand how and why different players might use different information (and what features of those sources determine which signals get acquired), two important network structures are explored in Sections 5 and 6. Before moving on to these cases, as a benchmark, it is useful to review briefly the properties of a symmetric equilibrium (of an asymmetric game) and provide conditions under which it arises.

¹¹Clearer signals are endogenously more public in the sense of having higher correlation coefficients in equilibrium. Note that the correlation between the observation of source i by m and m' is

$$\rho_{imm'} = \kappa_i^2 \left[\left(\kappa_i^2 + \frac{\xi_i^2}{z_{im}} \right) \left(\kappa_i^2 + \frac{\xi_i^2}{z_{im'}} \right) \right]^{-\frac{1}{2}}.$$

In the equilibrium described in this section by (6), and in those to follow, $z_{im} = \xi_i w_{im}$ when positive. But, from (7), $w_{im} = f_m(\xi_i) / \kappa_i^2$ when positive, where f_m is a decreasing (player-specific) function of ξ_i . It is straightforward to check that $\rho_{imm'} > \rho_{jmm'} \Leftrightarrow \xi_i < \xi_j$ if both m and m' acquire i and j . If either m or m' does not acquire some i , then $\rho_{imm'} = 0$. As will be seen throughout, players acquire a subset consisting of the most clear signals. So, in equilibrium, the clearer the signal the more endogenously public it is.

4. A SYMMETRIC BENCHMARK

The general structure (captured by $\bar{\Gamma}$) admits a great deal of asymmetry. Typically, therefore, information use and acquisition differ across players. However, there is an important class of asymmetric networks for which the equilibrium is symmetric.

Suppose that players care equally about coordination: $\beta_m = \beta$ for all m , and so $\bar{\Gamma} = \beta\Gamma$. No assumption is made on the connections $\gamma_{mm'}$ and so very asymmetric networks are permitted. Nonetheless, the equilibrium is symmetric: each player uses the same weight $w_i = w_{im}$ on signal i and the constant arising from the constraint is the same for each m , $c = c_m$. To see this, insert the symmetric weights into the first-order condition (5):

$$w_i = \beta \sum_{m' \neq m} \gamma_{mm'} w_i \rho_i + c_m \psi_i = \beta \rho_i w_i + c_m \psi_i \quad \Rightarrow \quad c_m = c \quad \forall m \quad \Rightarrow \quad w_i = \frac{c \psi_i}{1 - \beta \rho_i}.$$

c can be solved by summing these weights across i , and using the equality $\sum_{i=1}^n w_i = 1$. The following proposition summarizes these facts using the clarity-accuracy notation.

Proposition 4 (Information Use and Symmetric Coordination Motives). *If players share the same aggregate coordination motive, so that $\beta_m = \beta$ for all m ,*

$$w_{im} = w_i \quad \forall m \quad \text{where} \quad w_i = \frac{1}{(1 - \beta)\kappa_i^2 + \xi_i^2} \bigg/ \sum_{j=1}^n \frac{1}{(1 - \beta)\kappa_j^2 + \xi_j^2}. \quad (9)$$

In the symmetric equilibrium players use information in proportion to its precision-weighted publicity, a result familiar from Myatt and Wallace (2014, Proposition 1), for instance. (9) serves as a useful benchmark for the networks of later sections.

Now consider the same setting but where players acquire information endogenously. Applying the first-order conditions in (6) from Lemma 2 and inserting $w_{im} = w_i$ and $z_{im} = z_i$ for all m and i , whenever $z_i > 0 \Leftrightarrow w_i > 0$, then

$$w_i = \beta w_i + \frac{c - \xi_i}{\kappa_i^2} \quad \text{and} \quad z_i = \xi_i w_i.$$

So, for $i \in N_\star \equiv \{j : z_j > 0\} \subseteq \{1, \dots, n\}$, equilibrium weights are given by

$$w_i = \frac{c - \xi_i}{(1 - \beta)\kappa_i^2}, \quad \text{whereas for } j \notin N_\star \quad w_j = 0.$$

The constant $c_m = c$ (for all m) can be found by summing over $i \in N_\star$. Once again, using the average clarity notation from Section 3.2, $c = \bar{\xi}_\star + (1 - \beta) / \sum_{i \in N_\star} 1/\kappa_i^2$.

Proposition 5 (Information Acquisition with Symmetric Coordination Motives). *If players share the same aggregate coordination motive then $w_{im} = w_i$ and $z_{im} = z_i$, where*

$$w_i = \frac{1}{\kappa_i^2} \left\{ \frac{1}{\sum_{j \in N_\star} 1/\kappa_j^2} - \frac{\xi_i - \bar{\xi}_\star}{1 - \beta} \right\} \quad \text{and} \quad Z = \bar{\xi}_\star - \frac{1}{1 - \beta} \sum_{j \in N_\star} \frac{(\xi_j - \bar{\xi}_\star)^2}{\kappa_j^2} \quad (10)$$

for $i \in N_\star$ and $Z_m = Z$ is the total information acquisition for player m . Moreover, $N_\star = \{i : \xi_i < \bar{\xi}_\star + (1 - \beta) / \sum_{j \in N_\star} 1/\kappa_j^2\}$ is uniquely defined. The players use a (possibly strict) subset of the signals, consisting of the clearest. Signals $j \notin N_\star$ are ignored: $w_j = z_j = 0$.

The parallels between these results and those presented in Propositions 1–3 are plain. Indeed, these earlier propositions may be applied directly to obtain (9) and (10). Using the expression for the inverse of $(I - \bar{\Gamma})$ derived from the discussion in Footnote 9,

$$(I - \bar{\Gamma})^{-1}\mathbf{1} = \sum_{k=0}^{\infty} \bar{\Gamma}^k \mathbf{1} = \sum_{k=0}^{\infty} \beta^k \Gamma^k \mathbf{1} = \sum_{k=0}^{\infty} \beta^k \mathbf{1} = \frac{1}{1 - \beta} \mathbf{1}.$$

The third equality holds because Γ is a row-stochastic matrix ($\sum_{m'=1}^M \gamma_{mm'} = 1$ for all m) and so $\Gamma\mathbf{1} = \mathbf{1}$. Thus (7) and (8) directly imply (10). A similar exercise can be conducted for (9). Essentially, $\beta_m = \beta$ for all m gives every player the same Bonacich centrality. Information use and acquisition is determined only by the properties of the signals, and the results reduce to the symmetric-player world of Myatt and Wallace (2012).

A property of both this symmetric equilibrium (Proposition 5) and also in the presence of asymmetries (Proposition 2) is that the clearest (lowest ξ_i) information sources are those that are used. A clear information source is equivalent to one that is relatively cheap to acquire. (A lower marginal cost in the linear cost function is equivalent to a lower value of ξ_i .) Such cheap-to-acquire sources are used even if they do not accurately reflect the state of the world (that is, κ_i^2 is high) and so are related to the “fake news” sources discussed by Gentzkow and Allcott (2017). Moreover, relatively central players are those who are more susceptible to such fake news.

4.1. Asymmetric Coordination Motives. A necessary condition for asymmetric behaviour is that players differ in their desire to coordinate. This section describes briefly one situation in which such differences are present.

To proceed, suppose that $\gamma_{mm'} = 1/(M - 1)$ for all $m \neq m'$ so that there are no asymmetries in the connections between players. However, suppose that the aggregate coordination motives of players differ: ordering players appropriately, suppose that $0 < \beta_1 < \beta_2 < \dots < \beta_M$.¹² Given that aggregate coordination motives are the only source of asymmetry, it is unsurprising that they determine the centralities of the various players.

Lemma 3 (Coordination and Centrality). *Suppose that there are no asymmetries in the connections between players. Players with a stronger coordination motive are more central (in the sense of Bonacich) to the network: writing ζ_m for the m th element of $\zeta \equiv [I - \rho\bar{\Gamma}]^{-1}\mathbf{1}$, the players' centralities satisfy $\zeta_1 < \zeta_2 < \dots < \zeta_M$ for any $1 \geq \rho > 0$.*

With this in hand, earlier results apply immediately. Proposition 2 notes that the equilibrium weight placed on an endogenously acquired signal deviates from that signal's relative accuracy according to its relative clarity and according to the relevant player's Bonacich centrality. Similarly, Proposition 3 can also be applied directly.

Corollary (to Propositions 2 and 3). *Suppose that players differ only in their aggregate desire to coordinate, and consider an equilibrium in which players acquire and use the same set of information sources. A player with a stronger coordination motive makes more use of relatively clear information, and acquires less information overall.*

¹²The restriction to coordination (rather than anti-coordination) shortens proofs and speeds exposition.

5. A CORE-PERIPHERY NETWORK

That (at least two) players are differently influenced by others in aggregate is a necessary condition for asymmetry in the network structure to feed through into asymmetric information use and acquisition across players. Equivalently, players must have different Bonacich centralities. This section and the next present general formulations for two such networks, commonly found in the literature and analytically tractable, to explore how asymmetries in centrality affect asymmetries in information use and acquisition.

This section studies information use and acquisition across a two-type network, where the specification also allows for significant asymmetry within each group of types.

5.1. A Two-Type Network of Players. Partition the players into two subsets A and B of size M_A and $M_B = M - M_A$ respectively. Suppose that these subsets satisfy

$$\beta_m = \begin{cases} \beta_A & m \in A \\ \beta_B & m \in B \end{cases}, \quad \sum_{m' \in A} \gamma_{mm'} = \begin{cases} \omega_{AA} & m \in A \\ \omega_{BA} & m \in B \end{cases}, \quad \text{and} \quad \sum_{m' \in B} \gamma_{mm'} = \begin{cases} \omega_{AB} & m \in A \\ \omega_{BB} & m \in B \end{cases}.$$

These requirements say that two members of a group share the same aggregate concern for coordination with groups A and B respectively. For example, if players m and m' are both members of group A , then $\sum_{m'' \in B} \beta_m \gamma_{mm''} = \sum_{m'' \in B} \beta_{m'} \gamma_{m'm''}$. Similar claims apply when referring to intra-group coordination with other members of A . Furthermore, members of each group share the same aggregate coordination motive. Nevertheless, the opportunity for further substantial asymmetry remains.

This definition encompasses many important network structures. For instance, core-periphery networks fall under this specification, as do, therefore, star networks.

For a star network, suppose that player M is the hub player, connected to all other $M - 1$ players, who in turn are connected only to player M . The usual specification has $\beta_m \gamma_{mm'} = \beta_{m'} \gamma_{m'm}$ if m and m' are connected, and $\gamma_{mm'} = 0$ otherwise. This fits the definition above with $A = \{1, \dots, M - 1\}$, $B = \{M\}$, and with aggregate coordination motives satisfying $\beta_B = (M - 1)\beta_A$. Figure 1 illustrates such a specification.

However, the definition is broader than that: general core-periphery networks fit the construction here, as do many other network forms.¹³ The key advantage of networks with this two-type structure is that solving for the weights (on signals) essentially boils down to inverting a 2×2 matrix, for which an explicit solution is available.

5.2. Information Use. Players' information use is symmetric within each group. To see why, consider a strategy profile that satisfies intra-group symmetry. Now consider (for example) a member of group A . This player notes that all members of group B act in the same away. Thus, the desire to coordinate with all of them according to the relevant

¹³See Goyal (2007, Chapter 4, p. 80) for an example with $M_A = 8$ players on the periphery, $M_B = 4$ in the core, and $\beta_m \gamma_{mm'} = \beta_{m'} \gamma_{m'm}$ for connected players. The "windmill" networks of Dziubiński and Goyal (2017, p. 345) fall into this definition (at least, those with the same number of players in each clique do).

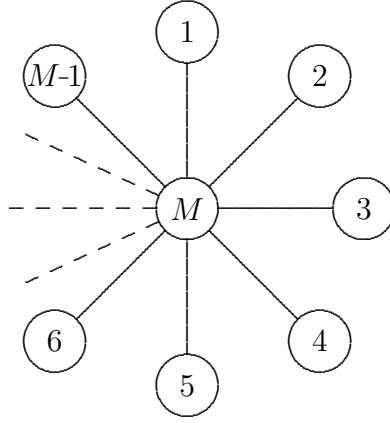


FIGURE 1. The Star Network with $M - 1$ Spokes

Lines represent undirected links from m to m' with $\beta_m \gamma_{mm'} = \beta_{m'} \gamma_{m'm}$. If there is no line then $\gamma_{mm'} = 0$. Players $\{1, \dots, M - 1\}$ on the spokes care about the actions of other spoke players only through an indirect connection via the hub player $\{M\}$.

row of the adjacency matrix $\bar{\Gamma}$ is equivalent to placing weight $\beta_{A\omega_{AB}}$ on one representative member of group B ; the same is true when thinking about co-members of group A . From this, it follows that all members of A will choose best replies symmetrically.

