The Determinants of Product Lines

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Abstract. We study the determinants of product lines in a multi-product Cournot model which allows for cost asymmetries and which eliminates strategic motivations (such as a desire to soften competition) for product-line choice. We identify two distinct forces that lead firms to offer multiple qualities. We then investigate how the interaction of asymmetric competition with these forces shapes equilibrium product lines. Our model predicts a rich variety of possible outcomes, ranging from head-to-head competition (in which all firms offer all products) to complete separation of product lines (so that no product is offer by two or more firms) and including the intermediate case of partial separation. In an international trade context, we predict whether a disadvantaged foreign producer will specialize in higher or lower qualities. If cost factors are the main drivers of quality-based discrimination (so that there are decreasing returns to quality) then foreign firms sell only lower qualities. However, if demand factors are the main drivers (so that, other things equal, the demand for higher qualities is less elastic) then foreign firms sell only high qualities.

We study the determinants of the (vertically differentiated) product lines of multi-product competitors who engage in quantity competition. We are motivated by two observations.

Our first observation is that multi-product firms may differ in their production costs. For instance, a firm that exports to a given country may face higher costs (owing to tariffs or transportation) than a domestic firm. Our second observation is that product-line configurations amongst rivals may differ substantially in their degree of overlap. For instance, some rivals choose to compete head-to-head by offering the same product qualities while other rivals offer completely separate product lines.

Our goal is to determine when and why such different configurations occur in equilibrium. Moreover, we seek to identify precisely which firms will offer which product line. For example, if there is product-line separation in equilibrium, then we seek to identify whether a cost-disadvantaged firm will focus on high quality or on low quality.

A primary finding is that—despite there being no strategic reason (such as a desire to soften price competition) for firms to limit their product lines—firms may, nevertheless, avoid head-to-head competition. That is, although firms do not observe the product line

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choices of their competitors prior to making their own product line and output choices, a firm nonetheless may choose to omit from its own product line some products that its rivals sell. Asymmetric costs are necessary for this outcome to emerge. As cost asymmetry varies equilibrium product lines may either be fully separated, fully overlapping, or partially overlapping. Thus, our motivating observations stated above are closely related: the degree of cost asymmetry determines the degree of product-line overlap.

Another finding is that the structure of a firm's product line depends both on its costs relative to rivals and also on which of two distinct forces for price discrimination is dominant in the industry under consideration. Even when all firms sell all products, the identity of the dominant force determines the market shares of different firms over different ranges of quality. We distinguish between *cost-driven* and *elasticity-driven* price discrimination (defined below). When price discrimination is cost-driven, firms with less advantageous cost structures tend to sell only lower-quality products while firms with more advantageous cost structures tend to sell only higher-quality products. On the other hand, if discrimination is elasticity-driven then cost-disadvantaged firms tend to sell only higher-quality products while cost-advantaged firms tend to sell all qualities. An application is to international trade: these forces determine whether exporters (facing tariffs that make them higher cost than domestic firms) specialize in high or instead low-quality products, and whether domestic firms sell only high-quality goods or instead both low and high-quality goods.

To describe our results and the underlying intuitions in more detail, we begin by more carefully describing the differences between cost-driven and elasticity-driven discrimination.

Price discrimination being cost-driven means that a monopolist offers a full product line because providing higher quality is significantly more costly than providing lower quality. For example, if a business-class seat on a transcontinental flight is substantially more spacious than an economy seat, then it is also much more costly to provide.

In contrast, price discrimination being elasticity-driven means that a monopolist offers a full product line because higher quality products have less-elastic demand than lower quality products, although higher quality might not be significantly more expensive to supply. For example, if the main quality gain from flying business class is a free glass of wine or tastier snack then the provision of higher quality is not particularly expensive. Nonetheless, because some consumers are willing to pay a significant premium for this extra quality, a monopolist may optimally choose to deny this perk to economy ticket-holders.

The computer industry provides other examples of these two motivations for price discrimination. For instance, when considering faster-access memory devices, faster microprocessors, larger hard drives, or larger and higher resolution flat-panel displays, there may be substantive costs over lower-quality versions, so that the decision to offer multiple versions is cost-driven. On the other hand, sometimes a firm offers a higher-quality version that entails no significant costs, as when Intel or IBM sold lower-quality microprocessors and printers by first building a high-quality version and then disabling certain features.[1]

Settling on the example of airlines, consider competing airlines each of which can sell both economy and business class tickets.[2] These products are substitutes: an increase in the output of one reduces the price of the other. However, a (higher quality) business class ticket is equivalent to a bundle of a (lower quality) basic economy ticket together with an additional quality upgrade from economy to business class. Usefully, the bundle's components are neither complements nor substitutes.[3] Hence multi-product Cournot competition over two (or more) qualities mimics single-product Cournot markets for economy class and upgrades respectively. A complication is that an airline cannot sell more upgrades than basic economy tickets. If competition in the market for upgrades leads to a push against this constraint, then (in essence) all this airline's economy tickets are upgraded; this is equivalent to the elimination of economy class from a product line. It follows that a gap in a product line, such as the omission of economy class, arises when a supplier has a strong incentive to expand the supply of the upgrades relative to the supply of the basic product.

Suppose for the sake of developing intuition that there are only two airlines and that price discrimination in the airline industry is entirely cost-driven. Moreover suppose that any cost advantage of one firm over the other is more pronounced in the market for upgrades. Because price discrimination is cost-driven, the shape (although perhaps not level) of demand for the economy ticket is the same as for the upgrade market. Just like in single-product Cournot markets, the firm with lower costs has more market share in the upgrade market than in the economy market. Following the discussion just above, if this difference is sufficiently large, the constraint that the lower-cost firm cannot sell more upgrades than basic economy tickets will bind, meaning that the lower-cost firm sells only business-class tickets. In turn, this high production of business-class tickets may drive the higher-cost firm out of the upgrade market, so that it specializes in economy-class tickets. This explains why cost-driven price discrimination can lead to completely non-overlapping product lines, even in the absence of any strategic motivation to soften competition.

Now suppose instead that price discrimination in the airline market is entirely elasticity-driven: assume that an upgrade to business class has no significant cost, but also assume that the demand curve for this upgrade is much less elastic than the demand curve for economy travel. In a single-product Cournot setting the market share of higher-cost firms

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[1] In the early 1990s Intel disabled the mathematics coprocessor of its premium 486DX microprocessor to create a lower-quality 486SX version. Similarly, IBM used software that limited the printing speed of its 4019 LaserPrinter to create an economy version. In both cases, owners of the economy versions could, at a later date, purchase an upgrade to restore the full features of the premium version.

[2] An airline is able to adjust its capacity on any given route by, for example, using larger or smaller airplanes.

[3] A local change in the upgrade supply influences the price of the upgrade to business class but this change does not influence the marginal buyer (and hence price) of an economy ticket. Similarly, a local change to the economy supply does not influence the marginal buyer of the upgrade.

actually increases with demand elasticity (but of course the lower-cost firm still has a larger market share than the higher-cost firm). An intuition is that, in order to take advantage of the less-elastic demand, the lower-cost firm (which has a higher per-unit margin) reduces its market share. The higher-cost firm not only gains market share but may in fact increase its output. When operating in a multi-product world, this means that the higher-cost firm's constraint that it cannot sell more upgrades than basic economy tickets may bind, causing it to sell only business-class tickets. At the same time, this expansion of upgrade output leads the lower-cost firm to reduce its own supply of upgrades, thereby loosening that firm's constraint—it will continue to sell both qualities.

Our discussion above emphasizes that an attractive feature of our approach (involving quantity-setting and potentially asymmetric costs) is that it can explain a complete range of outcomes including full product-line overlap, partial product-line overlap, and full product-line separation.[4] In contrast, standard models that predict non-overlapping product lines, such as price-setting models in which ex-ante symmetric firms commit to product lines before setting their prices, do not easily also predict partially overlapping or fully overlapping products lines. Rather, in such two-stage models, no two firms will sell the same products—equilibrium outcomes exhibit complete separation of product lines.

Our approach not only suggests that such distinct outcomes may occur in equilibrium, but also pins down which particular firms sell which particular products (as already discussed, this is determined by the interaction between the forces of price discrimination and asymmetric costs). In contrast, standard models of two-stage competition make no such predictions. That is, standard models that predict that firms will not compete head-to-head do not predict which firm will focus on low quality and which on high quality.

Another benefit of our model is that it in addition to explaining gaps in product lines, it can explain the relative market presence of different firms. By this we mean simply that there may be less extreme outcomes than firms actually removing a product. For example, even if all firms sell all products, some firms may concentrate their output more in certain ranges of quality. As we discuss in more detail later, exactly the same forces that we have already discussed (asymmetric costs and the underlying forces driving price discrimination) predict which types of firms will have higher relative presence in low versus high quality products.

In later sections we provide a more detailed assessment of the application to international trade that we have already mentioned. Further applications and results emerge in other specific circumstances. One such case is where cost asymmetries take a particular form: all suppliers have the same production costs, but differ in the maximum qualities that they are capable of producing. This may be so if specialized knowledge is required to produce higher quality levels, and some firms have more such knowledge than others. We show that a gap in a supplier's product line contains the maximum feasible quality of a less-capable rival,

[4]If firms are symmetric, then (under familiar regularity conditions) they each offer the same product line.

and that such gaps typically exist. Thus, we tie gaps in the product lines of more-capable firms to the technological limitations of their inferiors.

We also allow different technological capabilities of the form just described (maximum feasible qualities) to arise endogenously. When suppliers choose their maximum quality level and quantities at the same time, then (subject to mild regularity conditions), no two competitors choose the same capability. For example, the highest quality is only offered by a single supplier. This supplier balances the additional fixed cost of offering that highest quality step against the monopoly profit from the the associated upgrade. In contrast, the nearest competitor (with the second highest maximum quality) recognizes that offering that highest quality generates only the duopoly profit from that upgrade.

This paper builds upon classic studies of second-degree discriminatory pricing (Spence, 1977; Mussa and Rosen, 1978; Mirman and Sibley, 1980; Itoh, 1983; Maskin and Riley, 1984; Goldman, Leland, and Sibley, 1984), relates to more recent studies of multi-product supply by a monopolist (Anderson and Dana Jr, 2009; Anderson and Çelik, 2015; Johnson and Myatt, 2016), and complements a renewed contemporary interest in price discrimination (Aguirre, Cowan, and Vickers, 2010; Cowan, 2007, 2012; Armstrong, 2015).[5]

Most notably other work has studied Cournot competition by suppliers who offer multiple qualities (Gal-Or, 1983; De Fraja, 1996; Johnson and Myatt, 2003, 2006a,b, 2016). De Fraja (1996) considered the possibility of incomplete product lines but, in common with others (Gal-Or, 1983; Johnson and Myatt, 2006b, 2016; Anderson and Çelik, 2015), restricted to symmetric competitors. Johnson and Myatt (2003, 2006a) allowed for asymmetric competitors, but did not systematically analyze the determinants of product lines. Our contribution is to characterize how (possibly endogenously chosen) asymmetric technological capabilities lead to particular distinctive features of product lines. Unlike other well-known studies (Brander and Eaton, 1984; Champsaur and Rochet, 1989) the single-stage game form means that product lines are not chosen strategically to soften price competition.

In the international trade literature, the empirical work of Bernard, Redding, and Schott (2010) has emphasized multi-product firms. Theoretical contributions include Eckel and Neary (2010), Bernard, Redding, and Schott (2011), and Nocke and Yeaple (2014). Similar to us, they are interested in what determines the product lines of potentially heterogenous firms. Unlike us, these papers consider monopolistically competitive firms selling horizontally differentiated products and focus on the role that globalization (leading to larger market size) plays in determining product portfolios. We contribute by predicting whether exporting firms will focus on high-quality or low-quality product lines.

[5]It is related somewhat more distantly to recent contributions to a literature that studies product bundling (Armstrong, 2010, 2013; Armstrong and Vickers, 2010; Chen and Riordan, 2013).

1. A Multi-Product Cournot Oligopoly

Here we describe the components of our model. On the demand side, buyers choose between different quality variants. On the supply side, multi-product firms compete in quantities.

1.1. **Demand.**

A buyer ("he") with type θ is willing to pay at most $v(\theta, q)$ for a single unit of a product with quality q, where $v(\theta, q)$ is increasing in both of its arguments, is twice differentiable, and satisfies the usual sorting condition: $\partial v(\theta, q)/\partial\theta\partial q > 0$.

Faced with a menu of qualities, a buyer θ purchases a single unit of the product that offers him the greatest positive surplus. Amongst a unit mass of potential buyers, for $z \in [0, 1]$ we write $\theta(z)$ for the buyer type with z others above him, where $\theta(z)$ is a strictly decreasing and differentiable function of z. Hence, if z units of a single quality q were to be supplied, then the market-clearing price (and so the inverse-demand curve) would be $v(\theta(z), q)$. A special case is when preferences are multiplicative, so that $v(\theta(z), q) = \theta(z)q$. For this case, the inverse-demand curve for a higher quality is a scaled version of that for a lower quality.

1.2. **Supply.**

Feasible quality levels are drawn from a set $q_1 < \cdots < q_I$ indexed by $i \in \{1, \ldots, I\}$. This finite set is not restrictive: we allow I to be arbitrarily large. For some results we do indeed make I large by requiring the quality increments $q_i - q_{i-1}$ to become small for all i. Our results can also be derived within a continuum-of-qualities specification.