Given that this is the case, the coordination motives of the M players within $\bar{\Gamma}$ can be summarized via the much simpler 2×2 adjacency matrix $\bar{\Omega}$ where

$$\bar{\Omega} \equiv \begin{bmatrix} \beta_A \omega_{AA} & \beta_A \omega_{AB} \\ \beta_B \omega_{BA} & \beta_B \omega_{BB} \end{bmatrix}.$$

Hence $(I - \rho \bar{\Omega})^{-1} \mathbf{1}$ reports the Bonacich centralities of the two player groups.

The relative use of (exogenously provided) information by members of the two groups is determined by how public each signal is: whether one group rather than the other makes more use of a signal depends upon whether the correlation coefficient of that signal exceeds a critical value. To find that critical value, define

$$\phi(\rho) = (1 - \beta_A \omega_{AA} \rho)(1 - \beta_B \omega_{BB} \rho) - \beta_A \beta_B \omega_{AB} \omega_{BA} \rho^2 \quad (11)$$

which is the determinant of $(I - \rho \bar{\Omega})$. Next, define the weighted averages ψ_+ and ρ_+ as

$$\psi_+ = \sum_{i=1}^n \frac{\psi_i}{\phi(\rho_i)} \quad \text{and} \quad \rho_+ = \sum_{i=1}^n \frac{\psi_i \rho_i}{\phi(\rho_i)}. \quad \text{Finally, let } \hat{\rho} \equiv \frac{\rho_+}{\psi_+}. \quad (12)$$

The maintained assumption $|\beta_m| < 1$ guarantees $\phi(\rho) > 0$ for all $\rho \in [0, 1]$. Moreover, $\phi(\rho) \leq 1$ for all $\rho \in [0, 1]$. The ‘‘average publicity’’ term $\hat{\rho}$ is also between zero and one. As the formal result below confirms, whether a signal is more heavily used by members of A rather than B turns on whether or not ρ_i exceeds $\hat{\rho}$.

Despite any other intra-group asymmetries, and as noted above, players in A use the same weight on a signal i : define $w_{im} = w_{iA}$ for all $m \in A$; symmetrically for $m \in B$. The

proof of Proposition 6 shows that the equilibrium signal-use weights satisfy

$$w_{iJ} = \frac{\psi_i}{\psi_+} \frac{1}{\phi(\rho_i)} \left[\frac{\rho_i}{\hat{\rho}} + \left(1 - \frac{\rho_i}{\hat{\rho}}\right) \frac{1 - \beta_J \hat{\rho}}{\phi(\hat{\rho})} \right]$$

for $i \in \{1, \dots, n\}$ and $J \in \{A, B\}$. It is then straightforward to find

$$\Delta w_i \equiv w_{iA} - w_{iB} = \frac{\psi_i}{\psi_+} \frac{\beta}{\phi(\rho_i)} \frac{(\rho_i - \hat{\rho})(\beta_A - \beta_B)}{\phi(\hat{\rho})}.$$

The last term determines the sign of this expression, and therefore whether players in A or B use information source i more. The next proposition is immediate.

Proposition 6 (Relative Information Use by the Player Types). *If players in B care more about coordination than players in A , so that $\beta_B > \beta_A$, then players in B place more weight on a signal if and only if it is relatively public:*

$$w_{iA} < w_{iB} \quad \Leftrightarrow \quad \rho_i > \hat{\rho} \equiv \rho_+ / \psi_+.$$

The intuition is as before: relatively central players are those with stronger coordination motives, and they find that relatively public information is more useful for coordination because correlated signals reveal more about the actions of others.

5.3. Information Acquisition. The intuition above carries over to the case when players choose which signals (and how much of each) to acquire. Indeed, it is reinforced and compounded by the endogenous acquisition decisions made by the players.

Moving to a world in which signals are acquired endogenously, a first observation is that either players in A acquire a subset of those signals acquired by players in B or vice versa. An examination of the weight given to each signal i which is acquired in (7) provides some general intuition. Take the most central player. For this player ξ_i is sufficiently small such that the term inside the brackets in (7) is positive. Thus, it must be positive for all other players. Essentially, if the most central player uses a signal, so does everyone else. Of course, this argument ignores the fact that the equilibrium conditions in (7) apply only when every player acquires the same set of signals. However, the broad intuition carries over to the two-type setting.

Define the set of signals acquired in equilibrium by players in A and B respectively as $N_A = \{i : z_{iA} > 0\}$ and $N_B = \{i : z_{iB} > 0\}$, where $z_{iA} = z_{im}$ for $m \in A$ and similarly for z_{iB} . Similarly, define total acquisition as $Z_A = Z_m$ for $m \in A$ and $Z_B = Z_m$ for $m \in B$.

Lemma 4 (Nested Attention). *Either $N_A \subseteq N_B$ or $N_B \subseteq N_A$ or both.*

The intuition that if more central players (that is, players who are more influenced by the actions of others) acquire a particular signal then so will everyone else is robust.

Proposition 7 (The Attention Paid to Information Sources). *In a two-type network, if members of B care more about coordination than members of A , then players in B will acquire a (possibly weak) subset of the signals acquired by players in A : $\beta_B \geq \beta_A \Rightarrow N_B \subseteq N_A$. Moreover, this subset consists of the clearest (lowest ξ_i) signals in N_A .*

Not only do more central players place relatively high weight on relatively public signals, but they will also ignore entirely signals which are insufficiently clear. They do so even in circumstances when other players on the network pay attention to such (relatively private) information sources. In addition, the fact that more central players care more about the actions of others and less about the fundamental θ per se leads them not just to acquire fewer signals, but to acquire less information overall.

Proposition 8 (Total Information Acquisition in a Two-Type Network). *Suppose $\beta_B > \beta_A$, so that players in B are more concerned with coordination than players in A . $N_B \subseteq N_A$ by Proposition 7. Players in A acquire more information than players in B , $Z_A \geq Z_B$, and place less weight on relatively clear signals ($w_{iA} \leq w_{iB}$ if and only if ξ_i is sufficiently small). If $N_B = N_A$ then $w_{iA} \leq w_{iB} \Leftrightarrow \xi_i \leq \bar{\xi}$ where $\bar{\xi}$ is the accuracy-weighted average of ξ_i across the (common) set of acquired information sources.*

To see these propositions in action, recall the star network illustrated in Figure 1. Consider a simple example with just $n = 3$ information sources, with $\xi_1 < \xi_2 < \xi_3$, so that information sources are ordered by their clarity: 1 is the clearest. Suppose further that $\xi_3 > \xi_1 + (1 - \beta_A)\kappa_1^2$. This is sufficient for neither players on the spokes ($m \in A$) nor the hub player ($m \in B \equiv \{M\}$) to acquire a signal from source 3. Clarity determines whether a signal is acquired, and in this case source 3 is insufficiently clear for acquisition.

Suppose, on the other hand, that $\xi_2 < \xi_1 + (1 - \beta_A)\kappa_1^2$. Then certainly players in the spokes will acquire a signal from the second source. Proposition 7 can be applied: $\beta_B = (M - 1) \times \beta_A$ and so $N_B \subseteq N_A$. Whether the signals acquired by the hub player constitute a strict subset of those acquired by the spoke players or not depends critically upon M . In particular, if M is sufficiently small, so that the hub player is “not too central” then $N_B = N_A$ and the players acquire the same set of signals. However, if

$$M > M^* \equiv 1 + \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\beta_A[\beta_A\kappa_1^2 + (\xi_2 - \xi_1)]} > 2$$

then $w_{2B} = z_{2B} = 0$: the hub player M does not acquire a signal from information source 2. Instead, the hub player places all weight on the single clearest signal from source 1. In this case, the equilibrium values of the weights for spoke players in A are

$$w_{1A} = \frac{\beta_A\kappa_1^2 + \kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad w_{2A} = \frac{(1 - \beta_A)\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad \text{and} \quad w_{3A} = 0.$$

As mentioned, $w_{1B} = 1$ and $w_{2B} = w_{3B} = 0$. Thus $N_B = \{1\} \subset N_A = \{1, 2\}$. Now consider total information acquisition (or equivalently, the total cost paid for acquired information) by the different types of player. From (6), $Z_B = \xi_1$ trivially. $Z_A = \xi_1 w_{1A} + \xi_2 w_{2A}$ and so it is straightforward to verify the statement in Proposition 8 that $Z_A > Z_B$ (so long as $w_{2A} > 0$). Spoke players acquire more information than the hub player does.

5.4. Application: Asymmetric Cournot Competition. Consider now the model of asymmetric Cournot competition described in Section 1.4. Writing q_m for the action

choice of player m (indicating a quantity) the inverse-demand curve for product m is

$$p_m = \theta\alpha_m - \sum_{m'=1}^M \delta_{mm'} q_{m'}.$$

Now divide the suppliers into two groups A and B . Products within a group are perfect substitutes, but products from different groups are imperfectly substitutable. Specifically, if m and m' are within the same group then set $\delta_{mm'} = 1$, but if they are in different groups then set $\delta_{mm'} = \delta$ for some $\delta \in (0, 1)$. For a supplier within group A , for example,

$$m \in A \quad \Rightarrow \quad p_m = \theta\alpha_m - \sum_{m' \in A} q_{m'} - \delta \sum_{m' \in B} q_{m'}.$$

This specification endogenously generates a desire for coordination which satisfies

$$\beta_m = -\frac{\sum_{m' \neq m} \delta_{mm'}}{2\delta_{mm}} = -\frac{1}{2} \begin{cases} (M_A - 1) + \delta M_B & m \in A \\ (M_B - 1) + \delta M_A & m \in B \end{cases}$$

and so $\beta_A > \beta_B \Leftrightarrow |\beta_A| < |\beta_B| \Leftrightarrow M_A < M_B$.

Members of the smaller bloc (for example, group A if $M_A < M_B$) face less direct undifferentiated competition, and so the strength of the anti-coordination motive is weaker for them: $|\beta_A| < |\beta_B|$. This, of course, means that their overall coordination motive is stronger and so they place greater emphasis on relatively clear and relatively public information. In contrast, members of the larger group use a larger number of relatively private information sources in order to achieve anti-coordination.

The two-type network explored powerfully illustrates the core message of the paper. Not only do more central players use relatively clear (public) information more, they acquire more of it at the expense of (potentially quite accurate) relatively private signals, which they choose to ignore. Relatively central players acquire relatively little information overall; that which they do acquire is typically public in nature.

6. A HIERARCHY NETWORK

This section turns attention to a hierarchy network in which most players share the same aggregate concern for coordination, but nevertheless acquire and use information differently owing to their positions within the hierarchy.

6.1. The Hierarchy. Suppose players are arranged in a linear hierarchy. Player 1 (at the top) does not care about coordination: $\beta_1 = 0$. Others share the same coordination motive: $\beta_m = \beta$ for all $m > 1$. Player 2 is connected to player 1 only, player 3 is connected to player 2 only, and so on: for $m > 1$, $\gamma_{mm'} = 1$ if $m' = m - 1$ and is zero otherwise. This is a directed and asymmetric network. Players “further down the chain” care more about coordination, not directly, but rather through their indirect connections to those above.

Although the results of this section will focus on the above story for simplicity, a more general network structure can be accommodated. In particular, suppose that each level

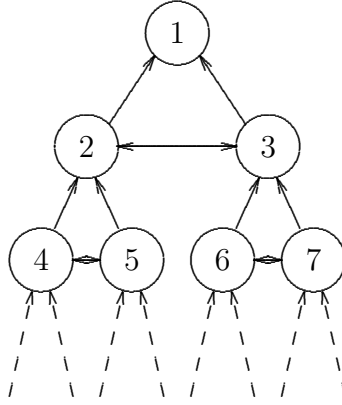


FIGURE 2. The Hierarchy Network with Two Members per Group

Arrows represent directed links from m to m' with $\gamma_{mm'} > 0$. If there is no arrow then $\gamma_{mm'} = 0$. Members of each (two player) group care about the actions of those within their group and those of a single player in the level directly above only.

in the hierarchy contains multiple players. Level $\ell \geq 2$ contains $(g + 1)^{\ell-2}$ groups, each containing $g + 1$ players whose payoffs depend upon the actions of all others within their group and exactly one player from the level above, $\ell - 1$. In level 1, there is a single player (player 1) who is unconnected to any other player.

A simple version is illustrated in Figure 2. Here, $g = 1$, $\gamma_{mm'} = \frac{1}{2}$ for all connections, and each group member is linked to exactly the same player in the level above. The payoff weighting attached to members of one's own group versus that attached to the player in the higher level may in general be different. For instance, set $\gamma_{mm'} = \gamma$ if m and m' are connected and in the same level and $\gamma_{mm'} = \gamma'$ if m' is the player from the level above m to whom m is linked (such that the normalization $\sum_{m' \neq m} \gamma_{mm'} = 1$ continues to hold). Else $\gamma_{mm'} = 0$. Note that it does not matter precisely to which player (or players) in level $\ell - 1$ the players from a single group in ℓ are connected: the equilibrium weights are the same for members of each level so long as the aggregate influence the actions of those above has upon them is the same. Nor will it matter precisely how many groups there are in any given level, nor their size: again, the aggregate influence the group's actions have upon each of its members is the only feature that matters.