These qualities are supplied by R multi-product Cournot firms, indexed by $r \in \{1, \ldots, R\}$. Firm r is able to manufacture a product of quality q_i at a constant (with respect to quantity) marginal cost of $c_{ir} = c_r(q_i)$. The function $c_r(q)$ is an increasing and differentiable function of q, and so $\Delta c_{ir} \equiv c_{ir} - c_{(i-1)r} \geq 0$; for each firm, a higher quality product costs weakly more to produce than a lower quality product. In some cases, we will consider a symmetric industry, and when we do so we drop the subscript r.

We write $c(q) \equiv \frac{1}{R}\sum_{r=1}^{R} c_r(q)$ for the industry average cost of producing a unit of quality q. That is, $c(q)$ is the average cost, across all firms, of producing a single unit of a product of quality q. Sometimes it will be more convenient notationally to denote $c_i \equiv c(q_i)$, meaning that c_i is the average (across all firms) cost of producing a single unit of quality q_i.

Later in the paper we consider a specification in which each firm pays a fixed cost that increases with the highest quality that it offers, but for now we ignore fixed costs.

The firms simultaneously choose their supplies of all qualities. z_{ir} is the supply of quality q_i offered by firm r, and $z_i \equiv \sum_{r=1}^{R} z_{ir}$ is the industry supply of that quality. A firm's *product line* is the set of qualities that it offers in strictly positive supply.

1.3. **Prices.** Market-clearing prices (denoted p_i for quality q_i) ensure that higher buyer types purchase higher qualities. The marginal buyer of quality q_i (the type $\theta(Z_i)$ with $Z_i \equiv \sum_{j=i}^{I} z_j$ others above him) is indifferent between qualities q_i and q_{i-1}, and so

$$\Delta p_i \equiv p_i - p_{i-1} = P_i(Z_i) \quad \text{where} \quad P_i(Z_i) \equiv v(\theta(Z_i), q_i) - v(\theta(Z_i), q_{i-1}). \tag{1}$$

Z_i is the output of products with quality q_i or higher. This can be interpreted as the supply of upgrades to this quality level (and beyond). Similarly, Δp_i is the price of an upgrade from quality q_{i-1} to the next step q_i. We set $q_0 = p_0 = 0$ for convenience.

From an upgrades perspective, quality q_i is a combination of the i upgrades from Δq_1 to Δq_i. An upgrade's price Δp_i depends only on its own supply, and so the advantage of thinking in terms of upgrades is that they are neither substitutes nor complements.

We write Z_{ir} for the quantity of the ith upgrade offered by firm r, so that $Z_{ir} \equiv \sum_{j=i}^{I} z_{jr}$. From its choice of I upgrade supplies we can recover its supply of each quality via $z_{ir} = Z_{ir} - Z_{(i+1)r}$. The supplies of the qualities must be weakly positive, and so a profile of upgrade supplies must satisfy the constraints $Z_{1r} \geq \cdots \geq Z_{Ir}$. If a constraint binds (so that $Z_{ir} = Z_{(i+1)r}$) then the corresponding quality is omitted from the firm's product line ($z_{ir} = 0$).

1.4. **Profits.** Firm r seeks to maximize its profit across all qualities:

$$\begin{aligned}
\text{Multi-Product Profit of Firm } r &= \sum_{i=1}^{I} z_{ir}(p_i - c_{ir}) \\
&= \sum_{i=1}^{I} (Z_{ir} - Z_{(i+1)r}) \left(\sum_{j=1}^{i} (\Delta p_j - \Delta c_{jr}) \right) \\
&= \sum_{i=1}^{I} Z_{ir}(\Delta p_i - \Delta c_{ir}) \\
&= \sum_{i=1}^{I} Z_{ir} \left(P_i(Z_i) - \Delta c_{ir} \right), \qquad (2)
\end{aligned}$$

where we recall that $P_i(Z_i) = v(\theta(Z_i), q_i) - v(\theta(Z_i), q_{i-1})$.

The ith element of this summation depends only upon Z_{ir} and Z_i. These are (respectively) firm r's supply and the industry supply of the ith upgrade. Thus a firm's objective is to maximize the sum of profits from I separate Cournot upgrade markets. If the monotonicity constraints on upgrade supplies do not bind, then a Nash equilibrium of a multi-product Cournot game replicates single-product Nash equilibria in each upgrade market.[6] Furthermore, if such constraints are non-binding then all firms offer complete product lines.

[6]Johnson and Myatt (2006a) described sufficient conditions for the existence of multi-product equilibria.

2. Product Lines in a Symmetric Oligopoly

As a benchmark we now briefly consider a symmetric oligopoly and review the conditions under which firms offer complete product lines (so that $z_{ir} > 0$ for every i and r). The conditions (which concern the returns to quality and the relationship between quality and demand elasticity) for this benchmark were reported by Johnson and Myatt (2006a).

2.1. **Optimal Supply in Upgrade Markets.** A preliminary step is to ensure that the single-product Cournot games (one for each upgrade) are well behaved. $P_i(Z_i)$ is the market-clearing price (that is, the inverse demand function) in the ith upgrade market. We assume throughout that the associated marginal revenue function is decreasing in output. That is, fixing Z_{ir}, we assume that $P_i(Z_i) + Z_{ir}P_i'(Z_i)$ is decreasing in industry output Z_i.

Under this regularity condition, Johnson and Myatt (2006a, Proposition 1) characterized a unique equilibrium when firms are symmetric. In this equilibrium, firms produce equal shares of each upgrade. Let $Z_i^\star$ denote the equilibrium industry supply of upgrade i. In an equilibrium with complete product lines, so that $Z_1^\star > \cdots > Z_m^\star > 0$, these supplies satisfy the usual Cournot first-order condition in each upgrade market, so that

$$P_i(Z_i^\star) + \frac{Z_i^\star P_i'(Z_i^\star)}{R} = \Delta c_i, \tag{3}$$

where we have dropped the subscript r from Δc_{ir} in light of the assumed symmetry.

2.2. **Complete Product Lines.** To pin down whether product lines are complete, we ask when the I solutions to equation (3) are strictly ordered. The profit margin $P_i(Z_i^\star) - \Delta c_i$ associated with upgrade i is central to this. If it is more responsive to changes in output then optimal supply is lower. To measure responsiveness, we define the *profit margin elasticity*:

$$\eta_i(Z) \equiv -\left[\frac{\partial \log(P_i(Z) - \Delta c_i)}{\partial \log Z}\right]^{-1} = -\frac{P_i(Z) - \Delta c_i}{Z P_i'(Z)}. \tag{4}$$

Equation (3) is equivalently $\eta_i(Z_i^\star) = 1/R$. The profit-margin elasticity is *decreasing in quality* if $\eta_i(Z) > \eta_{i+1}(Z)$ for all i and Z.[7] Conceptually, this means that each individual firm has stronger incentives to raise output at lower quality levels, market supply being equal. Thus, decreasing profit margin elasticity implies that $Z_i^\star > Z_{i+1}^\star$ for all i.

The profit margin elasticity condition can also be recast in more familiar terms:

$$\eta_i(Z) = \left[1 - \frac{\Delta c_i}{P_i(Z)}\right]\epsilon_i(Z) \quad \text{where} \quad \epsilon_i(Z) \equiv -\left[\frac{\partial \log P_i(Z)}{\partial \log Z}\right]^{-1} = -\frac{P_i(Z)}{Z P_i'(Z)}, \tag{5}$$

so that $\epsilon_i(Z)$ is the usual elasticity of demand. For $\eta_i(Z) > \eta_{i+1}(Z)$ it is sufficient that $\epsilon_i(Z)$ is decreasing in i and $\Delta c_i / P_i(Z)$ is increasing in i. The first statement says that consumers are less price sensitive in their evaluation of higher upgrades. Recalling that $P_i(Z) =$

[7] This comparison of profit-margin elasticities applies for outputs where the profit margins are strictly positive. For Z where $P_i(Z) \leq \Delta c_i$ we may proceed by defining $\eta_i(Z) = 0$ and adjusting our definition accordingly.

$v(\theta(Z), q_i) - v(\theta(Z), q_{i-1})$, the second condition says that higher upgrades are relatively more expensive to produce (where the cost of production is measured relative to the marginal buyer's willingness to pay) so that there are *decreasing returns to quality*.

These observations yield a benchmark result (Johnson and Myatt, 2006a, Proposition 6).

Proposition 1 (Complete Product Lines)**.** *If the elasticity of the upgrade profit margin is decreasing in quality, then symmetric multi-product Cournot oligopolists offer complete product lines. If there are (i) decreasing returns to quality, and (ii) the elasticity of demand for upgrades is decreasing in quality, then this condition is satisfied.*

This statement simplifies if $v(\theta(z), q) = \theta(z)q$. When this is so, the elasticity of demand is independent of quality, and so $\epsilon_i(Z) = \epsilon_{i+1}(Z)$ for all i and Z. Furthermore, there are decreasing returns to quality if and only if $\Delta c_i/\Delta q_i$ is increasing in i.

Corollary. *If valuations are multiplicative in type and quality,* $v(\theta(z), q) = \theta(z)q$*, and* $\Delta c_i/\Delta q_i$ *is increasing in* i*, then symmetric firms offer complete product lines.*

The two conditions reported in Proposition 1 are jointly sufficient but are not individually necessary for the members of a symmetric industry to supply complete product lines. Most particularly, although the assumption of decreasing returns to quality is very sensible for many products, there are other products where it is not, meaning that the incremental cost of raising quality from a given level is quite small compared to the cost of supplying that given level. For example, the cost of serving an economy-class airline traveler a glass of wine or a sandwich may be very small compared to the basic cost of transporting that traveler, and yet such perks may be reserved for business-class ticket-holders. "Damaged goods" provide an even more extreme violation of decreasing returns to quality: a low quality good is obtained by damaging a high quality good, as when a speedy microprocessor or printer is converted into a lower quality good by intentionally slowing it down (Deneckere and McAfee, 1996; Varian, 1997; Shapiro and Varian, 1998; McAfee, 2007).

Despite a lack of decreasing returns to quality, complete product lines may nonetheless emerge in a symmetric industry. The reason is that the second condition in Proposition 1 may hold, so that the elasticity of demand for upgrades is decreasing in quality. More broadly, the desire to discriminate (in the second degree sense) using quality may be driven by either (or both) of two forces: decreasing returns to quality or decreasing elasticity of demand for upgrades. The first force corresponds to *cost-driven discrimination*, whereas the second force generates *elasticity-driven discrimination*.

This distinction matters. That is, whether the desire to practice quality-based discrimination is driven primarily by one force or instead the other leads to important predictive differences regarding how equilibrium product lines are affected by asymmetry in the cost structures of firms. Such asymmetries are the focus of our work in the next sections, to which we now turn.

3. Firm Asymmetries, Market Presence, and Product Line Gaps

Here we investigate how the market shares of firms across different quality ranges are influenced by the interaction between demand elasticities and any asymmetries among firms' cost structures. This interaction leads different firms to concentrate their outputs, either partly or wholly, within product ranges of different qualities.

Of particular interest are cases where such differences in market shares are extreme enough to lead to gaps in the product lines offered by different firms. As we will show, the relationship between demand elasticities and cost asymmetries that explains such gaps is somewhat subtle. For example, in some cases it is firms with cost advantages that may omit a lower-quality product, while in other cases it is firms with cost disadvantages that may omit a lower-quality product (while still selling the high quality product).

Throughout this section, we focus on situations in which symmetric firms would offer complete product lines. From Proposition 1 it would be sufficient to assume that there are decreasing returns to quality (so that discrimination is cost-driven) and that the elasticity of demand for upgrades is decreasing in quality (so that discrimination is elasticity-driven). As discussed at the end of Section 2, however, even when one of these conditions fails, complete product lines may nonetheless emerge. Therefore, to maintain generality, we do not necessarily assume that both of these conditions hold.

Immediately below we explain in detail how the emergence of product-line gaps is connected to changes in the market shares of firms across different quality-upgrade markets. Afterwards, we describe the three forces that may cause appropriate changes in market shares, thereby causing product-line gaps or, in less extreme cases, causing different firms to concentrate their output in different quality ranges. We close this section by offering predictions related to our observations about product lines.

In the next section (Section 4) we build upon our work here. There we present three explicit specifications that generate complete product lines by all firms (and so head-to-head competition) in a symmetric industry, but lead to diverse outcomes when firms are asymmetric.

3.1. **Three Forces that Cause Product-Line Gaps.** Recall that firm r omits product i when it upgrades all products of quality q_i to at least quality q_{i+1}, so that $Z^\star_{ir} = Z^\star_{(i+1)r}$. Thus, to study product line gaps, we study when these upgrade constraints bind.

To this end, imagine (for a moment) that firms are not constrained to obey upgrade constraints, and let $Z^\dagger_i$ denote the equilibrium industry output in upgrade market i in this unconstrained situation.[8] Similarly let $Z^\dagger_{ir}$ denote firm r's output in this scenario. Under

[8]Specifically, $Z^\dagger_i$ is the equilibrium industry output in a single-product Cournot industry in which the inverse-demand curve is $P_i(Z_i)$ and where firm r faces a constant marginal cost of Δc_{ir}.

the maintained assumption of this section, $\{Z_i^\dagger\}$ forms a strictly decreasing sequence.[9] If the market share (of each upgrade market) enjoyed by firm r is not changing as quality increases then this firm's upgrade output sequence $\{Z_{ir}^\dagger\}$ is also strictly decreasing in i. This means that, in the underlying Cournot game in which upgrade constraints are obeyed, there is an equilibrium in which $Z_{ir}^\star = Z_{ir}^\dagger$, and so firm r sells each and every product.

Therefore, for the actual multi-product Cournot oligopoly in which firms do obey upgrade constraints, a necessary condition for gaps to appear is that market shares of some firms change when considering the unconstrained market.

To investigate when this is so, note that, as in a standard single-product Cournot market, the unconstrained market share of firm r in the supply of upgrade i is

$$\frac{Z_{ir}^\dagger}{Z_i^\dagger} = \epsilon_i(Z_i^\dagger) - \left(\epsilon_i(Z_i^\dagger) - \frac{1}{R}\right)\frac{\Delta c_{ir}}{\Delta c_i}, \quad \text{where} \quad \Delta c_i \equiv \frac{\sum_{r=1}^{R} \Delta c_{ir}}{R} = c_i - c_{i-1}. \tag{6}$$

(This equation is readily obtained by manipulating the firms' first-order conditions.)

There are three distinct forces that influence the market shares given in Equation (6). Each force may potentially raise a firm's market share moving from one upgrade market to the upgrade market with the next highest quality, so causing a monotonicity constraint to bind and thus inducing that firm to drop the corresponding quality from its product line.

We now identify and discuss each of these three forces separately. As we will explain, which force is most likely to be active is closely connected to the underlying incentives for quality-based price discrimination to exist in a symmetric or monopoly market.

3.1.1. *Quality-Dependent Comparative Costs.* Recall that Δc_{ir} is firm r's upgrade cost to quality q_i and (as defined in equation (6)) Δc_i is the industry-average cost of upgrading to quality q_i. Thus $\Delta c_{ir}/\Delta c_i$ is a measure of the comparative cost position of firm r.

The first force that may lead to a product-line gap is an improvement in a comparative cost position (relative to the industry average) in the production of higher upgrades. If

$$\frac{\Delta c_{ir}}{\Delta c_i} > \frac{\Delta c_{(i+1)r}}{\Delta c_{i+1}}, \tag{7}$$

[9] To reiterate, we are focusing on situations in which a symmetric industry in which Δc_i is the common marginal cost for all firms generates an equilibrium with complete product lines. We recall again (the analysis preceding Proposition 1 applies) that we require the elasticity of the upgrade profit margin to be decreasing in quality. If there are both decreasing returns to quality and the elasticity of demand for upgrades is decreasing in quality, then this holds. However, even if one of these conditions is violated, there may still be complete product lines so long as the other condition is sufficiently strongly satisfied.

then firm r's comparative cost position is better for the higher upgrade (that is, better for upgrade $i+1$ than for upgrade i).[10] Equation (6) indicates that, other things equal, this means that its market share will increase, so that $Z^\dagger_{(i+1)r}/Z^\dagger_{(i+1)} > Z^\dagger_{ir}/Z^\dagger_i$.

This force is present, for example, when a firm enjoys a technological advantage for higher quality upgrades, but a lesser advantage (or a disadvantage) for lower quality upgrades. To take an extreme case, suppose there are two qualities and all firms are capable of producing quality q_1 at the same cost, but all firms other than r find producing quality q_2 prohibitively expensive (perhaps because r has developed a unique capability). Then r's comparative cost position is improving, suggesting an increased market share for the higher quality good that may lead it to remove the lower quality good from its product line.

Specification 1 in Section 4 below explores in detail the influence of such changes in comparative costs. Furthermore, in Section 5 we consider situations of the kind discussed above where only some firms are able to manufacture higher qualities, and where these technological capabilities are endogenously determined.

3.1.2. *Quantity-Dependent Demand Elasticity.* To identify the other two forces that may alter market shares and hence lead to product-line gaps, return attention to Equation (6) and suppose that comparative costs are fixed, so that $\Delta c_{ir}/\Delta c_i$ does not vary with quality.[11] This neutralizes the first force (discussed just above) so that the the only way that firm r's market share can increase is if the elasticity $\epsilon_i(Z^\dagger_i)$ differs from $\epsilon_{i+1}(Z^\dagger_{i+1})$. That is, the elasticity of demand in the lower upgrade market (evaluated at the appropriate aggregate industry supply $Z^\dagger_i$) must differ from the elasticity of demand in the higher upgrade market (evaluated at the appropriate, and potentially different, industry supply $Z^\dagger_{i+1}$).

There are two distinct ways in which demand elasticity can differ across different quality upgrade markets. One possibility is that the shape of elasticity differs across upgrade markets, so that even at some common aggregate output Z, $\epsilon_i(Z)$ differs from $\epsilon_{i+1}(Z)$. We will call this the third force, and defer its discussion for the moment.

The other possibility, and what we will call the second force that may lead a firm to gain market share in higher upgrade markets, is that $\epsilon_i(Z)$ changes with Z. To isolate this force, we assume temporarily that the shape of elasticity is the same across upgrade markets, so that $\epsilon_i(Z) = \epsilon_{i+1}(Z) = \epsilon(Z)$ for all Z and some $\epsilon(Z)$. This corresponds to a multiplicative specification, $v(\theta(z), q) = \theta(z)q$.

[10]As an aside, we note that an increasing comparative cost position is consistent with decreasing returns to quality. The reason is that decreasing returns to quality is a statement only about how the (appropriately quality-normalized) costs of a single firm change with quality, whereas increasing comparative cost is a statement about how a firm's cost position relative to other firms changes with quality.

[11]Specifically, for each firm r, $\Delta c_{ir}/\Delta c_i$ does not vary with the quality index i. The average cost across all firms within any upgrade market may still depend on quality (for example, it may be higher in higher upgrade markets) but that dependence is such that all firms' cost change by the same proportional amount.

To see why this second force matters, note that the presence of complete product lines implies that aggregate outputs differ in the two upgrade markets, so that $Z_i^\dagger > Z_{i+1}^\dagger$. Assuming the typical case of demand, in which $\epsilon(Z)$ is decreasing in Z, we use Equation (6) to discern the change in firm r's market share moving from market i to $i+1$.[12] Precisely, to consider how a decrease in aggregate output affects firm r's market share, we examine

$$-\frac{\partial}{\partial Z_i^\dagger}\left[\frac{Z_{ir}^\dagger}{Z_i^\dagger}\right] = \left(1 - \frac{\Delta c_{ir}}{\Delta c_i}\right)\frac{\partial \epsilon(Z_i^\dagger)}{\partial Z_i^\dagger} > 0 \quad \Leftrightarrow \quad \Delta c_{ir} < \Delta c_i. \tag{8}$$

Thus, moving to higher upgrade markets raises the market shares of firms that are more efficient than the industry average. To be precise, the market shares of firms that are more efficient than the industry average is higher in higher upgrade markets if (i) industry output falls as quality increases, (ii) comparative costs are fixed, and (iii) $\epsilon(Z)$ is strictly decreasing in supply Z, where $\epsilon(Z)$ is the common (across different qualities) upgrade demand elasticity.

Such an increase in market share may be accompanied by an increase in an advantaged firm's actual supply of the higher quality upgrade. If such an increase is sufficiently large then this firm may drop quality q_i from its product line. For modest increases in output, this firm continues to sell both products but (by upgrading more units) increases its supply of higher quality goods while lowering its supply of lower quality goods.

Another viewpoint is the following. The use of complete product lines by all firms ensures that industry output remains characterized by Equation (3).[13] Additionally, under our current experiment of fixing the shape of elasticity across upgrade markets, we have assumed a multiplicative structure for preferences, so that $P_i(Z) = \Delta q_i \theta(Z)$. Therefore, if $Z_i^\dagger > Z_{i+1}^\dagger$, then it must be that the average quality-adjusted upgrade cost in market $i+1$ is higher than in market i, meaning $\Delta c_{(i+1)r}/\Delta q_{i+1} > \Delta c_{ir}/\Delta q_i$. Finally, because comparative costs are not changing, so that $\Delta c_{ir}/\Delta c_i = \Delta c_{(i+1)r}/\Delta c_{i+1}$, it must be that each firm's quality-adjusted upgrade cost has increased by the same proportional amount.

To understand the impact of the second force, therefore, it is equivalent to ask how market shares respond in a single-product world when each firm suffers the same proportional increase in its marginal cost. Such a scaling of costs is more damaging to firms with higher marginal costs. This is because the absolute magnitude of cost increases is higher for firms that had higher costs to begin, and lower for firms that had lower costs to begin. Therefore, such cost increases lower the market shares of less efficient firms and raise the market

[12]For example, in a single-product setting, $\epsilon(Z)$ is decreasing in Z if the underlying distribution function of valuations generating demand is log-concave or not too log-convex. The borderline case is constant-elasticity demand, which is log-convex and in which $\epsilon(Z)$ is constant in Z.

[13]When all firms offer complete product lines, $P_i(Z_i) + Z_{ir}P_i'(Z_i) = \Delta c_{ir}$ holds for each firm r in each upgrade market i. Summing over all firms and dividing by the number of firms R returns Equation (3).

shares of more efficient firms.[14] An extreme demonstration of this intuition arises when there are only two firms, and their costs are scaled upwards by a large amount. In such a case, the less efficient firm may drop out of the market, sending its market share to zero, while the more efficient firm claims the entire market for itself.

Specification 3 in Section 4 below pursues the ramifications of this second force in more detail. But here it is worth mentioning explicitly that there are several circumstances where this second force will be at work. It arises most cleanly when all firms have cost functions that are proportional shifts of one another. This can arise in an international-trade context. Consider, for example, a situation in which domestic and foreign producers have identical technologies, but where foreign producers face higher costs. The iceberg transportation cost specification (Samuelson, 1954) commonly used in many influential models of trade supposes that foreign producers must pay ad valorem tariffs (or equivalently a fraction of imported output is lost in transit). This corresponds to a scaling up of production costs for foreign producers. Thus, if elasticity is constant with quality, which implies that discrimination is cost-driven, then domestic producers may eliminate lower qualities from their product lines, and so foreign imports will dominate the supply of those lower qualities.[15]

3.1.3. *Quality-Dependent Demand Elasticity.* To describe the third force that may influence market shares and cause gaps in the product lines of firms, we reflect on the second force, discussed above. That force applies most cleanly when firms' comparative capabilities do not change with quality and when the shape of demand elasticity is the same across different qualities. For multiple products to emerge under such conditions, there must be decreasing returns to quality. For market shares to change, elasticity must depend on quantity within any given upgrade market.

But there are real-world examples where it may not seem plausible that there are meaningful decreasing returns to quality, and yet firms offer multiple qualities. This is most clearly true when (for a common production cost) different versions are obtained by turning features on or off. Such versioning is commonly employed in the sale of technology products and software services (Deneckere and McAfee, 1996; Varian, 1997; Shapiro and Varian, 1998; McAfee, 2007). The third force can explain such an observation: when demand is less elastic in higher upgrade markets, a monopolist (or, indeed, the member of a symmetric industry) has incentives to restrict demand in such markets, even in the absence of any cost concerns. This may lead to the sale of multiple quality-differentiated products.

[14] Formally, we already demonstrated that a decrease in aggregate output would increase the market share of more-efficient firms. To complete the argument here we note that the maintained assumption of decreasing marginal revenue implies that an upward scaling of all firms' costs lower aggregate output.

[15] Outside this trade context, costs may similarly be scaled if some firms command proportionately lower input prices than others (perhaps because vertical integration allows them to obtain their inputs at marginal cost while their rivals pay more, or because they have better bargaining power over suppliers).

Amongst asymmetric rivals, this same force may lead some firms to remove some products from their product lines. To see this most clearly, suppose that (i) comparative costs do not vary with quality, and (ii) elasticity is constant with respect to supply within any upgrade market, so that $\epsilon_i(Z) = \epsilon_i$ for some constant ϵ_i. Once again using (6), we see that an increase in quality i that reduces ϵ_i has the following effect on firm r's market share.

$$-\frac{\partial}{\partial \epsilon_i}\left[\frac{Z_{ir}^{\dagger}}{Z_i^{\dagger}}\right] = -\left(1 - \frac{\Delta c_{ir}}{\Delta c_i}\right) > 0 \quad \Leftrightarrow \quad \Delta c_{ir} > \Delta c_i. \tag{9}$$

Therefore, as demand becomes less elastic in higher quality upgrade markets (so that ϵ_i becomes smaller), it is actually the less-efficient firms that gain market share, and more-efficient firms that lose market share. Of course, firms that are more efficient still have higher overall market shares, relative to their less efficient competitors, within any particular upgrade market, and indeed this observation forms the basis of the intuition for why market shares change as they do as quality increases. To wit, by virtue of having higher market shares, more-efficient firms have more to gain by restricting their output when demand is less elastic in higher upgrade markets. This leads their market shares to fall and the market shares of less-efficient firms to rise. In line with earlier intuitions, if the market-share increase of a less-efficient firm is significant enough, its upgrade constraint may bind, causing it to remove lower quality products.

Hence, the third force pushes less-efficient firms to focus on higher-quality products. Another way to illustrate this is to take an extreme case. Suppose demand in the baseline market is highly elastic. This drives price close to the industry-wide average of marginal cost in that market, which in turn makes it difficult for a less efficient firm to maintain positive market share. Nonetheless, if the demand for a luxury upgrade is highly inelastic, overall upgrade output may be low enough so that the less efficient firm may operate profitably. However, because its output in the low quality upgrade market would be close to zero if that market were in isolation, its upgrade constraint is likely to bind when both qualities are considered together. In particular, *the less efficient firm may sell only the high quality*, while the more efficient firm may sell both versions. Returning to the international-trade context (in which foreign suppliers suffer a disadvantage via the usual iceberg trade costs) this implies that foreign imports will be restricted to higher-quality variants, so that domestic firms supply all low-quality products and also a share of higher-quality ones.