This framework can be generalized even further with no important qualitative consequences for the results.¹⁴ Here, then, the focus will be on a simple case where each level is identified with a single player: $g = 0$ and $\ell = m$. Note that other than player 1, who has $\beta_1 = 0$, each player $m > 1$ has coordination preference parameter $\beta_m = \beta$. In this sense, a hierarchy constitutes a minimal departure from symmetry.

6.2. Information Use. Applying the first-order conditions of (5) in Lemma 1,

$$w_{jm} = \beta w_{j(m-1)} \rho_j + c_m \psi_j \quad \text{for } m > 1,$$

¹⁴Appendix B provides a recipe for doing so in the case where each level $\ell > 1$ contains several groups of $g + 1$ players. Aside from a technicality or two, the proofs involve nothing more than a change of variables.

and $w_{j1} = c_1 \psi_j$ for $m = 1$. Summing over j for $m = 1$ immediately yields

$$c_1 = \frac{1}{\sum_{i=1}^n \psi_i} \quad \text{and so} \quad w_{j1} = \frac{\psi_j}{\sum_{i=1}^n \psi_i} \equiv \hat{\psi}_j.$$

Player 1, entirely unaffected by those players below on the hierarchy, uses precision-weighted information. The objective is to explore information use for those players lower down the hierarchy. Players sufficiently far down the hierarchy behave as if the network was symmetric (see Proposition 4). Essentially, such players have the same centrality.

Proposition 9 (Information Use in a Hierarchy). *Consider a hierarchy: (i) $\beta_1 = 0$ and (ii) for $m > 1$, $\beta_m = \beta$ and $\gamma_{mm'} = 1$ only if $m' = m - 1 \geq 1$ and is zero otherwise. Then*

$$w_{j1} = \frac{1}{\kappa_j^2 + \xi_j^2} \bigg/ \sum_{i=1}^n \frac{1}{\kappa_i^2 + \xi_i^2} \quad \text{and} \quad w_{jM} \rightarrow \frac{1}{(1-\beta)\kappa_j^2 + \xi_j^2} \bigg/ \sum_{i=1}^n \frac{1}{(1-\beta)\kappa_i^2 + \xi_i^2}$$

as $M \rightarrow \infty$. Player 1 uses each information source in proportion to its precision, whereas players very far “down the chain” use weights approximately proportional to the precision-weighted publicity of each source. Moreover, c_m , the constant proportional to the multiplier on the constraint associated with the weights is decreasing in m .

Moving down the chain of the hierarchy is equivalent to following a chain of iterative best replies, which naturally converges (further down the chain) to the equilibrium use of information in a game where all players share the same coordination motive.

6.3. Information Acquisition. Players within the hierarchy typically acquire different sets of signals. Without loss, order the information sources by clarity so that $\xi_1 < \xi_2 < \dots < \xi_n$, and define $N_m = \{i : z_{im} > 0\} \subseteq \{1, \dots, n\}$. Further, let $n_m = \max\{i \in N_m\}$. $n_m \leq n$ is the least clear signal that player m acquires and uses.

The objective is to show that $N_m = \{1, \dots, n_m\} \subseteq N_{m-1}$ for all $m > 1$: that is, lower players in the hierarchy acquire (weakly) fewer signals than higher players, and that these consist of precisely the n_m clearest (lowest ξ_i) signals. Certainly player 1 acquires a subset consisting of the clearest signals. To see this, note that from (6), $w_{j1} = (c_1 - \xi_j)/\kappa_j^2$ for any $j \in N_1$. Now c_1 is constant across i , so if for any $j > 1$, $\xi_j < c_1$ then $\xi_{j-1} < c_1$. But then $w_{j-1,1} > 0$ and hence $z_{j-1,1} > 0$. Indeed, c_1 can be directly calculated from (6) and

$$\sum_{i \in N_1} w_{i1} = 1, \quad \text{so} \quad c_1 = \frac{1 + \sum_{i \in N_1} \xi_i / \kappa_i^2}{\sum_{i \in N_1} 1 / \kappa_i^2}.$$

The proof to the next proposition, which characterizes the acquired sets of signals for each player in equilibrium, confirms N_1 is uniquely determined. Thus $N_1 = \{1, \dots, n_1\}$ as required. The following lemma is a first step.

Lemma 5 (Shrinking Signal Acquisition). *If player m does not acquire signal j then nor does any later player $m' > m$ in the hierarchy. That is $z_{jm} = 0 \Rightarrow z_{jm'} = 0$ for all $m' > m$.*

The next proposition uses this lemma to characterize the sets of signals acquired by each player on the hierarchy network in equilibrium. Players “further down” the hierarchy

acquire fewer, relatively clear, signals. In fact, they acquire subsets of signals acquired by players above them in the hierarchy, consisting of the most clear. As mentioned above, player m chooses to acquire the clearest n_m signals only, and $n_{m+1} \leq n_m$ for all m .

Proposition 10 (Information Acquisition in a Hierarchy). *Each player in the hierarchy acquires a (weak) subset of the signals acquired by the player above. These consist of the most clear (lowest ξ_j) signals. That is, for all $m \geq 1$, there is a unique n_m such that $w_{jm} > 0 \Leftrightarrow z_{jm} > 0$ for all $j \leq n_m$ and $w_{jm} = 0 \Leftrightarrow z_{jm} = 0$ for all $j > n_m$ with $n_{m+1} \leq n_m$.*

Proposition 10 states that players further down the chain use a weak subset of the signals used by those above. This subset can be strict, as a simple example suffices to show. Consider, for instance, an M player chain with just $n = 3$ information sources ordered in terms of their clarity $\xi_1 < \xi_2 < \xi_3$. Let

$$\xi_1 < \xi_1 + \frac{\kappa_1^2}{1 + \beta} < \xi_2 < \xi_1 + \kappa_1^2 < \xi_3.$$

(Note $\beta < 1$, so this chain of inequalities is feasible.) With this example, $n_1 = 2$ and $n_m = 1$ for all $m \geq 2$. The weights on the signals are

$$w_{11} = \frac{\kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad w_{21} = \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2}, \quad \text{and} \quad w_{31} = 0.$$

Furthermore, $w_{1m} = 1$ and $w_{2m} = w_{3m} = 0$ for all $m \geq 2$. The top player in the hierarchy ignores information source 3, but acquires signals from sources 1 and 2. Lower players acquire a signal from source 1 only (and trivially must place weight 1 upon it, therefore). The constants (which do not converge until $m = 3$) satisfy

$$c_1 = \frac{\kappa_1^2 \kappa_2^2 + \xi_1 \kappa_2^2 + \xi_2 \kappa_1^2}{\kappa_1^2 + \kappa_2^2}, \quad c_2 = \kappa_1^2 + \xi_1 - \frac{\beta \kappa_1^2}{\kappa_1^2 + \kappa_2^2} [\kappa_2^2 + (\xi_2 - \xi_1)], \quad \text{and} \quad c_m = (1 - \beta) \kappa_1^2 + \xi_1,$$

for all $m \geq 3$. It's straightforward to check $c_1 > c_2 > c_m$ where $m \geq 3$. Note $c_2 \neq c_3$, even though $n_2 = n_3$ and $w_{i2} = w_{i3}$ for all i . The two players acquire the same signals and use the same weights, but player 3 values information less than player 2 (in some sense, at least). Total acquisition, equivalently the total cost paid for information, is given by

$$Z_1 = \xi_1 \frac{\kappa_2^2 + (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2} + \xi_2 \frac{\kappa_1^2 - (\xi_2 - \xi_1)}{\kappa_1^2 + \kappa_2^2} \quad \text{and} \quad Z_m = \xi_1,$$

for all $m \geq 2$. A quick calculation confirms that $Z_1 > Z_m \Leftrightarrow \xi_1 < \xi_2$, as assumed. Thus the player at the top of the hierarchy acquires more information than those below. Indeed, this feature applies more generally to any hierarchy network—total information acquisition is decreasing in m : $\sum_{i=1}^n z_{im} \equiv Z_m \leq Z_{m-1}$ for all $m > 1$. In other words, with linear costs $C_m(z_m) = \sum_{i=1}^n z_{im}$ in place, players further down the hierarchy acquire less information (or equivalently pay less for information acquisition) and hence place a lower value on information, confirming the intuition in the above example.

Proposition 11 (Total Information Acquisition in a Hierarchy). *Players lower down in the hierarchy acquire (weakly) less information than those above: $Z_m \leq Z_{m-1}$ for all $m > 1$. Player $m+1$ places more weight on signal j than player m does ($w_{jm+1} > w_{jm}$) and*

acquires more of j ($z_{jm+1} > z_{jm}$) if and only if j is clear enough. If players m and $m + 1$ use the same set of signals ($N_m = N_{m+1}$) then player $m + 1$ places more weight on signal j than player m does (and acquires more of it) if and only if j is clearer than average.

This reemphasizes the main message: more central players (in this case, those further down the hierarchy) acquire less information, acquire relatively clear (or public) information, and use relatively clear (or public) information more intensively. The set of sources they acquire is a subset of those acquired by less central (higher) players.

7. RELATED LITERATURE AND CONCLUDING REMARKS

This paper links two strands of literature: studies of the social value of information in games after Morris and Shin (2002), and studies of coordination (or anti-coordination) games played on a network following Ballester, Calvó-Armengol, and Zenou (2006).

The contribution of Morris and Shin (2002) generated a large literature investigating information use in quadratic-payoff coordination games and the welfare consequences thereof. In such games, payoffs depend upon the distance of players' actions from some unknown fundamental and their distance from some aggregate of the actions of others (the average, for instance). The typical setting is one in which all players receive public (perfectly correlated) and private (completely uncorrelated) signals about the fundamental. Their actions are influenced more by public information: such signals are relatively important for higher-order beliefs, and so inform players about the likely actions of others. Such models have been applied widely, for instance to investment games with complementarities, business cycles, oligopoly games, political leadership, and financial markets (Angeletos and Pavan, 2004, 2007; Myatt and Wallace, 2014, 2015, 2017; Dewan and Myatt, 2008, 2012; Allen, Morris, and Shin, 2006).

This paper allows for the endogenous acquisition of multiple information sources. The information structure was introduced (and applied to a model of political leadership) in Dewan and Myatt (2008, 2012). It was extended (in a "beauty contest" model) by Myatt and Wallace (2012), and has been applied in a Lucas-Phelps island economy (Myatt and Wallace, 2014), to Cournot and price-setting industries (Myatt and Wallace, 2015, 2017, respectively), and beyond (see Pavan, 2016, for a recent application of this structure). In this approach, different signals may be acquired by the players at some cost (or ignored): the publicity of each information source is an equilibrium phenomenon rather than exogenously fixed at the outset (for an alternative approach to information acquisition in coordination games, see Hellwig and Veldkamp, 2009). Several recent contributions, including the paper by Leister (2017) discussed below, allow players to choose the precision of a single private signal (see for instance Colombo, Femminis, and Pavan, 2014; Llosa and Venkateswaran, 2013). Almost all of these papers focus on a world with symmetric players (often modelled as a continuum, so that no single player's action has any

effect upon the aggregate).¹⁵ Relative to this literature, a novelty here is that the model admits a very general class of asymmetries in preferences, captured by the weights of links in the network on which players are arranged. It does so while retaining a broader information structure. This allows an exploration of how network position (or preference diversity) affects the qualities of the information optimally acquired and used.

The model of Ballester, Calvó-Armengol, and Zenou (2006) is closely related. They studied a general class of quadratic-payoff games where players' preferences are described by the weights attached to the actions of players to whom they are linked. In a complete information setting, players' (weighted Bonacich) centralities determine their equilibrium actions. This connects an older literature (Katz, 1953; Bonacich, 1987) on indices of network centrality (or status) to equilibrium play in a broad class of games. The current paper shows that this connection naturally extends to situations of incomplete and dispersed information. Player centrality and an appropriate measure of signal publicity combine to determine information use and acquisition in such games.

The structure of Ballester, Calvó-Armengol, and Zenou (2006) has been taken up by much of the network literature, and provides the starting point of the aforementioned paper by Leister (2017).¹⁶ This recent contribution studies a quadratic-payoff game in which players are arranged on a network and have access to a single, “perfectly private”, signal. Players can control the precision of this signal at some cost. The novelty relative to that paper is therefore (most obviously) that multiple information sources with different underlying properties (accuracy and clarity) are available to players. More subtly, in the model here, players are able to ignore various of their signals entirely, whereas Leister (2017) assumes enough structure to ensure an interior equilibrium. On the other hand, the analysis of Leister (2017) admits a wider range of cost functions; considers a variant in which the precision choices of players are publicly observed prior to play; and focuses on the consequences for welfare. The messages that emerge are complementary, and it remains an open (and welcome) question as to whether the conclusions reached there carry over to the more general information structure studied here.

Along similar lines, a contemporaneous study by Denti (2017) models “flexible” information acquisition by players arranged on a network. Here, players design their signal's correlation structure with the signals received by others and the state. In equilibrium, the cost of acquiring such information is shown to take the entropy form of the rational inattention literature (Sims, 2003, 2006) if and only if better information is more costly in (a strengthened version of) the Blackwell sense. As in the current paper, Bonacich centrality combines with—now optimally chosen with entropic costs—signal correlation

¹⁵Exceptions include Myatt and Wallace (2017), where an application-specific asymmetry arises from supplier size differences in a price-setting industry, Leister (2017), and Denti (2017), both discussed below.

¹⁶The structure is extended in Belhaj, Bramoullé, and Deroïan (2014); König, Testone, and Zenou (2014) study the stochastic stability of equilibria in a network-formation game in which payoffs take this form. For applications to monopoly (and oligopoly) pricing in which the network architecture reflects consumption externalities (and with quadratic utilities along these lines), see Fainmesser and Galeotti (2016a,b). None of these papers studies the way in which players acquire and use heterogeneous information.

to determine the network effects of information acquisition. As a result of the non-convexities generated by entropic costs, however, and unlike here, multiplicity may arise (as in the related but symmetric example of Section 9 in Myatt and Wallace, 2012).

Somewhat more distantly related is a recent paper by Golub and Morris (2017). Their focus is on the convergence of higher-order expectations (to “consensus expectations”) rather than asymmetric coordination games per se. The network represents the extents to which agents use others’ expectations to form their own (higher-order) expectations. Nonetheless, as they point out (Section 3.1), there is a close relationship between this model and the study of asymmetric coordination games of the sort played here. Their model differs in some technical details (for example, they consider finite state and signal spaces, bounded actions, and focus on the case $\beta \rightarrow 1$) and information acquisition is not modelled, but nonetheless the connection to centrality remains, and their “tyranny of the uninformed” (that the least informed player’s expectations are the most influential in determining the consensus) provides an interesting counterpoint to the results here.

There is also much work on information transmission and communication in networks. Once again, a variant of the structure in Ballester, Calvó-Armengol, and Zenou (2006) is often used as the starting point. A recent example is the work of Herskovic and Ramos (2015), in which a network-formation game is played by players who each initially have access to a single (uncorrelated) signal about the fundamental. Rather than (as here) investigating the impact of network structure on information use, that paper (and much of the literature from which it proceeds) explores the impact that information use has on network structure. Interestingly, publicity is key here also: players with particularly “good” information attract others who link with them in order to observe a (noisy) signal of their information. The more who link, the more public the signal becomes, the more useful it is to others trying to coordinate.¹⁷ Thinking of such players as “opinion makers”, Herskovic and Ramos (2015) relate their result to the origins of leadership.

Earlier work in this vein includes Calvó-Armengol and de Martí (2007, 2009), who study communication and information transmission on networks. In these papers the network describes the communication links between players, rather than the extent to which the various players care about the actions of their opponents. The underlying game studied is symmetric; players receive a single (private) signal, and these signals are then communicated to others via the network structure. In an asymmetric setting, Calvó-Armengol, de Martí, and Prat (2015) study communication in which players can control the precision with which they send and receive signals to and from others (at some

¹⁷So, in Herskovic and Ramos (2015), the better informed become the more influential. This contrasts with the “tyranny of the uninformed” result of Golub and Morris (2017) mentioned above (and the distinction is discussed in some detail in the latter). It is interesting to compare these results with the observation of the current paper, that relatively centrally located players tend to focus on fewer, relatively public, signals. In other words, those who are more influenced by others acquire less information.

cost).¹⁸ Again, players exogenously receive only a single, private, signal; although the aggregated information is endogenously public via the communication process.^{19,20}

Relative to the social value of information literature, this paper studies a general class of preference asymmetries via the network on which players are arranged. Relative to the literature on games and communication in networks, the model here focuses on the acquisition and use of information sources independent of the network structure itself. Instead, the model incorporates a rich correlation structure over multiple different sources whose publicity and precision are affected by the acquisition decisions of the players themselves. A key contribution is to identify a connection between information acquisition (and use), the signal's publicity, and the players' centrality in the network.

APPENDIX A. PROOFS OF LEMMAS AND PROPOSITIONS

Proof of Lemma 1. This follows from the arguments in the main text. Uniqueness follows from the invertibility of the matrix described in the proof to Proposition 1, below. \square

Proof of Proposition 1. $|\rho_i \beta_m| < 1$ for all m and so $[\mathbf{I} - \rho_i \bar{\Gamma}]$ has full rank for all i . Using vector notation, the first-order condition of (5) in Lemma 1 can be written as $\mathbf{w}_i = \psi_i [\mathbf{I} - \rho_i \bar{\Gamma}]^{-1} \mathbf{c}$. Summing over signals gives $\mathbf{1} = \sum_{j=1}^n \psi_j [\mathbf{I} - \rho_j \bar{\Gamma}]^{-1} \mathbf{c}$. There is a unique solution to the optimization problem if and only if $\sum_{j=1}^n \psi_j [\mathbf{I} - \rho_j \bar{\Gamma}]^{-1}$ is invertible: $\mathbf{c} = [\sum_{j=1}^n \psi_j [\mathbf{I} - \rho_j \bar{\Gamma}]^{-1}]^{-1} \mathbf{1}$.²¹ Then, as required, the equilibrium weights may be written in vector notation as

$$\mathbf{w}_i = \psi_i [\mathbf{I} - \rho_i \bar{\Gamma}]^{-1} \left[\sum_{j=1}^n \psi_j [\mathbf{I} - \rho_j \bar{\Gamma}]^{-1} \right]^{-1} \mathbf{1} = \psi_i \sum_{k=0}^{\infty} (\rho_i)^k \bar{\Gamma}^k \left[\sum_{j=1}^n \psi_j \sum_{l=0}^{\infty} (\rho_j)^l \bar{\Gamma}^l \right]^{-1} \mathbf{1},$$

where the second equality follows the discussion in Footnote 9, and which further justifies the discussion immediately following the proposition. \square

Proof of Lemma 2. Form the relevant Lagrangian from (4) and differentiate with respect to w_{im} and z_{im} for interior solutions. Apply the relevant constraints (noting $z_{im} \geq 0$ for all i). Complementary slackness conditions apply for i such that $z_{im} = w_{im} = 0$. \square

¹⁸Interestingly, players are unable to ignore entirely another player's signal by assumption. The current paper, the paper by Herskovic and Ramos (2015) discussed above, and the work of Currarini and Feri (2015), who study bilateral information sharing on networks, suggest this may not be entirely innocuous.

¹⁹Similar questions relating to information transmission in networks are addressed by Hagenbach and Koessler (2010) and Galeotti, Ghiglino, and Squintani (2013). In those papers, communication is modelled as "cheap talk" and payoff asymmetries enter through biases on the fundamental motive (rather than via the coordination motive). The focus is, therefore, upon the potential for credible communication.

²⁰In related work, Galeotti and Goyal (2010) present a model in which the players' payoffs depend on information acquired from their neighbours. Their focus is on the outcome of a network formation process when equilibrium play of the game is itself network-dependent (using the networked public-good provision game of Bramoullé and Kranton 2007, later generalized in Bramoullé, Kranton, and D'Amours 2014). A recent experimental treatment of these network formation issues can be found in Goyal, Rosenkranz, Weitzel, and Buskens (2017), while Galeotti, Goyal, Jackson, Vega-Redondo, and Yariv (2010) investigate a variant in which players have incomplete information about the structure of the network.

²¹This is not immediate: the sum of many invertible matrices is not necessarily itself invertible. It can be guaranteed by restricting the β_m parameters (that they be small enough: the inverse exists if $\beta_m = 0$ for all m , so continuity of the matrix inverse function guarantees such positive values can be found).

Proof of Proposition 2. The first-order conditions in (6) may be written $\mathbf{w}_i = \bar{\Gamma} \mathbf{w}_i + (1/\kappa_i^2)[\mathbf{c} - \xi_i \mathbf{1}]$ so long as $w_{im} > 0$ for every m , that is $i \in N_*$. Restricting to the case where if i is acquired by any player m then i is acquired by all others too, so that $w_{im} > 0$ for all m ,

$$[\mathbf{I} - \bar{\Gamma}] \mathbf{w}_i = \frac{1}{\kappa_i^2} [\mathbf{c} - \xi_i \mathbf{1}] \quad \Rightarrow \quad \mathbf{w}_i = \frac{1}{\kappa_i^2} [\mathbf{I} - \bar{\Gamma}]^{-1} [\mathbf{c} - \xi_i \mathbf{1}] \quad \Rightarrow$$

$$\mathbf{1} = \sum_{i \in N_*} w_i = \sum_{i \in N_*} \frac{1}{\kappa_i^2} [\mathbf{I} - \bar{\Gamma}]^{-1} [\mathbf{c} - \xi_i \mathbf{1}] = [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{c} \sum_{i \in N_*} \frac{1}{\kappa_i^2} - [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} \sum_{i \in N_*} \frac{\xi_i}{\kappa_i^2},$$

which, using the definition of $\bar{\xi}_*$ given in (7), can be solved explicitly for \mathbf{c} :

$$\mathbf{c} = \frac{1}{\sum_{i \in N_*} 1/\kappa_i^2} [\mathbf{I} - \bar{\Gamma}] \mathbf{1} + \bar{\xi}_* \mathbf{1}.$$

Now \mathbf{c} can be substituted back into the expression for \mathbf{w}_i , giving

$$\mathbf{w}_i = \frac{1}{\kappa_i^2} \left\{ \frac{1}{\sum_{j \in N_*} 1/\kappa_j^2} \mathbf{1} - (\xi_i - \bar{\xi}_*) [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} \right\},$$

The fact that players acquire the clearest signals follows from inspection. \square

Proof of Proposition 3. Multiply (7) through by ξ_i to obtain

$$\mathbf{z}_i = \frac{\xi_i}{\kappa_i^2} \left\{ \frac{1}{\sum_{j \in N_*} 1/\kappa_j^2} \mathbf{1} - (\xi_i - \bar{\xi}_*) [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} \right\},$$

then sum over $i \in N_*$, and note the identity $\sum_{i \in N_*} \xi_i (\xi_i - \bar{\xi}_*) / \kappa_i^2 \equiv \sum_{i \in N_*} (\xi_i - \bar{\xi}_*)^2 / \kappa_i^2$, by the definition of $\bar{\xi}_*$ in (7), yielding the expression in (8). \square

Proof of Proposition 4. Follows directly from arguments in the main text. \square

Proof of Proposition 5. The first part and (10) follow directly from arguments in the main text. The formulation of N_* follows immediately from inspection of the first equation in (10). For N_* unique, see the proof in Myatt and Wallace (2012, Proposition 2). \square

Proof of Lemma 3. For expositional simplicity (and without loss of generality) set $\rho = 1$. The vector of Bonacich centralities is $\zeta = [\mathbf{I} - \bar{\Gamma}]^{-1} \mathbf{1} = \sum_{k=0}^{\infty} \bar{\Gamma}^k \mathbf{1}$ where $\zeta^k \equiv \bar{\Gamma}^k \mathbf{1}$. ζ^1 satisfies $\zeta_m^1 = \beta_m$ and so (i) $\zeta_1^1 < \dots < \zeta_M^1$ and (ii) $(\zeta_1^1 / \beta_1) \geq \dots \geq (\zeta_M^1 / \beta_M)$. This is an induction basis. As an induction hypothesis suppose that, for $k \geq 1$, both (i) and (ii) hold. Now, for any $m < M$,

$$\zeta_m^{k+1} = \frac{\beta_m}{M-1} \sum_{m' \neq m} \zeta_{m'}^k \quad \text{and so} \quad \zeta_m^{k+1} < \zeta_{m+1}^{k+1} \quad \Leftrightarrow \quad \beta_m \sum_{m' \neq m} \zeta_{m'}^k < \beta_{m+1} \sum_{m' \neq m+1} \zeta_{m'}^k$$

$$\Leftrightarrow \quad \beta_m \zeta_{m+1}^k - \beta_{m+1} \zeta_m^k < (\beta_{m+1} - \beta_m) \sum_{m' \neq m, m+1} \zeta_{m'}^k.$$

The right-hand side is positive, and so a sufficient condition for this to hold is $\beta_m \zeta_{m+1}^k \leq \beta_{m+1} \zeta_m^k$ or equivalently $(\zeta_{m+1}^k / \beta_{m+1}) \leq (\zeta_m^k / \beta_m)$, which holds from the induction hypothesis. Further,

$$\frac{\zeta_m^{k+1}}{\beta_m} \geq \frac{\zeta_{m+1}^{k+1}}{\beta_{m+1}} \quad \Leftrightarrow \quad \sum_{m' \neq m} \zeta_{m'}^k \geq \sum_{m' \neq m+1} \zeta_{m'}^k \quad \Leftrightarrow \quad \zeta_{m+1}^k \geq \zeta_m^k,$$

which also holds owing to the induction basis. By the principle of induction, (i) and (ii) hold for all k . This in turn implies that $\zeta_m = \sum_{k=0}^{\infty} \zeta_m^k$ is strictly increasing in m . \square