3.2. **Predictions.** Here we briefly summarize the predictions that emerge from the analysis of the three forces considered above, each of which describes a way in which different firms concentrate their output in different quality ranges or, most simply but also extremely, remove certain products from their product lines.[16]

[16]There is another reason that gaps may appear. This is not out focus, simply because it is standard in single-product markets. An inefficient firm may have a gap in its product line because it finds it unprofitable

The first and second forces can result in market-share gains for a firm that either becomes comparatively more efficient as quality increases (the first force) or instead is simply more efficient within any upgrade market (the second force). This may lead cost-advantaged competitors to remove products from their product lines. This happens when the market-share gains for such firms are strong relative to any contraction of industry output in higher upgrade markets, causing monotonicity constraints to bind. These two forces are most obviously relevant when a monopolist (or a symmetric supplier) would primarily face cost-driven incentives for price discrimination (brought about by decreasing returns to quality). These first two forces are central to the explicit examples provided in Sections 4.1–4.4 below, and also to our study of endogenous technological capabilities in Section 5.

In contrast, the third force suggests that it may be less efficient firms that gain market share in higher quality upgrade markets, causing such firms to remove lower quality products from their product lines. This force is most applicable when members of a symmetric industry would offer multiple products despite the lack of decreasing returns to quality. The key condition here is that the elasticity of demand falls as quality increases. We pursue this prospect in several examples collected in Section 4.5.

We conclude this section by returning once more to our international trade example in which foreign importers face higher costs across all products. Which firms, foreign or domestic, specialize in which products? If quality-based price discrimination is primarily cost-driven, then the second force is most likely in effect, meaning that domestic firms will focus on high quality and foreign firms may focus on low quality. However, if such second-degree discrimination is primarily elasticity-driven, then we expect foreign importers to be focused on higher qualities, with domestic firms producing both low and high quality goods.

4. Specific Examples

We have identified the conditions that induce a monopolist (or, more generally, symmetric firms) to offer multiple quality-differentiated products: decreasing returns to quality, and decreasing (with respect to quality) demand elasticity. We have also identified three forces that induce some firms in an asymmetric industry either to delete qualities from their product lines, or otherwise to weaken their presence at particular quality levels. In this section, we consider three explicit specifications that show these forces at work.

to operate at or above some quality level, so that its upgrade supplies are zero. If product lines are complete for all competitors, then the price of the ith upgrade satisfies the familiar Cournot mark-up rule:

$$P_i(Z_i^\star) = \frac{R\epsilon_i(Z_i^\star)\Delta c_i}{R\epsilon_i(Z_i^\star) - 1}. \tag{10}$$

This must exceed the marginal cost for each firm, and so their costs cannot differ by too much. Specifically,

$$\frac{\max_r \Delta c_{ir}}{\Delta c_i} \leq \frac{R\epsilon_i(Z_i^\star)}{R\epsilon_i(Z_i^\star) - 1}. \tag{11}$$

If this condition is violated for some firm r, then it earns negative profits on upgrade i, and so it cannot find it optimal to sell the product of quality q_i.

4.1. **Constant Elasticity.** Here and in the next subsection we focus on a specification with multiplicative preferences, so that $v(\theta(z), q) = \theta(z)q$ and hence $P_i(Z_i) = \Delta q_i \theta(Z_i)$. From Proposition 1's corollary, the condition to ensure that symmetric competitors offer complete product lines is that there are decreasing returns to quality. Imposing this condition for each firm in an asymmetric industry means that

$$\frac{\Delta c_{1r}}{\Delta q_1} < \frac{\Delta c_{2r}}{\Delta q_2} < \cdots < \frac{\Delta c_{Ir}}{\Delta q_I} \tag{12}$$

for each $r \in \{1, \ldots, R\}$, which holds if the underlying cost function is convex ($c_r''(q) > 0$). Note that this assumption, as in the case of more general preferences, effectively involves not just costs but also consumer preferences.[17]

Multiplicative preferences ensure that the elasticity of demand does not vary with product quality. For our specification here we also assume that the elasticity of demand for any given upgrade does not vary with quantity, so that the elasticity of demand for any upgrade equals the constant value $\epsilon > 1$.[18] This specification therefore eliminates two of the three possible causes of product-line asymmetry, allowing us to isolate the effect of changing comparative costs as we move upward through the set of feasible qualities.

Specification 1 (Constant Elasticity Demand)**.** *Assume: (i) preferences are multiplicative, $v(\theta(z), q) = \theta(z)q$; (ii) the distribution of consumer types satisfies $\theta(z) = z^{-1/\epsilon}$; and (iii) $c_r(q)$ is convex in q for each firm $r \in \{1, \ldots, R\}$. Equivalently, the elasticity of demand is constant with respect to both quality and quantity, and there are decreasing returns to quality.*

Our goal here is to provide exact conditions under which changing comparative costs indeed lead some firms (in particular, those whose comparative costs are improving with quality) to remove lower-quality products. Under Specification 1, Equation (6) implies that if upgrade constraints were ignored then market shares and aggregate outputs would be

$$\frac{Z_{ir}^\dagger}{Z_i^\dagger} = \epsilon - \left(\epsilon - \frac{1}{R}\right)\frac{\Delta c_{ir}}{\Delta c_i} \quad \text{and} \quad Z_i^\dagger = \left(\frac{\Delta q_i}{\Delta c_i}\left(1 - \frac{1}{R\epsilon}\right)\right)^\epsilon, \tag{13}$$

recalling that $\Delta c_i = \frac{1}{R}\sum_{r=1}^R \Delta c_{ir}$. Because each firm has decreasing returns to quality (see Equation (12)), $\Delta q_i / \Delta c_i$ is decreasing in i, so that $Z_i^\dagger$ is also decreasing in i.

As anticipated from our earlier discussion, this means that there must be changes in the market shares if there are gaps in product lines. From Equation (13), such changes for firm r can only be driven by changes in $\Delta c_{ir}/\Delta c_i$, the comparative cost structure of r.

To confirm that an improvement in firm r's comparative cost advantage may cause its monotonicity constraints to bind, leading it to remove a product, it is helpful to suppose that

[17] In particular, for general preferences decreasing returns to quality are in reference to a particular consumer's preferences. In the case of multiplicative preferences, if there are decreasing returns for any type θ then there are decreasing returns for all consumer types.

[18] This implies that ϵ is also the elasticity for any stand-alone product. $\epsilon > 1$ ensures that demand is elastic.

quality differences between products are small (that is, $\max_i[\Delta q_i]$ is sufficiently small). If Δq_i is small then $\Delta c_{ir}/\Delta q_i \approx c'_r(q_i)$. When this is true for all products, and if all firms sell all products, then firm r's upgrade outputs can be written as

$$Z^{\star}_{ir} \approx Z_r(q_i) \quad \text{where} \quad Z_r(q) = \left(1 - \frac{1}{R\epsilon}\right)^{\epsilon} \left(\epsilon - \left(\epsilon - \frac{1}{R\epsilon}\right)\frac{c'_r(q)}{c'(q)}\right)(c'(q))^{-\epsilon}. \tag{14}$$

If product lines are complete then this should be decreasing in q for each r. Computing the derivative of $Z_r(q)$ with respect to q gives us the following proposition.

Proposition 2. *Suppose Specification 1 (Constant Elasticity Demand) holds.*

(a) If comparative capabilities are unchanging, so that $c_r(q)/c(q)$ is independent of q, and if costs are not too high for any firm, by which we mean that $c_r(q)/c(q) < R\epsilon/(R\epsilon - 1)$ for all r, then there exists an equilibrium in which all firms offer complete product lines.

(b) Unless the following inequality holds for all r and q, then (if the quality increments are sufficiently small) there must be a gap in the product line of a firm:

$$\frac{c''_r(q)}{c''(q)} + \frac{R\epsilon^3}{(R\epsilon^2 - 1)} \geq (1 + \epsilon)\frac{c'_r(q)}{c'(q)} \tag{15}$$

Conversely, if product lines are complete, this inequality does hold for all r and i.

The key to the omission of qualities is a violation of Equation (15). When r's comparative costs are decreasing, $c''_r(q)/c''(q)$ is smaller than $c'_r(q)/c'(q)$.[19] For instance, suppose that $c'_r(q) = c'(q)$ and $c''_r(q)/c''(q) \approx 0$, which means that r has the same costs as the industry average for the upgrade to q, but that its upgrade costs are not significantly increasing locally (compared to the increase in industry-average costs). Violation of (15) thus reduces to $R > 1 + (1/\epsilon)$, a condition which holds because demand is elastic ($\epsilon > 1$) and because there is more than one firm $R > 1$.

Under the assumptions on firm r's costs just given, we conclude that if the underlying quality increments are small and if all firms other than r were offering complete product lines, then r would find it optimal to eliminate a quality. Therefore, in equilibrium, some products are not offered by all firms.

This confirms the argument given earlier that it is the increase in the cost advantage of a firm that drives it to remove certain products from its line. Indeed, we emphasize that nothing in the logic provided requires that r has lower costs for all quality levels. So long as r's comparative costs are significantly improving, so that $c''_r(q)/c''(q) \approx 0$, then even if r has a cost disadvantage for quality q, so that $c'_r(q)/c'(q) > 1$, appropriate values of R and ϵ would ensure that r would have the incentive to remove a product from its product line.

[19]One might wonder from (15) whether simply having large cost asymmetry $c'_r(q)/c'(q)$ is sufficient to violate this inequality, contradicting our claim that changes in comparative costs are crucial. However, if comparative costs are not changing, then (15) cannot be violated if (14) gives r a positive market share for upgrade i.

4.2. **Linear Demand.** Here we focus on how demand elasticity that depends on quantity can lead firms to offer non-overlapping or minimally overlapping products (despite there being no strategic reason for doing so). That is, we focus on the second force that can lead firms to remove products from their product lines.

To ensure that the other two forces play no role, we suppose that (i) the comparative costs of each firm r, $\Delta c_{ir}/\Delta c_i$, do not change with quality, and (ii) preferences are multiplicative, so that elasticity $\epsilon_i(Z)$ does not depend on quality for any fixed any output level Z. To ensure that symmetric competitors would offer all products, we continue to assume that (iii) there are decreasing returns to quality for each firm, so that Equation (12) holds for each r.

The assumptions above imply that cost function of firm r satisfies $c_r(q) = \gamma_r c(q)$ for some $\gamma_r > 0$, with $\frac{1}{R}\sum_{r=1}^{R} \gamma_r = 1$, and with $c(q)$ being a convex function that denotes the average cost of quality across all firms. Firm r has lower than average costs if and only if $\gamma_r < 1$.

Linear demand is one of the simplest examples of demand whose elasticity depends on output, and so we adopt this common specification here. Specifically, types are uniformly distributed: $\theta \sim U[0,1]$, or equivalently $\theta(z) = 1 - z$. This yields linear inverse-demand curves of the form $P_i(Z_i) = \Delta q_i(1 - Z_i)$ for each upgrade. The elasticity of demand $\epsilon_i(Z) = \epsilon(Z) = (1 - Z)/Z$ is decreasing in output.

Specification 2 (Linear Demand)**.** *Assume: (i) preferences are multiplicative, $v(\theta(z), q) = \theta(z)q$; (ii) the distribution of consumer types satisfies $\theta(z) = 1 - z$; and (iii) $c_r(q) = \gamma_r c(q)$, where $c(q)$ is convex and $\frac{1}{R}\sum_{r=1}^{R} \gamma_r = 1$. Equivalently, the elasticity of demand is constant with respect to quality, there are decreasing returns to quality, demand within upgrade markets is linear, and the comparative cost structures of firms are unchanging.*

To check whether each firm sells each product, we check both whether the appropriate monotonicity constraints hold and also whether each firm supplies positive output of each upgrade. Under Specification 2, the unconstrained output of the ith upgrade for firm r is

$$Z_{ir}^{\dagger} = \frac{1 + [R - (R+1)\gamma_r](\Delta c_i/\Delta q_i)}{R+1}, \tag{16}$$

so long as this yields a positive solution for each firm. If it does not, then some firm r would not find it profitable to offer all products given that all other firms would. The solution is indeed strictly positive for everyone if and only if, for all $r \in \{1, \ldots, R\}$,

$$\gamma_r < \frac{R}{R+1} + \frac{1}{(R+1)\Delta c_i/\Delta q_i}. \tag{17}$$

This constraint is tightest for the least efficient firm (the firm r with the largest value of γ_r) and highest upgrade (upgrade I), and if it fails then at least that firm omits the highest quality from its product line (and possibly others) simply because the associated upgrade price is insufficient to cover its costs.

For all firms to sell all products, the unconstrained upgrade outputs given in Equation (16) must form a strictly decreasing sequence for each firm, which is so if and only if

$$(R+1)\gamma_r > R. \tag{18}$$

This is easiest to violate for the most efficient firm (the firm r with the smallest value γ_r), and if it is violated then there must be a gap in a product line. As discussed earlier, the reason this condition is most easily violated for the most efficient firm is that firms that are more efficient than average gain market share when the aggregate output Z in any upgrade market is smaller; this is a consequence of elasticity $\epsilon(Z)$ being decreasing.

The conditions (17) and (18) are necessary and sufficient for firms to sell complete product lines under this linear specification, where it is enough to check (17) for the least efficient firm and (18) for the most efficient firm. However, considerably more can be said about the equilibrium structure of product lines, as reported in the following proposition.

Proposition 3. *Suppose Specification 2 (Linear Demand) holds. There exists an equilibrium in which all suppliers offer complete product lines if and only if*

$$\frac{R}{R+1} \leq \min_r\{\gamma_r\} \quad \textit{and} \quad \max_r\{\gamma_r\} \leq \frac{R}{R+1} + \frac{1}{(R+1)\Delta c_I/\Delta q_I}. \tag{19}$$

But, suppose that the first inequality is strictly violated. Then there is at most one product that all firms offer—product lines are asymmetric.