Proof of Proposition 6. A solution will be found by assuming $w_{jm} = w_{jA}$ and $c_m = c_A$ for all $m \in A$; $w_{jm} = w_{jB}$ and $c_m = c_B$ for all $m \in B$. Then, using (5) in Lemma 1,

$$\begin{aligned} w_{jA} &= \beta_A[\omega_{AA}w_{jA} + \omega_{AB}w_{jB}]\rho_j + c_A\psi_j; \\ w_{jB} &= \beta_B[\omega_{BA}w_{jA} + \omega_{BB}w_{jB}]\rho_j + c_B\psi_j. \end{aligned}$$

These equations solve readily to yield w_{jA} and w_{jB} in terms of c_A and c_B . For example,

$$w_{jA} = \psi_j \frac{\beta_A\omega_{AB}\rho_j c_B + (1 - \beta_B\omega_{BB}\rho_j)c_A}{(1 - \beta_A\omega_{AA}\rho_j)(1 - \beta_B\omega_{BB}\rho_j) - \beta_A\beta_B\omega_{AB}\omega_{BA}\rho_j^2}. \quad (13)$$

Clearly, an equivalent is readily available for w_{jB} simply by swapping A and B in the above wherever they occur. Now, summing over j , and using $\sum_{i=1}^n w_{im} = 1$ for all m ,

$$1 = \beta_A\omega_{AB}\rho_+c_B + (\psi_+ - \beta_B\omega_{BB}\rho_+)c_A = \beta_B\omega_{BA}\rho_+c_A + (\psi_+ - \beta_A\omega_{AA}\rho_+)c_B, \quad (14)$$

where ρ_+ and ψ_+ are given in (12). Equating the two expressions in (14), collecting terms, and noting that $\omega_{AA} + \omega_{AB} = \omega_{BA} + \omega_{BB} = 1$,

$$\frac{c_A}{c_B} = \frac{\psi_+ - \beta_A\rho_+}{\psi_+ - \beta_B\rho_+}, \quad (15)$$

from which $c_A > c_B \Leftrightarrow \beta_A < \beta_B$ is immediate. Now using (13), cancelling the common denominator and the ψ_j terms, $w_{jA} < w_{jB}$ if and only if

$$\beta_A\omega_{AB}\rho_j c_B + (1 - \beta_B\omega_{BB}\rho_j)c_A < \beta_B\omega_{BA}\rho_j c_A + (1 - \beta_A\omega_{AA}\rho_j)c_B.$$

Collecting terms and rewriting, this holds if and only if

$$\frac{1 - \beta_B\rho_j}{1 - \beta_A\rho_j} < \frac{c_B}{c_A} = \frac{1 - \beta_B\rho_+/\psi_+}{1 - \beta_A\rho_+/\psi_+},$$

where the last equality follows from (15). Assuming $\beta_A < \beta_B$ the first ratio is decreasing in ρ_j , so the inequality is equivalent to $\rho_j > \rho_+/\psi_+ \equiv \hat{\rho}$. \square

Proof of Lemma 4. The first-order conditions for w_{iA} and w_{iB} (when positive) from (6) are

$$w_{iA} = \beta_A[\omega_{AA}w_{iA} + \omega_{AB}w_{iB}] + \frac{c_A - \xi_i}{\kappa_i^2} \quad \text{and} \quad w_{iB} = \beta_B[\omega_{BB}w_{iB} + \omega_{BA}w_{iA}] + \frac{c_B - \xi_i}{\kappa_i^2}. \quad (16)$$

The above first-order conditions apply if $i \in N_A \cap N_B \equiv N_{A \cap B}$. For $i \in N_A \cap \neg N_B \equiv N_{A/B}$,

$$w_{iA} = \beta_A\omega_{AA}w_{iA} + \frac{c_A - \xi_i}{\kappa_i^2} \quad \text{and} \quad w_{iB} = 0.$$

Clearly, for $i \in N_B \cap \neg N_A \equiv N_{B/A}$ the expressions are reversed. So, first, suppose that there exists $i \neq j$ such that $i \in N_{A/B}$ and $j \in N_{B/A}$. Then

$$w_{iA} = \frac{1}{1 - \beta_A\omega_{AA}} \frac{c_A - \xi_i}{\kappa_i^2} > 0 \quad \text{and} \quad w_{jB} = \frac{1}{1 - \beta_B\omega_{BB}} \frac{c_B - \xi_j}{\kappa_j^2} > 0, \quad (17)$$

whereas $w_{iB} = w_{jA} = 0$. For a player in B to not use signal i , the right-hand side of the first-order condition given above in (16) must be weakly negative. That is,

$$\beta_B[\omega_{BB}w_{iB} + \omega_{BA}w_{iA}] + \frac{c_B - \xi_i}{\kappa_i^2} \leq 0.$$

In equilibrium, then, $\xi_i \geq \beta_B\omega_{BA}w_{iA}\kappa_i^2 + c_B = \beta_B\omega_{BA}(c_A - \xi_i)/(1 - \beta_A\omega_{AA}) + c_B$. Equivalently,

$$\xi_i \geq \frac{\beta_B\omega_{BA}c_A + (1 - \beta_A\omega_{AA})c_B}{1 - \beta_A\omega_{AA} + \beta_B\omega_{BA}}.$$

Now $w_{iA} > 0$, so $c_A > \xi_i$. This in turn implies that, for signal i ,

$$c_A > \xi_i \geq \frac{\beta_B \omega_{BA} c_A + (1 - \beta_A \omega_{AA}) c_B}{1 - \beta_A \omega_{AA} + \beta_B \omega_{BA}}, \quad \text{and so } c_A > c_B. \quad (18)$$

However, the very same exercise for signal $j \neq i$ can be conducted, yielding

$$c_B > \xi_j \geq \frac{\beta_A \omega_{AB} c_B + (1 - \beta_B \omega_{BB}) c_A}{1 - \beta_B \omega_{BB} + \beta_A \omega_{AB}}, \quad \text{and so } c_B > c_A.$$

Clearly, then, there cannot be both an $i \in N_{A/B}$ and a $j \in N_{B/A}$ in equilibrium. \square

Proof of Proposition 7. Suppose $N_B \subseteq N_A$. B -types use a (possibly weak) subset of the signals used by A -types. (16) and the first expression in (17) provide the first-order conditions for $i \in N_{A \cap B}$ and $i \in N_{A/B}$ respectively. $w_{iB} = 0$ for all $i \in N_{A/B}$ and all other weights are zero.

Consider the implication $N_B \subset N_A \Rightarrow \beta_B > \beta_A$ first. Suppose indeed that $N_B \subset N_A$. Then $N_{A \cap B} = N_B$. Substitute the first-order conditions for w_{iA} into those for w_{iB} in (16) for all $i \in N_B$ (recalling that $N_B \subset N_A$). This exercise yields

$$w_{iB}(1 - \beta_B \omega_{BB}) = \frac{\beta_B \omega_{BA}}{1 - \beta_A \omega_{AA}} \left[\beta_A \omega_{AB} w_{iB} + \frac{c_A - \xi_i}{\kappa_i^2} \right] + \frac{c_B - \xi_i}{\kappa_i^2}.$$

Rearranging to solve for w_{iB} , and using $\phi \equiv \phi(1)$ from (11),

$$\begin{aligned} \phi w_{iB} &= \beta_B \omega_{BA} \frac{c_A - \xi_i}{\kappa_i^2} + (1 - \beta_A \omega_{AA}) \frac{c_B - \xi_i}{\kappa_i^2} \\ w_{iB} &= \frac{1}{\phi \kappa_i^2} \left[\beta_B \omega_{BA} c_A + (1 - \beta_A \omega_{AA}) c_B - \xi_i (\beta_B \omega_{BA} + (1 - \beta_A \omega_{AA})) \right]. \end{aligned}$$

Now, summing over N_B , and noting $\sum_{j \in N_B} w_{jB} = 1$,

$$\beta_B \omega_{BA} c_A + (1 - \beta_A \omega_{AA}) c_B = \frac{\phi}{\sum_{j \in N_B} 1/\kappa_j^2} + \bar{\xi}_B (1 - \beta_A \omega_{AA} + \beta_B \omega_{BA}), \quad (19)$$

where $\bar{\xi}_B$ is the accuracy-weighted average clarity over signals used by type- B players (explicitly written in Proposition 8). Thus, for such $i \in N_B$, weights for B types are

$$w_{iB} = \frac{1/\kappa_i^2}{\sum_{j \in N_B} 1/\kappa_j^2} + \phi_B \left(\frac{\bar{\xi}_B - \xi_i}{\kappa_i^2} \right), \quad \text{with } \phi_B \equiv \frac{1 - \beta_A \omega_{AA} + \beta_B \omega_{BA}}{(1 - \beta_A \omega_{AA})(1 - \beta_B \omega_{BB}) - \beta_A \beta_B \omega_{AB} \omega_{BA}}. \quad (20)$$

The weights for type- A players depend on whether type- B players are using the signals or not. When they are, (16) applies for w_{iA} , when not, (17) applies. So

$$\begin{aligned} i \in N_{A/B} &\Rightarrow w_{iA} = \frac{1}{1 - \beta_A \omega_{AA}} \frac{c_A - \xi_i}{\kappa_i^2} \\ i \in N_{A \cap B} &\Rightarrow w_{iA} = \frac{1}{1 - \beta_A \omega_{AA}} \frac{c_A - \xi_i}{\kappa_i^2} + \frac{\beta_A \omega_{AB}}{1 - \beta_A \omega_{AA}} w_{iB}. \end{aligned} \quad (21)$$

Summing over all $i \in N_A = N_{A \cap B} \cup N_{A/B}$ and noting that $\sum_{j \in N_A} w_{jA} = \sum_{j \in N_B} w_{jB} = 1$,

$$1 = \frac{1}{1 - \beta_A \omega_{AA}} \left[c_A \sum_{j \in N_A} \frac{1}{\kappa_j^2} - \sum_{j \in N_A} \frac{\xi_j}{\kappa_j^2} \right] + \frac{\beta_A \omega_{AB}}{1 - \beta_A \omega_{AA}}.$$

Rearranging gives an expression for c_A . Now, by assumption, $N_B \subset N_A$ and so the chain of inequalities in the first expression of (18) holds for some i . Using (19) and

$$c_A = \frac{1 - \beta_A}{\sum_{j \in N_A} 1/\kappa_j^2} + \bar{\xi}_A,$$

(18) can be true for this i if and only if

$$\bar{\xi}_A + \frac{1}{\phi_A} \frac{1}{\sum_{j \in N_A} 1/\kappa_j^2} > \xi_i \geq \bar{\xi}_B + \frac{1}{\phi_B} \frac{1}{\sum_{j \in N_B} 1/\kappa_j^2}, \quad \text{where } \phi_A = \frac{1}{1 - \beta_A}. \quad (22)$$

Assume the converse of the required result, so that $\gamma_B \leq \gamma_A$. Then

$$\begin{aligned} \frac{1}{\phi_A} \leq \frac{1}{\phi_B} &\Leftrightarrow 1 - \beta_A \leq \frac{(1 - \beta_A \omega_{AA})(1 - \beta_B \omega_{BB}) - \beta_A \beta_B \omega_{AB} \omega_{BA}}{1 - \beta_A \omega_{AA} + \beta_B \omega_{BA}} \\ &\Leftrightarrow (1 - \beta_A \omega_{AA})^2 + (\beta_B \omega_{BA} - \beta_A \omega_{AB})(1 - \beta_A \omega_{AA}) \leq (1 - \beta_A \omega_{AA})(1 - \beta_B \omega_{BB}) \\ &\Leftrightarrow (1 - \beta_A \omega_{AA}) + (\beta_B \omega_{BA} - \beta_A \omega_{AB}) \leq (1 - \beta_B \omega_{BB}) \\ &\Leftrightarrow \beta_B \leq \beta_A. \end{aligned}$$

So, if $\beta_B \leq \beta_A$ then $1/\phi_A \leq 1/\phi_B$ and so

$$\bar{\xi}_A + \frac{1}{\phi_A} \frac{1}{\sum_{j \in N_A} 1/\kappa_j^2} \leq \bar{\xi}_A + \frac{1}{\phi_B} \frac{1}{\sum_{j \in N_A} 1/\kappa_j^2}. \quad (23)$$

But there must exist an i such that (18) holds, and therefore an $i \notin N_B$ such that (22) holds.

$$\xi_i \geq \bar{\xi}_B + \frac{1}{\phi_B} \frac{1}{\sum_{j \in N_B} 1/\kappa_j^2} \Leftrightarrow \xi_i \geq \bar{\xi}_{B \cup \{i\}} + \frac{1}{\phi_B} \frac{1}{\sum_{j \in N_B \cup \{i\}} 1/\kappa_j^2},$$

defining $\bar{\xi}_{B \cup \{i\}}$ in an appropriate way and using the usual argument via cross multiplication and addition of ξ_i/κ_i^2 to both sides. If $N_A = N_B \cup \{i\}$ then this expression along with (23) contradicts (22). If N_A contains further signals not in N_B , then let $i = \{\arg \min_j \xi_j \mid j \in N_{A/B}\}$ and apply the above argument. Then repeat the last part of the argument for $i + 1, i + 2$, etc., until all signals in $N_{A/B}$ are included. A contradiction is reached again: if $N_B \subset N_A$ then $\beta_B > \beta_A$, as required.

Now suppose $\beta_B \leq \beta_A$. Then N_B is not a subset of N_A by modus tollens. Apply Lemma 4: $N_A \subseteq N_B$. Thus, swapping A for B , if $\beta_B \geq \beta_A$ then $N_B \subseteq N_A$, proving the proposition's first statement. Note that application of (22) immediately gives the final statement of the proposition, that this subset consists precisely of the clearest signals in N_A . \square

Proof of Proposition 8. Proving the second statement, about the weights, first, recall the weights given in (20) and (21). Let $i \in N_{A \cap B}$ so that players in A and B acquire i ,

$$\begin{aligned} w_{iA} \leq w_{iB} &\Leftrightarrow \frac{1}{1 - \beta_A \omega_{AA}} \frac{c_A - \xi_i}{\kappa_i^2} + \frac{\beta_A \omega_{AB}}{1 - \beta_A \omega_{AA}} w_{iB} \leq w_{iB} \\ &\Leftrightarrow \frac{c_A - \xi_i}{\kappa_i^2} \leq (1 - \beta_A) w_{iB} \\ &\Leftrightarrow \frac{c_A - \xi_i}{\kappa_i^2} \leq \frac{1}{\phi_A} \left[\frac{1/\kappa_i^2}{\sum_{j \in N_B} 1/\kappa_j^2} + \phi_B \left(\frac{\bar{\xi}_B - \xi_i}{\kappa_i^2} \right) \right] \\ &\Leftrightarrow \phi_A \left[\frac{1}{\phi_A \sum_{j \in N_A} 1/\kappa_j^2} + (\bar{\xi}_A - \xi_i) \right] \leq \frac{1}{\sum_{j \in N_B} 1/\kappa_j^2} + \phi_B (\bar{\xi}_B - \xi_i) \\ &\Leftrightarrow \xi_i \leq \frac{1}{\phi_B - \phi_A} \left[\frac{1}{\sum_{j \in N_B} 1/\kappa_j^2} - \frac{1}{\sum_{j \in N_A} 1/\kappa_j^2} + (\phi_B \bar{\xi}_B - \phi_A \bar{\xi}_A) \right], \end{aligned}$$

where the fourth line follows from substitution for c_A and the final line from noting that $\phi_B > \phi_A$ if $\beta_B > \beta_A$. If $N_A = N_B$ then the summations are identical and cancel, and $\bar{\xi}_A = \bar{\xi}_B$, yielding the final result stated in the proposition. If $N_B \subset N_A$ then the signals unused by B ($i \in N_{A/B}$) are the least clear used by A , trivially confirming the result for such i .

Then, for the first statement in the result, again consider (20) and (21). Using the former of these, multiplying through by ξ_i and summing over all $i \in N_B$,

$$Z_B = \bar{\xi}_B - \phi_B \sum_{i \in N_B} \frac{(\xi_i - \bar{\xi}_B)^2}{\kappa_i^2}.$$

Similarly, multiply (21) through by ξ_i , and sum over $i \in N_A$,

$$Z_A = \frac{1}{1 - \beta_A \omega_{AA}} \left[\sum_{i \in N_A} \frac{\xi_i}{\kappa_i^2} c_A - \sum_{i \in N_A} \frac{\xi_i^2}{\kappa_i^2} \right] + \frac{\beta_A \omega_{AB}}{1 - \beta_A \omega_{AA}} Z_B,$$

since $w_{iB} = z_{iB} = 0$ for all $i \in N_{A/B}$. This is greater than or equal to Z_B if and only if

$$\begin{aligned} & \sum_{i \in N_A} \frac{\xi_i}{\kappa_i^2} c_A - \sum_{i \in N_A} \frac{\xi_i^2}{\kappa_i^2} \geq Z_B (1 - \beta_A) \\ \Leftrightarrow & \bar{\xi}_A - \phi_A \sum_{i \in N_A} \frac{(\xi_i - \bar{\xi}_A)^2}{\kappa_i^2} \geq \bar{\xi}_B - \phi_B \sum_{i \in N_B} \frac{(\xi_i - \bar{\xi}_B)^2}{\kappa_i^2}, \end{aligned}$$

which follows by substituting for c_A and rearranging. Now $\beta_B > \beta_A \Rightarrow \phi_B > \phi_A$. Moreover $N_B \subseteq N_A$. By the proof method of the later Proposition 11, $Z_A \geq Z_B$ as required. \square

Proof of Proposition 9. The objective is to examine the properties of w_{jM} as $M \rightarrow \infty$. First, w_{jm} is found for any j, m in terms of c_k with $k \in \{1, \dots, m\}$. For $m > 1$, repeated substitution yields

$$\begin{aligned} w_{jm} &= \beta \rho_j w_{jm-1} + \psi_j c_m \\ &= \beta \rho_j (\beta \rho_j w_{jm-2} + \psi_j c_{m-1}) + \psi_j c_m \\ &= \beta \rho_j (\beta \rho_j (\beta \rho_j w_{jm-3} + \psi_j c_{m-2}) + \psi_j c_{m-1}) + \psi_j c_m \\ &= \dots \\ &= (\beta \rho_j)^{m-1} w_{j1} + \psi_j \sum_{k=0}^{m-2} (\beta \rho_j)^k c_{m-k} \\ &= \psi_j \sum_{k=0}^{m-1} (\beta \rho_j)^k c_{m-k}, \end{aligned}$$

where the last line uses $w_{j1} = \psi_j c_1$. Now, making a change of variable for k ,

$$w_{jm} = \psi_j \sum_{k=1}^m (\beta \rho_j)^{m-k} c_k. \quad (24)$$

Using the fact that $\sum_{i=1}^n w_{im} = 1$, and (24), the sequence $\{c_m\}_{m=1}^M$ can be deduced:

$$1 = \sum_{i=1}^n \psi_i \sum_{k=1}^m (\beta \rho_i)^{m-k} c_k.$$

So, using this expression for m and $m+1$ yields

$$\begin{aligned} \sum_{i=1}^n \psi_i \sum_{k=1}^m (\beta \rho_i)^{m-k} c_k &= \sum_{i=1}^n \psi_i \sum_{k=1}^{m+1} (\beta \rho_i)^{m+1-k} c_k \\ &= \sum_{i=1}^n \psi_i \left[\sum_{k=1}^m (\beta \rho_i)^{m+1-k} c_k + c_{m+1} \right] \\ &= \sum_{i=1}^n \psi_i \beta \rho_i \sum_{k=1}^m (\beta \rho_i)^{m-k} c_k + \sum_{i=1}^n \psi_i c_{m+1} \\ \Rightarrow c_{m+1} \sum_{i=1}^n \psi_i &= \sum_{i=1}^n \psi_i (1 - \beta \rho_i) \sum_{k=1}^m (\beta \rho_i)^{m-k} c_k \quad \text{or} \\ c_{m+1} &= \sum_{i=1}^n \hat{\psi}_i (1 - \beta \rho_i) \sum_{k=1}^m (\beta \rho_i)^{m-k} c_k \\ &= \sum_{k=1}^m c_k \left[\sum_{i=1}^n \hat{\psi}_i (1 - \beta \rho_i) (\beta \rho_i)^{m-k} \right] \\ &= \sum_{k=1}^m c_k v_k^m, \quad \text{where } v_k^m \equiv \sum_{i=1}^n \hat{\psi}_i (1 - \beta \rho_i) (\beta \rho_i)^{m-k}. \end{aligned}$$

Now, note that $v_k^m = v_{k-1}^{m-1}$ and $v_k^{m-1} > v_k^m$ for all $m \geq k > 1$. Define $\Delta c_m \equiv c_m - c_{m-1}$. Then

$$\begin{aligned}\Delta c_{m+1} &= \sum_{k=1}^m c_k v_k^m - \sum_{k=1}^{m-1} c_k v_k^{m-1} = \sum_{k=1}^m c_k v_k^m - \sum_{k=2}^m c_{k-1} v_{k-1}^{m-1} \\ &= c_1 v_1^m + \sum_{k=2}^m c_k v_k^m - \sum_{k=2}^m c_{k-1} v_{k-1}^{m-1} \\ &= c_1 v_1^m + \sum_{k=2}^m \Delta c_k v_k^m.\end{aligned}$$

(The last line follows from $v_k^m = v_{k-1}^{m-1}$.) Now, by induction, it can be shown that $c_m < c_{m-1}$ for all $m > 1$, or equivalently that $\Delta c_m < 0$ for all $m > 1$. Suppose, first of all, that for some t , $\Delta c_t < 0$. Then, because $v_k^{t-1} > v_k^t$ for all $t \geq k > 1$,

$$\begin{aligned}\Delta c_{t+1} &= c_1 v_1^t + \sum_{k=2}^t \Delta c_k v_k^t = c_1 v_1^t + \sum_{k=2}^{t-1} \Delta c_k v_k^t + \Delta c_t v_t^t \\ &< c_1 v_1^{t-1} + \sum_{k=2}^{t-1} \Delta c_k v_k^{t-1} + \Delta c_t v_t^t = \Delta c_t + \Delta c_t v_t^t = (1 + v_t^t) \Delta c_t < 0,\end{aligned}$$

by the induction hypothesis. So, if $\Delta c_t < 0$ then $\Delta c_{t+1} < 0$. Now consider $m = 2$,

$$c_2 = \sum_{k=1}^1 c_k v_k^1 = c_1 v_1^1 = c_1 \sum_{i=1}^n \hat{\psi}_i (1 - \beta \rho_i) < c_1,$$

since $v_k^m < 1$ for all $m \geq k \geq 1$. So indeed $c_2 < c_1$ or $\Delta c_2 < 0$. Therefore, by induction, $\Delta c_m < 0$ for all m . In other words, $\{c_m\}_{m=1}^M$ is a decreasing sequence. It is bounded below. In particular, again using $\sum_{i=1}^n w_{im} = 1$, for all $m > 1$

$$w_{jm} = \beta w_{j,m-1} \rho_j + c_m \psi_j \quad \Rightarrow \quad c_m = \frac{1 - \beta \sum_{i=1}^n w_{i,m-1} \rho_i}{\sum_{i=1}^n \psi_i} \geq \frac{1 - \beta}{\sum_{i=1}^n \psi_i} > 0.$$

Moreover, the value of c_1 is known, and so

$$c_m \in \left[\frac{1 - \beta}{\sum_{i=1}^n \psi_i}, \frac{1}{\sum_{i=1}^n \psi_i} \right] \quad \text{for all } m.$$

Therefore $\{c_m\}_{m=1}^M$ converges as $M \rightarrow \infty$. It remains to establish that the sequence $\{w_{jm}\}_{m=1}^M$ converges as $M \rightarrow \infty$ for all j . In fact, subtracting $w_{j,m-1}$ from w_{jm} and using “ Δ ” notation $\Delta w_{jm} \equiv w_{jm} - w_{j,m-1} = \beta \rho_j \Delta w_{j,m-1} + \psi_j \Delta c_m$ for $m > 2$. Evaluating at M and taking $M \rightarrow \infty$,

$$\lim_{M \rightarrow \infty} \Delta w_{jM} = \beta \rho_j \lim_{M \rightarrow \infty} \Delta w_{j,M-1} + \psi_j \lim_{M \rightarrow \infty} \Delta c_M = \beta \rho_j \lim_{M \rightarrow \infty} \Delta w_{j,M-1},$$

since $\lim_{M \rightarrow \infty} \Delta c_M = 0$. Hence $\lim_{M \rightarrow \infty} \Delta w_{jM} = 0$. Thus the sequence $\{w_{jm}\}_{m=1}^M$ converges as $M \rightarrow \infty$ for all j . Define $c_\infty \equiv \lim_{M \rightarrow \infty} c_M$. From the M th first-order condition

$$w_{jM} = \beta w_{j,M-1} \rho_j + c_M \psi_j \quad \Rightarrow \quad \lim_{M \rightarrow \infty} (w_{jM} - \beta \rho_j w_{j,M-1}) = \psi_j c_\infty,$$

and so, defining $w_{j\infty} \equiv \lim_{M \rightarrow \infty} w_{jM}$, for all j ,

$$w_{j\infty} = \frac{\psi_j}{1 - \beta \rho_j} c_\infty \quad \Rightarrow \quad c_\infty = \left[\sum_{i=1}^n \frac{\psi_i}{1 - \beta \rho_i} \right]^{-1}.$$