The second part of this proposition indicates that if there is sufficient cost asymmetry, not only does the most efficient firm have incentives to prune qualities, but such asymmetry ensures that the final equilibrium outcome exhibits minimal overlap across the product lines of the entire industry—at most one product is sold by every firm. In other words, not only does sufficient cost asymmetry lead the most efficient firm to remove some lower-quality products, but the associated expansion of market share for those firms also drives out less-efficient firms from some upgrade markets—less-efficient firms can no longer profitably operate in all upgrade markets.

Below we extend the minimal overlap result to circumstances more general than linear demand and find even stronger results for the case of duopoly. Before proceeding, we note that the result just given about minimal product-line overlap can be generalized within the current context. In particular, suppose one considers only qualities higher than q_i for an arbitrary i, and suppose that only $R' < R$ firms have positive equilibrium output for such qualities. The methods used in the proof of Proposition 3 could be applied to find an analogue to the first inequality in the proposition. When that condition is violated, so that there is sufficient cost asymmetry amongst the active producers, then there is at most one product of quality at least q_i that all R' of these firms sell (again, even if all these firms could profitably supply all upgrade qualities).[20]

4.3. **Complete Product-Line Separation.** We now demonstrate even stronger results concerning the separation of product lines for a duopoly. We show that firms may entirely avoid head-to-head competition even though there is no strategic reason for so doing.

Although the sufficient condition we present for this requires neither multiplicative preferences nor any assumptions about the comparative costs of suppliers being constant with respect to quality, for the purpose of providing intuition in our initial discussion we assume temporarily that Specification 2 (Linear Demand) holds.

Label the two firms $r \in \{A, D\}$ where A is advantaged with lower costs, while D suffers a cost disadvantage. Specifically, $c_A(q) = \gamma_A c(q)$ and $c_D(q) = \gamma_D c(q)$ where $\gamma_A < 1 < \gamma_D$ and $\gamma_A + \gamma_D = 2$. If $\gamma_A > 2/3$ and if $\Delta c_2/\Delta q_2$ is small enough then both firms offer complete product lines (Proposition 3).

However, if $\gamma_A < 2/3$, then A's unconstrained duopoly outputs are increasing in quality (so that $Z^{\dagger}_{iA} < Z^{\dagger}_{(i+1)A}$). The need to satisfy the upgrade monotonicity constraints ($Z^{\star}_{iA} \geq Z^{\star}_{(i+1)A}$ must hold) suggests that upgrade constraints will bind (so that $Z^{\star}_{iA} = Z^{\star}_{(i+1)A}$). Intuitively, A eliminates some lower-quality products (such as quality q_i in this case).

What will be the first quality that A actually sells? If there is a quality for which D produces no upgrade supplies, then it is as if A is a monopolist for those upgrade levels and above. Because there are decreasing returns to quality, this means that A's unconstrained monopoly upgrade supplies form a decreasing sequence (even though its unconstrained duopoly outputs are increasing in quality). Additionally, under the assumed asymmetry, there is at most one product that both suppliers sell (Proposition 3). Thus, intuitively, at some quality weakly above the highest quality sold by D, A will begin offering complete products.

Proposition 4 confirms that the descriptions just given are correct under a more general set of circumstances that does not require the linear specification nor, for that matter, multiplicative preferences or any assumptions about returns to quality or elasticity. However, the result below will imply the result for linear demand as a simple corollary.

To this end, and building on earlier notation, let $Z^{\dagger}_{iA}$ and $Z^{\dagger}_{iD}$ denote the equilibrium outputs that would obtain in upgrade market i if all upgrade constraints were ignored (so that each upgrade market could be analyzed as a standalone market).

Proposition 4 (Duopoly). *Suppose $R = 2$, $\{Z^{\dagger}_{iD}\}$ is a strictly decreasing sequence wherever it is positive, and $\{Z^{\dagger}_{iA}\}$ is a strictly increasing sequence wherever $\{Z^{\dagger}_{iD}\}$ is positive.*

[20] There are two differences in the appropriate condition. Firstly, one must consider R' firms rather than R. Secondly, the average cost of these firms is lower than the average cost of all firms, and so the cost function $c(q)$ needs to be weighted appropriately.

(a) The low-cost firm sells only higher qualities: it offers all products at or above some quality level, but no qualities below that.

(b) The high-cost firm sells only lower qualities: it offers all products at or below some quality level, but no qualities above that.

(c) The firms split the market: the maximum quality of the high-cost firm is weakly lower than the minimum quality of the low-cost firm.

(d) If quality increments are sufficiently small, then there is gap (a range of omitted qualities) between the firms' two product lines.

Of course, the conditions in these proposition are placed on endogenous variables (the quantities that emerge from unconstrained single-product competition in each upgrade market). But we have already suggested what may lead to such an outcome. In particular, we know that when demand elasticity is decreasing in quantity, more-efficient firms tend to gain market share and this may in turn lead to the required conditions on firms' unconstrained duopoly outputs. We can check this insight by returning to the case of linear demand.

Corollary. *Suppose Specification 2 (Linear Demand) holds, and that $R = 2$. For supplier $r \in \{A, D\}$, where $c_r(q) = \gamma_r c(q)$, assume that $\gamma_A < 2/3 < 4/3 < \gamma_D$. Then the conditions and conclusions of Proposition 4 hold: there is minimal overlap in product lines.*

More general results are available. For example, if $\theta(z)$ is concave (and so the inverse-demand for each upgrade $P_i(Z_i)$ is concave) then $\gamma_A < 2/3 < 4/3 < \gamma_D$ is sufficient to imply the conditions and conclusion (the separation of product lines) of Proposition 4. And even if preferences are not multiplicative, the appropriate conditions on $\{Z_{iD}^{\dagger}\}$ and $\{Z_{iA}^{\dagger}\}$ may be satisfied. However, with non-multiplicative preferences we know that it may be the less-efficient firm that tends to gain market share in higher upgrade markets, and so that effect (driven by higher upgrade markets being less elastic) must be sufficiently small compared to the force that drives more-efficient firms to gain market share (elasticity being decreasing in aggregate output within a given upgrade market).

The avoidance of head-to-head competition occurs for reasons that are very different from those suggested in the existing literature on product differentiation. That literature has emphasized that suppliers design their product lines so as to soften competition with others. For example, D'Aspremont, Gabszewicz, and Thisse (1979) argued that suppliers will maximize horizontal differentiation to soften price competition. In a vertical differentiation framework, Shaked and Sutton (1982) showed that single-product suppliers select products of different qualities, and Champsaur and Rochet (1989) argued similarly that multi-product suppliers will choose non-overlapping product lines.

A key assumption within the articles just mentioned is that product lines are selected and observed by all players prior to competition in price. Thus, there is a strategic effect to product-line choice, which is that avoiding direct competition in products reduces the incentives of others to compete aggressively in prices. Motta (1993) argues that this also happens when suppliers choose quantities in the second stage.[21]

Our argument is very different. By assuming qualities and quantities are simultaneously selected, there is no scope for competition to be softened. Our conditions also ensure that symmetric suppliers (or a monopolist) would not omit qualities from their product lines. Instead, it is cost asymmetry that leads to minimally overlapping product lines.

Moreover, the equilibrium avoidance of head-to-head competition does not arise because one supplier has a cost advantage producing some products but a cost disadvantage producing others. Rather, the same supplier has lower costs for all products, yet nonetheless chooses to cede the supply of (complete) lower-quality products to its higher cost rival.

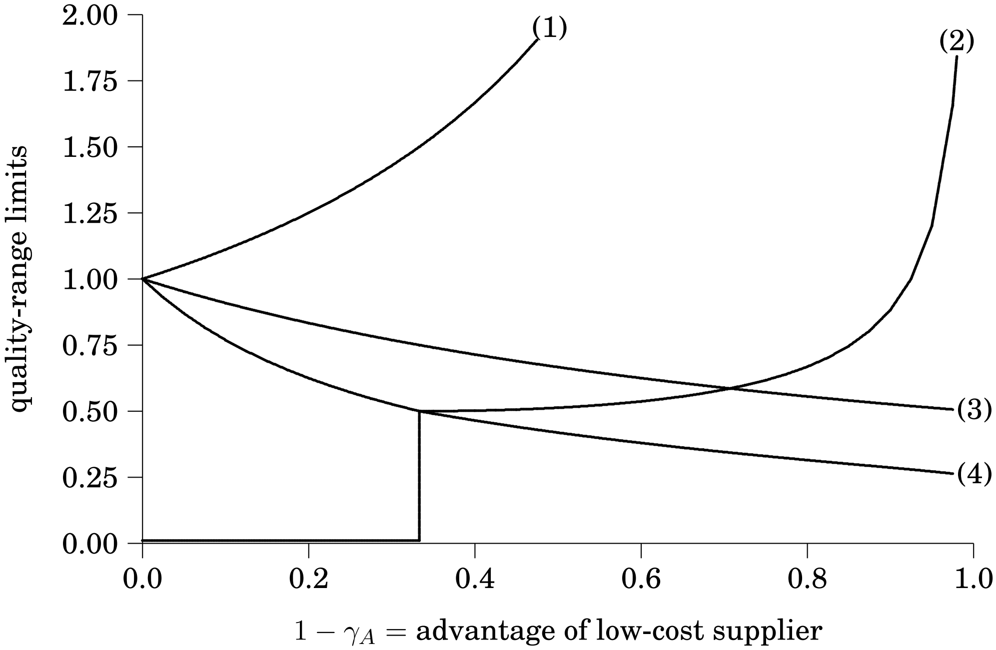
As noted this in Section 3, a natural application for our results here are markets for internationally traded goods. Adopting iceberg transportation costs, the disadvantaged supplier corresponds to a foreign supplier that loses a fraction of its goods in transit. The corollary to Proposition 4 predicts that there will be complete international separation in the supply of different quality ranges if trade frictions are sufficiently high. We note again that this applies in a world where quality-based discrimination is cost-driven.

4.4. **An Illustration of Product-Line Separation.** Figure 3 serves to illustrate Propositions 3 and 4. Using Specification 2 (Linear Demand), it shows the equilibrium product lines of the advantaged and disadvantaged firms. (Note that in this example there is a continuum of potential products.) As the cost advantage of the lower-cost firm increases, several notable changes occur in the two product lines.

Consider firstly the product line of the advantaged (that is, lower cost) firm. The highest quality offered, given by curve (1), increases as the cost asymmetry increases: it becomes profitable for it to be active in higher-quality upgrade markets. Secondly, once the cost advantage grows significant (so that $\gamma_A < 2/3$ and hence $1 - \gamma_A > 1/3$), the advantaged firm removes entirely a range of lower qualities. This can be seen by the jump in curve (2), which indicates the lowest-quality product of this firm.

Curve (4) demarcates the highest quality offered by the disadvantaged firm (who offers all products of lesser quality). At $1 - \gamma_A = 1/3$ (or equivalently $\gamma_D - 1 = 1/3$) the product lines of the duopolists no longer overlap. We also see that the disadvantaged firm offers fewer and fewer products as the cost asymmetry increases. Notably, it is driven out of upgrade markets that it would be active in as a monopolist (curve (3) shows the highest-quality product this firm would offer in the absence of competition). It is also driven out of some

[21]Other articles predict that head-to-head competition will arise. For example, Klemperer (1992) did so in a multi-product retailer setting in which consumers have shopping costs. De Fraja (1996) considered the simultaneous choice of product lines and quantities, as we do, and showed that when suppliers are symmetric in their capabilities, then any equilibrium is symmetric.



Notes. This figure illustrates the product lines offered by asymmetric duopolists when (i) preferences are multiplicative, so that $v(\theta, q) = \theta q$; (ii) types are uniformly distributed, so that $\theta \sim U[0, 1]$; and (iii) the marginal costs of quality q for the advantaged and disadvantaged firms are, respectively,

$$\frac{\gamma_A q^2}{2} \quad \text{and} \quad \frac{\gamma_D q^2}{2} \quad \text{where} \quad \gamma_A < 1 < \gamma_D \quad \text{and} \quad \gamma_A + \gamma_D = 2.$$

The four lines illustrate:

(1) The highest quality of the advantaged firm. This equals the maximum quality that it would offer as a monopolist.

(2) The lowest quality of the advantaged firm. For $1 - \gamma_A < \frac{1}{3}$ it offers a product range extended down to the lowest quality. However, for $1 - \gamma_A > \frac{1}{3}$ it prunes lower qualities from its product line.

(3) The highest quality that the disadvantaged firm would offer as a monopolist. This is higher than the maximum quality that it actually offers: it is forced out of higher qualities by its better competitor.

(4) The highest quality that the disadvantaged firm offers in the duopoly. This falls as its disadvantage worsens. For $1 - \gamma_A > \frac{1}{3}$ it is strictly below the minimum quality of the advantaged firm.

FIGURE 1. Product Lines under Changing Cost Advantage

markets that would be profitable for it as a duopolist, if upgrade constraints did not bind. That is, in the standalone equilibria of the distinct upgrade markets where upgrade constraints are ignored, there are some markets that the disadvantaged firm would be active in even though it produces zero output in those markets in the equilibrium of the full model (in which upgrade constraints must bind). The reason is that the upgrade constraints of the advantaged firm bind for some products, forcing it to increase output in some markets above the levels that would obtain in the standalone markets. This lowers the profitability for the disadvantaged firm in such markets, leading it to produce nothing.