Thus the weights converge to the familiar (from Section 4) expression

$$w_{j\infty} = \frac{\psi_j}{1 - \beta \rho_j} / \sum_{i=1}^n \frac{\psi_i}{1 - \beta \rho_i} \quad \text{for all } j.$$

Summarizing in the accuracy/clarity notation and substituting for ρ_j , ψ_j , and $\pi = 1 - \beta$ gives the expression in the statement of the proposition. \square

Proof of Lemma 5. The first-order conditions for $m > 1$ when $w_{jm} > 0$ may be derived from (6):

$$w_{jm} = \beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2}. \quad (25)$$

The first task is to show that $c_m \leq c_{m-1}$ for all $m > 1$. Consider (25). Sum over all $i \in N_m$, then

$$1 = \sum_{i \in N_m} w_{im} = \beta \sum_{i \in N_m} w_{im-1} + c_m \sum_{i \in N_m} \frac{1}{\kappa_i^2} - \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2}.$$

Note that $\sum_{i \in N_m} w_{im-1} \leq 1$. Thus, for all $m > 1$, c_m can be bounded below:

$$c_m \geq \frac{(1 - \beta) + \sum_{i \in N_m} \xi_i / \kappa_i^2}{\sum_{i \in N_m} 1 / \kappa_i^2} \equiv \Xi(N_m; \beta).$$

For $j \notin N_m$,

$$\beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 \quad \Rightarrow \quad \beta \sum_{i \in N^-} w_{im-1} + c_m \sum_{i \in N^-} \frac{1}{\kappa_i^2} - \sum_{i \in N^-} \frac{\xi_i}{\kappa_i^2} \leq 0,$$

where $N^- = \neg N_m \cap N_{m-1}$. For $j \in N^+ = N_m \cap N_{m-1}$ the condition in (25) applies, and

$$\sum_{i \in N^+} w_{im} = \beta \sum_{i \in N^+} w_{im-1} + c_m \sum_{i \in N^+} \frac{1}{\kappa_i^2} - \sum_{i \in N^+} \frac{\xi_i}{\kappa_i^2}.$$

For $j \in N^-$, $w_{jm} = 0$ and $N_{m-1} = N^+ \cup N^-$, so

$$1 \geq \sum_{i \in N_{m-1}} w_{im} \geq \beta \sum_{i \in N_{m-1}} w_{im-1} + c_m \sum_{i \in N_{m-1}} \frac{1}{\kappa_i^2} - \sum_{i \in N_{m-1}} \frac{\xi_i}{\kappa_i^2} = \beta + c_m \sum_{i \in N_{m-1}} \frac{1}{\kappa_i^2} - \sum_{i \in N_{m-1}} \frac{\xi_i}{\kappa_i^2}.$$

In other words, $c_m \leq \Xi(N_{m-1}; \beta)$ for all $m > 1$. Thus, $c_m \leq \Xi(N_{m-1}; \beta) \leq c_{m-1}$ for all $m > 2$. Moreover, from the earlier fact that $c_1 = \Xi(N_1, 0)$, and noting that $\Xi(\cdot, \beta)$ is decreasing in β , $c_m \leq \Xi(N_{m-1}; \beta) \leq c_{m-1}$ for all $m > 1$: $\{c_m\}_{m=1}^M$ is a decreasing sequence, as required.

Now consider the statement of the lemma. If m does not use j , then $w_{jm} = 0$. Therefore $\beta w_{jm-1} + (c_m - \xi_j) / \kappa_j^2 \leq 0$. As a consequence, $\xi_j \geq c_m$. For $m + 1$ to use j , w_{jm+1} must be strictly positive, and therefore (25) applies (evaluated for player $m + 1$) and is strictly positive. But $w_{jm} = 0$, so it must be that $c_{m+1} > \xi_j$. But then $\xi_j \geq c_m \geq c_{m+1} > \xi_j$, a contradiction. Repeating this argument for all $m' > m + 1$ yields the result. \square

Proof of Proposition 10. From the discussion in the main text, $N_1 = \{1, \dots, n_1\}$. To confirm that N_1 is unique, it is sufficient to confirm that $\Xi(N_1; 0)$ crosses the (rising) sequence of ξ_i s only once (which will be after n_1 and before $n_1 + 1$, by definition). First, take $j = \max\{i \in N\}$ such that $\xi_{j+1} > \Xi(N; 0) > \xi_j$, if such exists. Then, for instance, for any $k > j$,

$$\begin{aligned} \xi_k \geq \xi_{j+1} > \Xi(N; 0) &= \frac{1 + \sum_{i \in N} \xi_i / \kappa_i^2}{\sum_{i \in N} 1 / \kappa_i^2} \Leftrightarrow \xi_k \sum_{i \in N} \frac{1}{\kappa_i^2} > 1 + \sum_{i \in N} \frac{\xi_i}{\kappa_i^2} \\ \Leftrightarrow \xi_k \sum_{i \in N} \frac{1}{\kappa_i^2} + \frac{\xi_k}{\kappa_k^2} &> 1 + \sum_{i \in N} \frac{\xi_i}{\kappa_i^2} + \frac{\xi_k}{\kappa_k^2} \Leftrightarrow \xi_k \sum_{i \in N \cup \{k\}} \frac{1}{\kappa_i^2} > 1 + \sum_{i \in N \cup \{k\}} \frac{\xi_i}{\kappa_i^2} \\ \Leftrightarrow \xi_k > \frac{1 + \sum_{i \in N \cup \{k\}} \xi_i / \kappa_i^2}{\sum_{i \in N \cup \{k\}} 1 / \kappa_i^2} &= \Xi(N \cup \{k\}; 0). \end{aligned}$$

A symmetrical argument applies for $k \leq j$, so that $\xi_k \leq \xi_j < \Xi(N; 0) \Leftrightarrow \xi_k < \Xi(N \setminus \{k\}; 0)$. Thus, by continued application of these facts, no superset or strict subset of N can satisfy this

property. Therefore, there exists a unique $n_1 \geq 1$ such that $z_{j1} > 0 \Leftrightarrow w_{j1} > 0$ for all $j \leq n_1$ and $z_{j1} = w_{j1} = 0$ for all $j > n_1$. So $N_1 = \{1, \dots, n_1\}$ is unique as required.

Now, consider $m > 1$. In order to show $N_m = \{1, \dots, n_m\}$ with $n_m \leq n_{m-1}$ for all $m > 1$, note that $N_m \subseteq N_{m-1}$ from Lemma 5. Next, $N_m = \{1, \dots, n_m\}$ for all m is required. That is, each player uses a subset of signals consisting of the most clear (lowest ξ_j). This has been shown for $m = 1$. To see this for general m , consider the minimum m for which, for some j , $w_{jm} = 0$ but $w_{j(m-1)} > 0$. Now, again by Lemma 5, $w_{j(m-1)} > 0 \Rightarrow w_{jm'} > 0$ for all $m' < m - 1$. By way of a contradiction suppose that $j < n_m$ and there exists some $i > j$ for which $w_{im} > 0$. Then

$$\begin{aligned} \beta w_{jm-1} + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 &\Rightarrow \beta \left(\beta w_{jm-2} + \frac{c_{m-1} - \xi_j}{\kappa_j^2} \right) + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 \\ \Rightarrow \beta^2 w_{jm-2} + \beta \frac{c_{m-1} - \xi_j}{\kappa_j^2} + \frac{c_m - \xi_j}{\kappa_j^2} \leq 0 &\Rightarrow \beta^{m-1} w_{j1} + \sum_{k=2}^m \beta^{m-k} \frac{c_k - \xi_j}{\kappa_j^2} \leq 0 \\ &\Rightarrow \sum_{k=1}^m \beta^{m-k} \frac{c_k - \xi_j}{\kappa_j^2} \leq 0, \end{aligned}$$

where the penultimate line follows from repeated substitution for w_{jm-2} and the final line from the value of w_{j1} established in the main text. Rearranging,

$$\xi_j \geq \sum_{k=1}^m \beta^{m-k} c_k / \sum_{k=1}^m \beta^{m-k}.$$

Signal i is used by m , and so is used by all $m' < m$. The very same calculation can be made for i , therefore; because $w_{im} > 0$, the first-order condition applies, and

$$w_{im} = \sum_{k=1}^m \beta^{m-k} \frac{c_k - \xi_i}{\kappa_i^2}. \quad (26)$$

But $i > j$, so $\xi_i > \xi_j \geq \sum_{k=1}^m \beta^{m-k} c_k / \sum_{k=1}^m \beta^{m-k}$ implying $w_{im} = 0$, a contradiction. No “gap” can open up for the first time at any $m > 1$. Since there are “no gaps” at $m = 1$, there are no gaps for any m . Finally, observe that n_m is uniquely determined for each $m > 1$ (applying precisely the method used above for n_1). These facts together prove the statements in the proposition. \square

Proof of Proposition 11. Define

$$\bar{\xi}_m = \frac{\sum_{i \in N_m} \xi_i / \kappa_i^2}{\sum_{i \in N_m} 1 / \kappa_i^2}.$$

Now given the ordering of the ξ_i s and the facts proven earlier that $N_m \subseteq N_{m-1}$ for all $m > 1$, and there are “no gaps” for any m so that $N_m = \{1, \dots, n_m\}$, it is clear that this measure of “average clarity” declines: $\bar{\xi}_m \leq \bar{\xi}_{m-1}$ for all $m > 1$.²² Using this notation, construct the positive weights for player m . In particular, since $w_{im} > 0$ implies that $w_{im'} > 0$ for all $m' < m$. (26) applies whenever $w_{im} > 0$. Summing over all such i for player m and rearranging,

$$\sum_{k=1}^m \beta^{m-k} c_k \sum_{i \in N_m} \frac{1}{\kappa_i^2} = 1 + \sum_{k=1}^m \beta^{m-k} \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} = 1 + \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2}.$$

²²Of course, this measure is actually inversely related to average clarity (recall, $1/\xi_i^2$ is interpreted as information source i 's clarity). Therefore, as expected, the signals acquired by players further down the hierarchy have higher clarity on average than those acquired by players above them.

Therefore, dividing through both sides by $\sum_{i \in N_m} 1/\kappa_i^2$ and using the $\bar{\xi}_m$ notation,

$$\sum_{k=1}^m \beta^{m-k} c_k = \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1 - \beta^m}{1 - \beta} \bar{\xi}_m. \quad (27)$$

From the first order conditions, $z_{im} = \xi_i w_{im}$. So, if $w_{im} > 0$ then, from (26), for all $m > 1$,

$$z_{im} = \sum_{k=1}^m \beta^{m-k} \left(\frac{c_k - \xi_i}{\kappa_i^2} \right) \xi_i. \quad (28)$$

Now, total information use (or total cost of information use) is $Z_m = \sum_{i \in N_m} z_{im}$,

$$\begin{aligned} Z_m &= \sum_{k=1}^m \beta^{m-k} c_k \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} - \sum_{k=1}^m \beta^{m-k} \sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} \\ &= \left[\frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1 - \beta^m}{1 - \beta} \bar{\xi}_m \right] \sum_{i \in N_m} \frac{\xi_i}{\kappa_i^2} - \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} \\ &= \bar{\xi}_m - \frac{1 - \beta^m}{1 - \beta} \left(\sum_{i \in N_m} \frac{\xi_i^2}{\kappa_i^2} - \bar{\xi}_m^2 \sum_{i \in N_m} \frac{1}{\kappa_i^2} \right) = \bar{\xi}_m - \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)^2}{\kappa_i^2}, \end{aligned} \quad (29)$$

where the second equality follows from (27), the third from rearrangement and the definition of $\bar{\xi}_m$ and (29) from further rearrangement of the ‘‘variance-like’’ second term.

Now recall $i \in N_m$ if and only if $w_{im} > 0 \Leftrightarrow z_{im} > 0$. Using the recursive expression for w_{im} in (26), therefore, $i \in N_m$ if and only if

$$\begin{aligned} w_{im} > 0 &\Leftrightarrow \sum_{k=1}^m \beta^{m-k} \frac{c_k - \xi_i}{\kappa_i^2} > 0 \Leftrightarrow \sum_{k=1}^m \beta^{m-k} c_k > \frac{1 - \beta^m}{1 - \beta} \xi_i \\ &\Leftrightarrow \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1 - \beta^m}{1 - \beta} \bar{\xi}_m > \frac{1 - \beta^m}{1 - \beta} \xi_i \Leftrightarrow \xi_i < \bar{\xi}_m + \frac{1 - \beta}{1 - \beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2}. \end{aligned} \quad (30)$$

Now $N_m \subseteq N_{m-1}$ for all $m > 1$, and $z_{im} > 0 \Rightarrow z_{i-1m} > 0$ for all $m \geq 1$. Using these facts, the first statement of the proposition (concerning total acquisition) may be proved.