4.5. **Elasticity-Driven Discrimination.** In the examples considered so far, elasticity does not vary with quality and so we have imposed decreasing returns to quality in order to explain the presence of full product lines in a symmetric industry. We now consider a situation in which (for each particular quantity) elasticity does change as quality increases.

4.5.1. *Quality-Dependent Elasticities.* To simplify exposition we focus on an industry with two quality levels, and use L (low) and H (high) to label them. (Our findings are more general: the proof of Proposition 5 allows for I quality levels.) In the context of the airline example from our introductory remarks, these qualities correspond to economy-class (L) and business-class (H). We also specify linear demand for each quality. However, in a move away from Specification 2, the shape of the demand curve varies with quality.

Specification 3 (Quality-Dependent Elasticity). *For two quality levels $i \in \{L, H\}$, assume*

$$v(\theta(z), q_L) = q_L \left[\frac{1}{2} + \lambda_L \left(\frac{1}{2} - z\right)\right]$$

$$\textit{and} \quad v(\theta(z), q_H) = v(\theta(z), q_L) + (q_H - q_L)\left[\frac{1}{2} + \lambda_H \left(\frac{1}{2} - z\right)\right].$$

Costs satisfy $c_{ir} = \gamma_r c_i$ for $i \in \{L, H\}$ for each firm r, where $\frac{1}{R}\sum_{r=1}^{R} \gamma_r = 1$.

For simplicity of exposition we restrict the parameters so that it is profitable to supply at least some units of each product, and so that the entire market is not served.[22]

We also note that the regular linear-demand preferences of Specification 2 are obtained by setting $\lambda_L = \lambda_H = 1$. Moreover, the median buyer type, corresponding to $z = \frac{1}{2}$, is willing to pay $q_L/2$ for the low-quality product and and $q_H/2$ for the high-quality product. Hence, relative to this median buyer type there are increasing returns to quality if and only if

$$\frac{c_H - c_L}{q_H - q_L} < \frac{c_L}{q_L}.$$

Of course, elasticities of demand depend on quality. Specifically, the inverse-demand functions for the baseline low-quality product and the upgrade to high quality satisfy

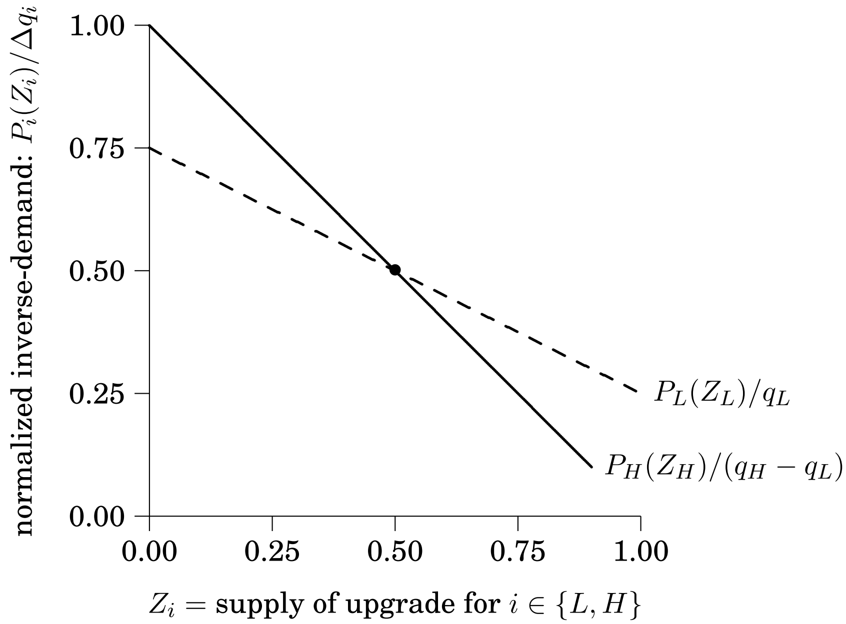
$$\frac{P_L(Z_L)}{q_L} = \frac{1}{2} + \lambda_L \left(\frac{1}{2} - Z_L\right) \quad \text{and} \quad \frac{P_H(Z_H)}{q_H - q_L} = \frac{1}{2} + \lambda_H \left(\frac{1}{2} - Z_H\right).$$

Using the notation $\epsilon_L(Z_L)$ and $\epsilon_H(Z_H)$ for the elasticities of the baseline and upgrade demand curves, $\epsilon_L(1/2) = 1/\lambda_L$ and $\epsilon_H(1/2) = 1/\lambda_H$. If $\lambda_H > \lambda_L$ then the inverse-demand curve for the upgrade is a clockwise rotation of that for the baseline low-quality product (Johnson

[22]For example, sufficient (but not necessary) conditions for this to be so are

$$\frac{1+\lambda_L}{2} > \frac{c_L}{q_L} > \frac{1-\lambda_L}{2} \quad \text{and} \quad \frac{1+\lambda_H}{2} > \frac{c_H - c_L}{q_H - q_L} > \frac{1-\lambda_H}{2}. \tag{20}$$

The first inequality ensures that the highest type of buyer is willing to pay the marginal production cost of the low-quality product, while the second inequality guarantees that the lowest type is not willing to pay this cost. The third and fourth inequalities have the same properties when applied to the upgrade to high quality.



Notes. Using Specification 3, this figure illustrates the quality-normalized inverse-demand curves for a low-quality product L and for the upgrade to a high-quality version H. Here, $\lambda_H = 1$ and $\lambda_L = \frac{1}{2}$: evaluated at the median type, the upgrade demand is less elastic than the baseline low-quality demand.

FIGURE 2. Quality-Dependent Elasticity: A Rotation of Demand

and Myatt, 2006b). Equivalently, when evaluated at the point corresponding to the median buyer type, elasticity is decreasing in quality if and only if $\lambda_H > \lambda_L$. Figure 2 illustrates.

Note that under this parameterization of this specification it is the case, as usual, that higher types are willing to pay more for both the baseline product and the upgrade. Additionally, higher types are willing to pay more for the upgrade than they are for the baseline product (normalizing by units of quality). However, lower types are willing to pay more for the baseline product than they are for the upgrade (again normalizing by units of quality). This divergence between the preferences of lower types and higher types over relative willingness to pay (for the upgrade versus the baseline product) does not hold in standard examples such as the multiplicative preferences case.

4.5.2. *Elasticity-Driven Discrimination in a Symmetric Industry.* We now evaluate the conditions needed for symmetric firms to engage in quality-based price discrimination.

If firms offer both products then the aggregate output in each upgrade market is equal to the corresponding output in a standalone symmetric Cournot oligopoly. Recall that in a symmetric Cournot oligopoly with linear demand and constant marginal cost, the equilibrium aggregate output is a fraction $R/(1+R)$ of the competitive output. That is, writing $\bar{Z}_i$ for $i \in \{L, H\}$ for the (competitive) output that equates price to marginal cost in the market for the ith upgrade, $Z_i^\dagger = [R/(1+R)]\bar{Z}_i$. For the two quality case,

$$\bar{Z}_L = \frac{1}{2} + \frac{1}{\lambda_L}\left(\frac{1}{2} - \frac{c_L}{q_L}\right) \quad \text{and} \quad \bar{Z}_H = \frac{1}{2} + \frac{1}{\lambda_H}\left(\frac{1}{2} - \frac{c_H - c_L}{q_H - q_L}\right).$$

The industry engages in quality-based second-degree discrimination if and only if $\bar{Z}_L > \bar{Z}_H$; that is, if and only if a competitive industry would so so.[23] This holds if and only if

$$\frac{1}{\lambda_L}\left[\frac{1}{2} - \frac{c_L}{q_L}\right] > \frac{1}{\lambda_H}\left[\frac{1}{2} - \frac{c_H - c_L}{q_H - q_L}\right]. \tag{21}$$

This is more easily satisfied when there are decreasing returns to quality, so that $(c_H - c_L)/(q_H - q_L) > c_L/q_L$. However, it can also hold if the returns to quality are increasing.

A leading example is when an increase in quality makes a negligible difference to production costs, so that $c_H \approx c_L$. This corresponds to the "damaged goods" examples described earlier. For this case, the condition required for the existence of both qualities is

$$\frac{\lambda_H}{\lambda_L}\left[\frac{1}{2} - \frac{c_L}{q_L}\right] > \frac{1}{2}.$$

If $c_L/q_L > \frac{1}{2}$ then this is never satisfied. However, if $c_L/q_L < \frac{1}{2}$ then if the demand for the upgrade is sufficiently inelastic relative to the demand for the baseline low-quality product (that is, if λ_H is large enough relative to λ_L) then both qualities are offered.

The condition $c_L/q_L < \frac{1}{2}$ implies that $(c_H - c_L)/(q_H - q_L) < \frac{1}{2}$ whenever there are increasing returns to quality. These conditions say that is efficient to supply both the low quality and also the upgrade to the median buyer type. In essence, this means that both the basic product and the upgrade have mass market appeal: the competitive supplies $\bar{Z}_L$ and $\bar{Z}_H$ are both greater than half of the potential addressable market. Inspecting Figure 2, this means that, for both cases, marginal cost intersects the lower part of the demand curve. If $\lambda_H > \lambda_L$, so that the (quality-normalized) upgrade demand is a clockwise rotation of the low-quality demand, then (other things equal) the competitive upgrade output is lower. The next proposition summarizes these observations.

Proposition 5. *Suppose Specification 3 (Quality-Dependent Elasticity) holds, that there are increasing returns to quality, and that it is efficient to supply the median buyer type both the baseline product and the upgrade (considered as separate products):*

$$\frac{1}{2} > \frac{c_L}{q_L} > \frac{c_H - c_L}{q_H - q_L}.$$

Given these assumptions, if the upgrade demand is sufficiently inelastic relative to the demand for the low quality,

$$\frac{\lambda_H}{\lambda_L} > \frac{\frac{1}{2} - (c_H - c_L)/(q_H - q_L)}{\frac{1}{2} - c_L/q_L},$$

[23]This is a particular instance of the finding that the equilibrium product line of a symmetric Cournot industry coincides with the socially optimal product line (Anderson and Çelik, 2015, Proposition 3).

then the firms within a symmetric industry will offer both product qualities. If this inequality fails then all firms offer only the high-quality product.

This result focuses on the case where there are increasing returns to quality and where the products have mass-market appeal. We can say more in other cases. For example, if $c_L/q_L > \frac{1}{2} > (c_H - c_L)/(q_H - q_L)$, so that the upgrade has mass-market appeal (the median type values it more than its cost) but the low-quality product does not, then symmetric firms only supply (for any values of λ_H and λ_L) the high-quality version of the product.

We now move on to consider the impact of asymmetries.

4.5.3. *Elasticity-Driven Discrimination in a Asymmetric Industry.* To illustrate the effect of asymmetric costs, we simplify exposition here by focusing on the case of duopoly. (The proof of Proposition 6 holds more generally, that is with R competing firms.) We label the two firms as $r \in \{A, D\}$ to indicate the advantaged and disadvantaged supplier, respectively. We assume that $c_{ir} = \gamma_r c_i$ for $i \in \{L, H\}$, where $\gamma_A < 1 < \gamma_D$ and where $(\gamma_A + \gamma_D)/2 = 1$.

The effects of asymmetric capability are easiest to see if we, once again, consider the "damaged goods" case where $c_H \approx c_L$. In this case, the upgrade is (close to) costless to produce. The higher relative costs of the disadvantaged firm D translate into a negligible absolute cost difference. Hence, treated as a standalone market, firms A and D can be expected to have (at least approximately) equal market share in the sale of the upgrade $q_H - q_L$. In contrast, for the basic low-quality product, firm D is at a disadvantage and so enjoys a minority market share. Of course, if the conditions of Proposition 5 hold then the aggregate industry output for the low-quality product will be higher than for the upgrade. If, however, γ_D grows sufficiently large (or, equivalently, γ_A becomes small enough) then the market share drop in market L will cause the monotonicity constraint of firm D to bind. At this point, firm D drops the low-quality version from its product line.

Returning to the more general case, the next proposition determines the degree of asymmetry required in order for the disadvantaged firm to restrict to the high-quality product.

Proposition 6. *Suppose Specification 3 (Quality-Dependent Elasticity) holds and $R = 2$. Further suppose that the conditions of Proposition 6 hold, so that (i) there are increasing returns to quality; (ii) it is efficient to serve the median buyer type both the baseline product and the upgrade (considered as separate products); and (iii) the members of a symmetric industry would sell both product qualities. If*

$$\frac{1}{6}\left[\frac{1}{\lambda_L} - \frac{1}{\lambda_H}\right] > \left(\gamma_D - \frac{2}{3}\right)\left[\frac{c_L/q_L}{\lambda_L} - \frac{(c_H - c_L)/(q_H - q_L)}{\lambda_H}\right]$$

then both firms supply the low-quality product. If not, then the disadvantaged firm drops the low quality from its product line, and sells only the high-quality product, while the advantaged firm continues to sell both qualities.

This stands in contrast to Proposition 4. Recall from that earlier result that, given sufficient asymmetry, the disadvantaged firm specialized in low-quality products. Here, that firm instead specializes in high-quality products. Notably, however, the advantaged firm retains a presence in the high-quality market (firm A sells both versions, and so the product lines overlap) and, moreover, enjoys greater market share in the sales of the high quality.

5. Asymmetric Technological Limitations

One force that shapes product lines is the change in firms' technological capability as we step through the quality levels. Here we study an interesting special case, when firms have different technological limitations. By this we mean that less capable firms cannot manufacture the highest-quality products, but face no cost disadvantage for those products that they can manufacture. Our key intuitions continue to hold, so that for example the decision by a firm to omit a quality from its product line is closely tied to the technological limitations of less-capable rivals. Moreover, we allow firms to choose the limitations of their technologies and show that asymmetries arise endogenously.