If $N_m = N_{m-1}$ then inspection of (29) is sufficient. The last term is (weakly) positive, and does not change from $m - 1$ to m , likewise the first term, but $\beta < 1$ so $\beta^m < \beta^{m-1}$, so $Z_m \leq Z_{m-1}$. The harder case is when $N_m \subset N_{m-1}$. Consider moving up the chain from player $m + 1$ to player m . Assume, in the first instance, that $N_m = N_{m+1} \cup \{j\}$, so that j is the (sole) signal that m acquires, but $m + 1$ does not.

First suppose that $\xi_j = \bar{\xi}_{m+1}$. Then $\bar{\xi}_m = \bar{\xi}_{m+1}$. Moreover, since $\bar{\xi}_m = \bar{\xi}_{m+1}$, from (29)

$$\begin{aligned} Z_m &= \bar{\xi}_{m+1} - \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} \\ &= \bar{\xi}_{m+1} - \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_{m+1}} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} - \frac{1 - \beta^m}{1 - \beta} \frac{(\xi_j - \bar{\xi}_{m+1})^2}{\kappa_j^2} \\ &= \bar{\xi}_{m+1} - \frac{1 - \beta^m}{1 - \beta} \sum_{i \in N_{m+1}} \frac{(\xi_i - \bar{\xi}_{m+1})^2}{\kappa_i^2} \geq Z_{m+1}, \end{aligned}$$

where the final equality follows from the supposition $\xi_j = \bar{\xi}_{m+1}$, and the inequality follows from $\beta^{m+1} < \beta^m$ and the (weak) positivity of the variance-like term.

Now treat Z_m as a function of ξ_j . Note that it is quadratic in ξ_j . Compute

$$\begin{aligned} \frac{dZ_m}{d\xi_j} &= \frac{d\bar{\xi}_m}{d\xi_j} - \frac{1-\beta^m}{1-\beta} \sum_{i \in N_m} \frac{d}{d\xi_j} \frac{(\xi_i - \bar{\xi}_m)^2}{\kappa_i^2} \\ &= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \left[\frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2} - 2 \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)}{\kappa_i^2} \frac{d\bar{\xi}_m}{d\xi_j} \right] \\ &= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \left[\frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2} - \frac{2/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} \sum_{i \in N_m} \frac{(\xi_i - \bar{\xi}_m)}{\kappa_i^2} \right] \\ &= \frac{1/\kappa_j^2}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} \frac{2(\xi_j - \bar{\xi}_m)}{\kappa_j^2}, \end{aligned}$$

where the final line (and the quantity $d\bar{\xi}_m/d\xi_j$) follow from the definition of $\bar{\xi}_m$. So it follows that Z_m is increasing in ξ_j if and only if $\xi_j < \hat{\xi}_m$ where

$$\hat{\xi}_m \equiv \bar{\xi}_m + \frac{1}{2} \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2}.$$

Summarizing, Z_m is a quadratic in ξ_j with its maximum at $\hat{\xi}_m$ and it is greater than or equal to Z_{m+1} when evaluated at $\xi_j = \bar{\xi}_{m+1}$. It is therefore greater than or equal to Z_{m+1} (which does not depend on ξ_j by assumption) for all $\xi_j \in [\bar{\xi}_{m+1}, \hat{\xi}_m + (\hat{\xi}_m - \bar{\xi}_{m+1})]$. Now

$$\hat{\xi}_m + (\hat{\xi}_m - \bar{\xi}_{m+1}) = 2\hat{\xi}_m - \bar{\xi}_{m+1} = \bar{\xi}_m + \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + (\bar{\xi}_m - \bar{\xi}_{m+1}),$$

where the last term is strictly positive. But, for j to be acquired by m and not by $m+1$, it must be that (30) holds for m and fails for $m+1$. That is

$$\bar{\xi}_{m+1} < \bar{\xi}_{m+1} + \frac{1-\beta}{1-\beta^{m+1}} \frac{1}{\sum_{i \in N_{m+1}} 1/\kappa_i^2} \leq \xi_i < \bar{\xi}_m + \frac{1-\beta}{1-\beta^m} \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2},$$

which implies ξ_j indeed lies (strictly) within the required range for Z_m to be larger than Z_{m+1} . This argument can be repeated for cases when N_{m+1} and N_m differ by more than one signal (in intermediate steps, starting with the highest ξ_j in N_m but not in N_{m+1} , and then the second highest, and so on). Therefore, $Z_m \geq Z_{m+1}$ for all m , as required.

For the second part of the proposition, first note that by substitution of (27) into (26),

$$w_{jm} = \frac{1}{\kappa_j^2} \left[\frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} (\xi_j - \bar{\xi}_m) \right] \quad (31)$$

whenever $w_{jm} > 0$. Information acquisition of signal j is then simply $z_{jm} = \xi_j w_{jm}$. Consider (31) evaluated at m and $m+1$.

$$\begin{aligned} w_{jm+1} > w_{jm} &\Leftrightarrow \frac{1}{\sum_{i \in N_{m+1}} 1/\kappa_i^2} - \frac{1-\beta^{m+1}}{1-\beta} (\xi_j - \bar{\xi}_{m+1}) > \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} - \frac{1-\beta^m}{1-\beta} (\xi_j - \bar{\xi}_m) \\ &\Leftrightarrow \frac{1}{\sum_{i \in N_{m+1}} 1/\kappa_i^2} - \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1-\beta^{m+1}}{1-\beta} \bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta} \bar{\xi}_m \\ &\quad > \frac{1-\beta^{m+1} - (1-\beta^m)}{1-\beta} \xi_j = \frac{\beta^m(1-\beta)}{1-\beta} \xi_j = \beta^m \xi_j \\ &\Leftrightarrow \xi_j < \left\{ \frac{1}{\sum_{i \in N_{m+1}} 1/\kappa_i^2} - \frac{1}{\sum_{i \in N_m} 1/\kappa_i^2} + \frac{1-\beta^{m+1}}{1-\beta} \bar{\xi}_{m+1} - \frac{1-\beta^m}{1-\beta} \bar{\xi}_m \right\} / \beta^m. \end{aligned}$$

Noting $z_{jm} = \xi_j w_{jm}$ proves the first part of the result so long as j is used by both m and $m + 1$. If j is not used by $m + 1$, then the result follows immediately ($m + 1$ uses a subset consisting of the clearest signals used by m). For the second part note that the last line in the above displayed inequality reduces to $\xi_j < \bar{\xi}_m = \bar{\xi}_{m+1}$ when $N_m = N_{m+1}$. \square

APPENDIX B. A GENERALIZED HIERARCHY NETWORK

Here, a more general version of the hierarchy network analysed in Section 6 is presented. The model introduced at the beginning of the section, for which Figure 2 illustrates an example, can be extended even further. Below, however, a recipe is provided for adapting Propositions 9–11 to the case where every level $\ell > 1$ contains several isolated groups, each containing $g + 1$ players. Any two players within a given group have $\gamma_{mm'} = \gamma_{m'm} = \gamma$. Each player m in level $\ell > 1$ is linked to precisely one player m' in layer $\ell - 1$ with $\gamma_{mm'} = 1 - g\gamma$. There is a single player in level 1 (player 1) who is linked to no-one. Further, suppose there are L levels in total.

First, an analogue to Proposition 9 is available. Define $w_{j\ell} \equiv w_{jm}$ and $c_\ell \equiv c_m$ for any player m residing in level ℓ . Applying Lemma 1, the optimal weight on signal j for a player in level ℓ is

$$w_{j\ell} = \beta\rho_j \{(1 - g\gamma)w_{j(\ell-1)} + g\gamma w_{j\ell}\} + c_\ell\psi_j,$$

for all $\ell > 1$. If $\ell = 1$ then $w_{j1} = c_1\psi_j$ as in the model described in Section 6. For $\ell > 1$,

$$\begin{aligned} (1 - \beta\rho_j g\gamma)w_{j\ell} &= \beta\rho_j(1 - g\gamma)w_{j(\ell-1)} + c_\ell\psi_j \\ w_{j\ell} &= \beta\rho_j^* w_{j(\ell-1)} + c_\ell\psi_j^*, \end{aligned} \quad (32)$$

where $\rho_j^* \equiv \rho_j(1 - g\gamma)/(1 - \beta\rho_j g\gamma)$ and $\psi_j^* \equiv \psi_j/(1 - \beta\rho_j g\gamma)$. This, however, is the very same expression as that of the opening statements in the proof to Proposition 9 in Appendix A, but with ρ_j replaced with ρ_j^* and ψ_j with ψ_j^* . The only caveat is that, at $\ell = 1$, $w_{j1} = c_1\psi_j$.

Taking account of this difference at $\ell = 1$ is all that is required to show an analogue for Proposition 9 (with a new value for π). Repeated substitution in (32) yields (for $\ell > 1$)

$$w_{j\ell} = (\beta\rho_j^*)^{\ell-1} w_{j1} + \psi_j^* \sum_{k=0}^{\ell-2} (\beta\rho_j^*)^k c_{\ell-k}.$$

Now $w_{j1} = \psi_j c_1 = \psi_j^* (1 - \beta\rho_j g\gamma) c_1 = \psi_j^* c_1 - \psi_j^* \beta\rho_j g\gamma c_1$. Therefore, (24) can be rewritten

$$w_{j\ell} = \psi_j^* \sum_{k=1}^{\ell} (\beta\rho_j^*)^{\ell-k} c_k - \psi_j^* (\beta\rho_j^*)^\ell g\gamma c_1$$

for any player in level $\ell \geq 1$. Now, other than the second term, this is precisely the same as (24). Following exactly the method of the proof to Proposition 9,

$$c_{\ell+1} = \sum_{i=1}^n \hat{\psi}_i^* (1 - \beta\rho_i^*) \sum_{k=1}^{\ell} (\beta\rho_i^*)^{\ell-k} c_k - g\gamma \sum_{i=1}^n \hat{\psi}_i^* (1 - \beta\rho_i^*) (\beta\rho_i^*)^\ell c_1,$$

where $\hat{\psi}_j^* = \psi_j^* / \sum_{i=1}^n \psi_i^*$. Noting that this last term is the analogue of v_0^ℓ , but where ψ_j is replaced with ψ_j^* , and ρ_j is replaced with ρ_j^* for all j , and abusing notation somewhat,

$$c_{\ell+1} = \sum_{k=1}^{\ell} c_k v_k^\ell - g\gamma v_0^\ell c_1 \quad \text{where} \quad v_k^\ell \equiv \sum_{i=1}^n \hat{\psi}_i^* (1 - \beta\rho_i^*) (\beta\rho_i^*)^{\ell-k}. \quad (33)$$

Following step-by-step the approach in the proof to Proposition 9 yields

$$\Delta c_{\ell+1} = c_1 v_1^\ell + \sum_{k=2}^{\ell} \Delta c_k v_k^\ell + g\gamma (v_0^{\ell-1} - v_0^\ell) c_1.$$

The last term is positive, given the definition of v_k^ℓ above. Showing that the sequence of c_k s is decreasing follows by induction. The only difficulty is the step at $\ell = 1$. But note, for all $\ell > 1$,

$$v_0^{\ell-1} - v_0^\ell = \sum_{i=1}^n \hat{\psi}_i^* (1 - \beta \rho_i^*)^2 (\beta \rho_i^*)^{\ell-1} < \sum_{i=1}^n \hat{\psi}_i^* (1 - \beta \rho_i^*)^2 (\beta \rho_i^*)^{\ell-2} = v_0^{\ell-2} - v_0^{\ell-1}.$$

Thus, the induction step follows even when adding this new term to Δc_{t+1} . Now, from (33), $c_2 = c_1 v_1^1 - g \gamma v_0^1 c_1 = c_1 (v_1^1 - g \gamma v_0^1)$. The first component in the parentheses is smaller than one. Therefore, $c_2 < c_1$. Once again, $\{c_k\}_{k=1}^L$ is a declining sequence, bounded, and so converges. The remainder of the proof is exactly the same, replacing ψ_j with ψ_j^* and ρ_j with ρ_j^* in each expression. Then, as before,

$$w_{j\infty} = \frac{\psi_j^*}{1 - \beta \rho_j^*} / \sum_{i=1}^n \frac{\psi_i^*}{1 - \beta \rho_i^*} \quad \text{for all } j.$$

Set $\pi = 1 - \beta(1 + g\gamma)$, and recall the maintained assumption that $|\beta(1 + g\gamma)| < 1$. Therefore, the weight attached to each signal j is precisely as given in Proposition 9, but where $\pi = 1 - \beta(1 + g\gamma)$ and $M = L$ denotes the final level in the hierarchy.

Variants of Lemma 5 and Proposition 10 continue to hold, replacing the player subscript m with the associated level ℓ and using the notation described above. All that is required is to replace β appropriately, and to take care to adjust the κ_i^2 parameters in the proof.

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