5.1. **Limits to Quality.** When referring to two different firms we use the labels r and s. We now use $c(q)$ to denote the common marginal cost of production for quality q incurred by any firm capable of supplying it: if firms r and s can both manufacture product i, then they have the same marginal cost $c_i = c(q_i)$. However, firm r can only manufacture qualities $q_i \leq \bar{q}_r$. Thus $\bar{q}_r$ describes the technological limitation of firm r. This maximum feasible quality $\bar{q}_r$ corresponds to one of the I total possible products: $\bar{q}_r = q_i$ for some $i \in \{1, 2, \ldots, I\}$.

One interpretation is that the production of higher qualities requires specialized technical expertise or intellectual property. From a theoretical perspective, this specification provides results that resonate with and extend those from our earlier analysis.

Within this context we ask two main questions. Firstly, if some product is omitted from at least one product line, can anything be said about the set of firms that omit that quality? Secondly, if some products are indeed omitted, which ones are?

5.2. **Product-Line Omissions and Firm Capability.** Proposition 7 answers the first question. To be precise, agree that quality q_i is omitted from firm r's product line if $q_i \leq \bar{q}_r$ and $z^\star_{ir} = 0$; firm r is capable of producing it but does not do so. Also, agree that firm r is more capable than firm s if $\bar{q}_r > \bar{q}_s$ and in this case firm s is less capable.

Proposition 7 (Product Line Omissions)**.** *If a quality is omitted from a product line, then the most capable firms are those who omit it. Equivalently, if a firm offers a quality, then any less-capable competitor also offers it so long as it is able to do so. Additionally, if two firms both offer a quality, then they offer identical supplies of all lower qualities.*

Thus, more-capable firms tend to offer fewer products than less-capable competitors, at least in the range of qualities that is feasible for those competitors.[24] Of course, because of the simultaneity of quantity and product-line decisions, this has nothing to do with a desire to soften competition amongst rivals. The prediction nonetheless is that the most-capable firms may avoid competing head-to-head with less-capable rivals.

The second result is that if r and s are both selling product i, then they actually produce the same upgrade quantities for all lower qualities; $Z^{\star}_{jr} = Z^{\star}_{js}$ for $j \leq i$. In fact, if r is more capable than s then $Z^{\star}_{ir} \geq Z^{\star}_{is}$ for any i. In summary: a more-capable firm produces weakly more upgrades than a less-capable rival for any given quality, but the same upgrade outputs beneath any quality that both competitors choose to produce.

This explains the first result of Proposition 7. Take a quality q_i that both a more-capable and a less-capable firm produce. Conceptually, the equality of upgrade quantities for i and below, combined with the more-capable firm's weakly higher upgrade output at higher quality levels, means that its upgrade constraint is more likely to bind for products of quality higher than q_i. Because this upgrade constraint binding for some product $k \geq i$ means that it does not sell product k, it follows that the more-capable firm will tend to sell fewer quality levels (in the range of quality that is feasible for the less-capable rival).

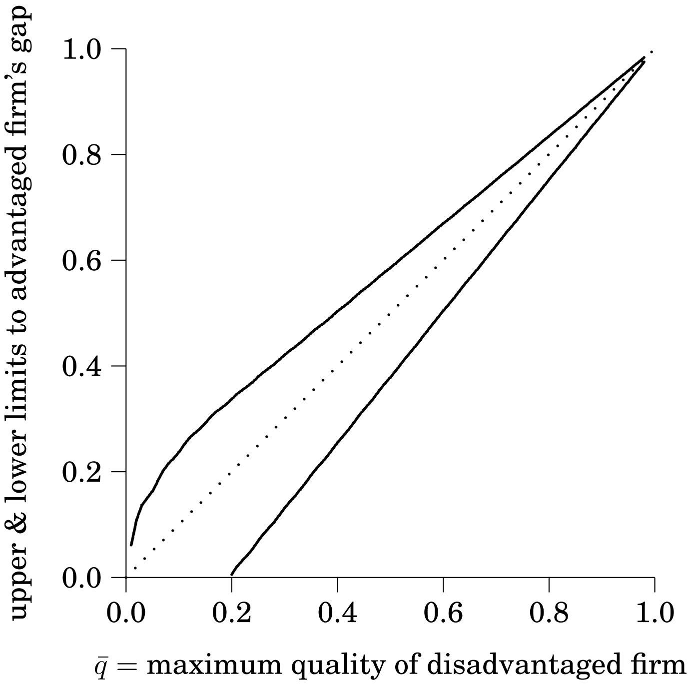
5.3. **Quality Ranges and Gaps in Product Lines.** The answer to our second question (which products are omitted?) is reported in the next proposition.

Proposition 8 (Gaps and the Capabilities of Rivals)**.** *A gap in a firm's product line includes the maximum feasible quality of a less-capable rival. Moreover, if the increments between feasible quality levels are sufficiently small, so that* $\max_i\{\Delta q_i\}$ *is not too large, then such gaps always exist: a firm omits any quality (and others that neighbor it) that corresponds to the maximum feasible quality of a less-capable rival, so long as that rival produces a strictly positive output of that maximum feasible quality.*

Putting this result together with Proposition 7, we see that more-capable firms are more likely to omit products than their less-capable competitors are, and that any such omitted products correspond to the maximum feasible quality of a less-capable rival. More precisely, an omitted product either equals the maximum feasible quality of a less-capable rival or is part of a gap of products encompassing that quality.

We have argued that changes in comparative cost advantages throughout the sequence of qualities tend to generate product-line gaps. Here, a more-capable firm's comparative cost advantage over a less-capable rival s jumps at $\bar{q}_s$. Hence, if no firms were omitting products, then the more-capable firm would have a strong incentive to expand just beyond $\bar{q}_s$. This causes its upgrade constraint to bind so that it no longer offers quality $\bar{q}_s$.

[24]Of course, more-capable firms may offer more distinct products in total, due to the fact that they are capable of producing more distinct products at the top end of the range of qualities.



Notes. This figure illustrates the product lines offered by asymmetric duopolists when (i) $v(\theta(z), q) = \theta(z)q$; (ii) $\theta(z) = 1 - z$; (iii) $c(q) = q^2/2$; and where (iv) a disadvantaged firm D faces a maximum quality limitation $\bar{q}_D$. The two solid lines indicate the upper and lower limits to the gap in the advantaged firm A's product line.

FIGURE 3. Product-Line Gaps with Technological Capabilities

If increments between neighboring qualities are sufficiently small then the incentive to raise upgrade outputs for qualities just beyond $\bar{q}_s$ is sufficiently strong that gaps definitely exist. Especially in this case, the equilibrium characterization is sharp, with gaps always existing and of course covering some less-capable firm's maximum feasible quality. The requirement for small quality increments ensures that one upgrade market looks similar to the next one. However, the higher upgrade market (just above the maximum feasible quality of the rival) contains one fewer competitor, which induces a higher output.

The arguments above can of course apply to the maximum feasible quality level of any less-capable rival. Thus, we have the following corollary. For this corollary, a "quality range" is a contiguous sequence of qualities that are offered in positive supply.

Corollary 1 (to Proposition 8)**.** *Suppose that all firms have distinct maximum feasible qualities, and order those firms so that $\bar{q}_1 < \cdots < \bar{q}_R$. The product line of firm r consists of at most r quality ranges separated by $r - 1$ gaps.*

For a duopoly ($R = 2$) Figure 3 illustrates the gap that appears in the product line of an advantaged firm. The horizontal axis measures the maximum feasible quality $\bar{q}_D$ of the disadvantaged firm, who sells all qualities up to some level less than or equal to $\bar{q}_D$. In contrast, the advantaged firm has a product-line gap which encompasses $\bar{q}_D$. When $\bar{q}_D$ is low, the advantaged firm excludes a set of products of the lowest quality. As $\bar{q}_D$ grows, it excludes a set of middle-quality products, but sells all products of lower and of higher quality. The product-line gap is the distance between the two solid curves.

5.4. **Endogenous Technological Limitations.** Above we presumed that there were exogenous differences in the maximum feasible qualities. But in some circumstances each firm has discretion in developing its own technological capability, and so it is unclear whether differences in maximum-feasible qualities will arise equilibrium.

Here we endogenize technological limitations. Each firm r simultaneously chooses $\bar{q}_r \in \{q_1, q_2, \ldots, q_I\}$ and its outputs of each quality that is less than or equal to $\bar{q}_r$. Firm r pays a fixed cost $F_r = F_r(\bar{q}_r)$ which is strictly increasing and differentiable in $\bar{q}_r$.

We are interested in the properties of fixed costs that endogenously lead to clear equilibrium outcomes and help us to predict either symmetric or instead asymmetric technological limitations and hence either symmetric or asymmetric equilibrium product lines. To this end we consider two cases. The first is when it is not too costly for any firm to be able to produce the highest-quality good I, so that $F_r(q_I)$ is small for each firm r. The second case is when it is prohibitively costly for any firm to be able to produce the highest-quality good, so that $F_r(q_I)$ is very large for each firm r.[25]

Proposition 9 (Endogenous Limitations)**.** *Suppose each firm r simultaneously choose its maximum feasible quality $\bar{q}_r$ and its outputs, bearing the fixed cost $F_r(\bar{q}_r)$.*

(a) If $F_r(q_I)$ is sufficiently small for each r, then all firms offer all products in equilibrium, and firms produce identical outputs for all quality levels.

(b) If the increments between the qualities are sufficiently small and if $F_r(q_I)$ is sufficiently large for each r, then firms choose different maximum feasible qualities: $\bar{q}_r \neq \bar{q}_s$ for $r \neq s$.

When fixed costs are not too significant, all firms invest enough to produce all I products and the equilibrium is symmetric. Unsurprisingly, the reason is that each firm finds it optimal to enter any upgrade market, even if others are already present. From this the symmetry of equilibrium output also follows, using Proposition 7.

The intuition for why firms offer asymmetric product lines, when the fixed costs of offering the highest-quality products are large and the quality increments are small, is as follows. Suppose that firms r and s chose the same maximum feasible quality: $\bar{q}_r = \bar{q}_s = q_k < q_I$ for some k. Then firm r, say, knows that if it instead increased its maximum quality by one step, it could be active in a higher upgrade market (market $k+1$). However, because by assumption s is not active in that market, by so increasing its maximum feasible quality r could ensure that it faces less competition in that upgrade market than in upgrade market

[25]To rule out trivialities, we suppose that the marginal upgrade costs are such that a monopolist could earn positive profits in the upgrade market for the highest quality if it did not need to bear fixed costs.

k. Because r must be covering its incremental fixed costs in upgrade market k, and because upgrade markets k and $k+1$ are very similar (this is where we use the assumption that quality increments are small, and we exploit the continuity of $F_r(\cdot)$ and $c(\cdot)$) firm r must find it profitable to expand into market $k+1$ and thereby face less competition.

Of course, fixing the technological capabilities of firms at their equilibrium levels, our earlier analysis implies that any decisions by firms to omit products are driven by the technological capabilities of less-capable firms and so Proposition 8 applies.

In summary, equilibria with either symmetric or asymmetric technological capabilities emerge under natural conditions. As already emphasized, in this environment firms are not choosing product lines strategically to soften the competition with their rivals.

5.5. **The Effect of Competition on the Maximum Available Quality.** We conclude this section by comparing the maximum feasible quality chosen (in equilibrium) by the most capable firm with the endogenous capability of a monopolist. To do this, we assume that the fixed-cost function $F_r(\cdot)$ is the same for all firms, so that $F_r(\cdot) = F(\cdot)$ for each r. Moreover, for simplicity of discussion we suppose that $F(\cdot)$ is convex.

Given our maintained assumptions, a monopolist will offer a complete product line for all qualities that it is capable of producing. It will choose its technological capability (that is, its maximum feasible quality) such that the additional fixed cost of the final step in its quality ladder is just equal to the monopoly profit obtained from the corresponding upgrade. Suppose that this optimal maximum quality, for a monopolist, is q_I.

Now consider the (endogenously) most capable competitor in an oligopoly. Further suppose that it offers strictly positive supplies of its top two qualities. This implies that it offers the monopoly supply of the highest upgrade in its product line. In deciding whether to produce this, it compares the monopoly upgrade profit to the associated fixed cost. This is the same comparison conducted by a monopolist. We can conclude that if the most capable competitor offers strictly positive supplies of its top two qualities then its (endogenously chosen) maximum feasible quality is equal to q_I.

However, the most capable firm also has a gap in its product line that encompasses the maximum quality of its nearest competitor. This gap can extend all the way up to its own maximum quality. In this case, the most capable firm offers its highest quality in positive supply but does not offer the quality below it. This, in turn, means that it is constrained: it must be supplying less than the monopoly output of the highest upgrade. This lessens its incentive to pay the associated fixed cost, which can result in it choosing a lower capability than that chosen by a monopolist operating under the same conditions.

From this we conclude that heightened competition can shrink the set of qualities available to buyers, in the sense that the endogenously chosen maximum quality of the most capable competitor may fall. This effect operates in addition to the absence of qualities from product lines owing to the gaps that surround the quality limits of inferior competitors.

6. Concluding Remarks

A standard explanation for why firms do not compete head-to-head is that they wish to soften competition. We have shown that, instead, a lack of head-to-head competition can be explained solely by appealing to asymmetries in the marginal cost structures of firms: those with lower costs tend to remove some products from their product lines.

More generally, we identified the circumstances required for gaps in product lines to arise, while maintaining assumptions on demand and costs such that a monopolist (or a symmetric competitor) would offer all qualities. Such gaps can be driven either by (i) changes in the comparative costs of firms, (ii) differences in the elasticity of demand as quality increases (for a given quantity), and (iii) changes in the demand elasticity within an upgrade market brought about by quantity changes within that market.

Omitted Proofs

Proof of Proposition 1. From Proposition 6 of Johnson and Myatt (2006a). □

Proof of Proposition 2. For part (a): note that $\Delta c_{ir}/\Delta c_i$ is independent of i. Next, using (13), $Z^{\dagger}_{ir}$ is positive whenever $c_r(q)/c(q) < R\epsilon/(R\epsilon - 1)$. From that same equation, aggregate output $Z^{\dagger}_{i}$ is strictly decreasing, and (because firm r's market share is constant) $Z^{\dagger}_{ir}$ is strictly decreasing. Taking $Z^{\dagger}_{ir} = Z^{\star}_{ir}$ completes this part of the proof. For part (b): if all firms offer complete product lines and if the increments between qualities are small, then $Z^{\dagger}_{ir} \approx Z_r(q_i)$ and so $Z_r(q)$ from (14) must be decreasing in q. Differentiating $Z_r(q)$ yields the appropriate condition for it to be decreasing. □

Proof of Proposition 3. The first part of the proposition (regarding conditions for there to exist an equilibrium in which all firms sell complete product lines) follows from work in the body of the text, and so here we prove that if $\min_r \gamma_r < R/(R+1)$ then there is at most one product that all firms offer. To do so we begin with the following preliminary observation. Take any firm r and any range of upgrade markets $\{i, \ldots, j\}$ that satisfies the properties that, when r is optimizing given the outputs of its rivals, (i) the upward constraint on Z_{ir} does not bind (so that either $i = 1$ or $Z^{\star}_{(i-1)r} > Z^{\star}_{ir}$), and (ii) the downward constraint on Z_{jr} does not bind (so that $Z^{\star}_{jr} > 0$ and either $j = I$ or $Z^{\star}_{jr} > Z^{\star}_{(j+1)r}$).

This means that r is free to either slightly increase or slightly decrease all its upgrade supplies in the range $\{i, \ldots, j\}$. Because it is assumed to be optimizing, this means that

$$\sum_{k=i}^{j} [\Delta q_k (1 - Z^{\star}_k - Z^{\star}_{kr}) - \Delta c_{kr}] = 0 \quad \Leftrightarrow \quad \sum_{k=i}^{j} [\Delta q_k (1 - Z^{\star}_k) - \Delta c_{kr}] = \sum_{k=i}^{j} \Delta q_k Z^{\star}_{kr}. \qquad (22)$$

Now suppose that for each firm s, its output in each upgrade market k in this range is replaced with the common value $\tilde{Z}_s \in [Z^\star_{is}, Z^\star_{js}]$ (which is independent of k) satisfying

$$\sum_{k=i}^{j} \Delta q_k Z^\star_{ks} = \tilde{Z}_s \sum_{k=i}^{j} \Delta q_k. \tag{23}$$

At these new output levels, (22) holds. This means that if firms were forced to choose the same outputs in upgrade markets $\{i, \ldots, j\}$, then a firm would choose the appropriate weighted average of its previous outputs, so long as its competitors were to do so.

We now use this observation to prove the result. Suppose for the sake of contradiction that there are at least two distinct products that everyone sells, and let the two such lowest quality products be l and $h > l$. Consider the upgrade ranges $\{1, \ldots, l\}$ and $\{l+1, \ldots, h\}$ and note that these ranges each satisfy the conditions assumed above (so that, for each firm, upgrades 1 and $l+1$ are unconstrained upwards and upgrades l and h are unconstrained downwards). Now consider a restricted model in which products of quality greater than h are eliminated, and in which each firm s is constrained to set $Z_{1s} = \cdots = Z_{ls}$ and $Z_{(l+1)s} = \cdots = Z_{hs}$. This restricted model is equivalent to a model with exactly two upgrades, A and B, where the quality of upgrade A is $\Delta q_A = \sum_{i=1}^{l} \Delta q_i$ and that of B is $\Delta q_B = \sum_{i=l+1}^{h} \Delta q_i$, and where the upgrade costs for s are $\Delta c_{As} = \sum_{i=1}^{l} \Delta c_{is}$ and $\Delta c_{Bs} = \sum_{i=l+1}^{h} \Delta c_{is}$. Using the observation above, it is an equilibrium in this model for s to supply an upgrade output $\tilde{Z}_{As}$ of A and $\tilde{Z}_{Bs}$ of B such that

$$\sum_{k=1}^{l} \Delta q_k Z^\star_{ks} = \tilde{Z}_{As} \sum_{k=1}^{l} \Delta q_k \quad \text{and} \quad \sum_{k=l+1}^{h} \Delta q_k Z^\star_{ks} = \tilde{Z}_{Bs} \sum_{k=l+1}^{h} \Delta q_k. \tag{24}$$

Moreover, $\tilde{Z}_{As} > \tilde{Z}_{Bs} > 0$ for each s. This means that all firms offer complete product lines (that is, both A and B). However, this cannot be the case. The reason is that this restricted model satisfies Specification 2 and so the first part of this proposition (which has already been proven) applies, meaning that if $\min_r \gamma_r < R/(R+1)$ then there is no equilibrium in which all firms sell complete product lines.

We have reached a contradiction. It follows that, in the original model there cannot be two products that all firms produce in positive supply, if $\min_r \gamma_r < R/(R+1)$. □

Proof of Proposition 4. Suppose that each product is sold by at least one of the firms. (If not, so that $Z^\star_{iL} = Z^\star_{(i+1)L}$ and $Z^\star_{iH} = Z^\star_{(i+1)H}$ for some i, then we amalgamate the identically supplied neighboring upgrades i and $i+1$ into a single upgrade. An equilibrium of the full game yields an equilibrium of the restricted game in which i is excluded from sale.)

Consider the lowest quality q_i that is omitted by H. Hence $Z^\star_{iH} = Z^\star_{(i+1)H}$. Since this is the lowest omitted quality, either $i = 1$ or H supplies quality q_{i-1} in which case $Z^\star_{(i-1)H} > Z^\star_{iH}$. In either case, H is free to raise Z_{iH} locally, and so must face at least a weak incentive to lower it: $Z^\star_{iH}$ must lie above H's reaction function in the standalone market for upgrade i.

Every product is offered by someone, and so L supplies q_i which implies $Z^\star_{iL} > Z^\star_{(i+1)L}$. Thus, L is free to lower Z_{iL} locally, and so must face at least a weak incentive to raise it: $Z^\star_{iL}$ must lie below L's reaction function. We conclude that $Z^\star_{iL} \leq Z^\dagger_{iL}$ and $Z^\star_{iH} \geq Z^\dagger_{iH}$ where $(Z^\dagger_{iH}, Z^\dagger_{iL})$ is the single-product equilibrium in the market for upgrade i.

Now suppose that H offers a product above quality q_i. Take the lowest such product, and label it as $j > i$. Since H offers quality q_j, but does not offer any product from i to $j-1$ inclusive, we know that $Z^\star_{iH} = Z^\star_{(i+1)H} = \cdots = Z^\star_{jH} > Z^\star_{(j+1)H}$. Hence quality q_{j-1} is not offered by H (in fact, it could be that $i = j-1$) and so q_{j-1} must be offered by L. Hence $Z^\star_{(j-1)L} > Z^\star_{jL}$. From this, L must have a weak incentive to lower Z_{jL} while H must have a weak incentive to raise Z_{jH}. We must be above L's reaction function and below H's reaction function. We conclude that $Z^\star_{jL} \geq Z^\dagger_{jL}$ and $Z^\star_{jH} \leq Z^\dagger_{jH}$. Assembling our observations: $Z^\dagger_{jH} \geq Z^\star_{jH} = \cdots = Z^\star_{iH} \geq Z^\dagger_{iH}$. However, this contradicts the assumption that $Z^\dagger_{iH} > Z^\dagger_{jH}$.

We conclude that (amongst the products that are sold) if H omits a quality then it omits all higher qualities: there is some i such that its product line consists of all products $j \leq i$. Equivalently: $Z^\star_{1H} > \cdots > Z^\star_{iH} > Z^\star_{(i+1)H} = \cdots = Z^\star_{mH} = 0$. We have established that claim (b) holds when all product qualities are sold by at least one firm.

We now characterize the qualities offered by the low-cost firm L. Writing i for the maximum quality offered by H, suppose that L offers quality q_j where $j < i$. Its upgrade supplies satisfy $Z^\star_{jL} > Z^\star_{(j+1)L}$. Hence it has a weak incentive to raise Z_{jL} and a weak incentive to lower $Z_{(j+1)L}$. Its rival H is locally unconstrained in these upgrade markets, and so H has weak incentives to lower Z_{jH} and raise $Z_{(j+1)H}$. We conclude that $Z^\star_{jL} \leq Z^\dagger_{jL}$, $Z^\star_{jH} \geq Z^\dagger_{jH}$, $Z^\star_{(j+1)L} \geq Z^\dagger_{(j+1)L}$, and $Z^\star_{(j+1)H} \leq Z^\dagger_{(j+1)H}$. From the inequalities concerning L, we have $Z^\dagger_{jL} \geq Z^\star_{jL} > Z^\star_{(j+1)L} \geq Z^\dagger_{(j+1)L}$, so that $Z^\dagger_{jL} > Z^\dagger_{(j+1)L}$. This contradicts our assumption that $Z^\dagger_{jL} < Z^\dagger_{(j+1)L}$ whenever $Z^\dagger_{jH} > 0$. Having reached this contradiction, we conclude that L offers no product $j < i$ Now suppose that L offers some product $j \geq i$. $Z^\star_{jL} > Z^\star_{(j+1)L}$ and so it is unconstrained upward in upgrade market $j+1$. Since $Z^\star_{(j+1)H} = 0$, this means that $Z^\star_{(j+1)L}$ must at least weakly exceed the monopoly supply of the upgrade $j+1$. This means that L is able to implement the monopoly output for every upgrade $j+1$ and higher. It is optimal to do so, and so L's upgrade supplies from j upward must form a strictly decreasing sequence. Summarizing: if L firms a product $j \geq i$, then it supplies every product above that. Moreover, its supplies of products $j+1$ upward are equal to the monopoly supplies. This establishes claims (a) and (c) when all products are sold by someone.

We now recall that we considered only equilibria in which each product is offered by at least one of the firms. (We did this by considering a restricted game following the amalgamation of identically supplied neighboring upgrades.) Our argument above shows that if L offers a product then it offers every feasible quality above that. Now consider the product line of firm H. Across the upgrade markets in which it is active, its opponent L offers a constant output. Hence, H's best reply to this (constant) output from its opponent is a strictly decreasing sequence of upgrade supplies.

Finally, we consider claim (d). Suppose that q_i is the highest quality offered by H, and suppose that it is also offered by L. This means that $Z^\star_{iL} \leq Z^\dagger_{iL}$, so that the L produces less than the (unconstrained) duopoly supply of upgrade i. This strictly exceeds $Z^\star_{(i+1)L}$, which is equal to the monopoly supply of upgrade $i+1$. Hence, if L is to offer product i then the duopoly supply of upgrade i must exceed the monopoly supply of upgrade $i+1$. If the size of the quality steps is large then this is possible. However, if quality increments (and specifically the quality increments Δq_i and Δq_{i+1}) are small, then (from continuity) upgrade market i and upgrade market $i+1$ are very similar. For sufficiently small quality increments the monopoly supply in upgrade market $i+1$ will exceed the duopoly supply in upgrade market $i+1$. This is a contradiction, and so claim (d) must hold. □

Proof of Proposition 5. Before proceeding, we expand Specification 3 to the full set of I quality levels by defining, for each $i \in \{1, \ldots, I\}$,

$$v(\theta(z), q_i) \equiv \sum_{j=1}^{i} \Delta q_j \left[\frac{1}{2} + \lambda_j \left(\frac{1}{2} - z\right)\right],$$

which implies that the inverse-demand curve for the ith upgrade satisfies

$$\frac{P_i(Z_i)}{\Delta q_i} = \frac{1}{2} + \lambda_i \left(\frac{1}{2} - Z_i\right).$$

Just as in the main text, we write $\bar{Z}_i$ for the competitive output where price is equal to the constant marginal cost of production. Notice that

$$P_i(\bar{Z}_i) = \Delta c_i \quad \Leftrightarrow \quad \bar{Z}_i = \frac{1}{2} + \frac{1}{\lambda_i}\left(\frac{1}{2} - \frac{\Delta c_i}{\Delta q_i}\right).$$

In the text we restricted (for simplicity of exposition) to cases in which it is profitable to supply some units of each product, but where it is not profitable to cover the entire market. Treating the ith upgrade as a standalone market, a sufficient condition for this to be true is that the solution for $\bar{Z}_i$ above satisfies $1 > \bar{Z}_i > 0$. These inequalities hold if and only if

$$\frac{1-\lambda_i}{2} < \frac{\Delta c_i}{\Delta q_i} < \frac{1+\lambda_i}{2}.$$

Specializing to the two-quality case, $i \in \{L, H\}$, yields the inequalities in Equation (20).

If all members of an oligopoly are to offer complete product lines then $\bar{Z}_i$ must be decreasing in i. Equivalently, for all $i \in \{1, \ldots, I-1\}$ we require

$$\frac{1}{\lambda_i}\left(\frac{1}{2} - \frac{\Delta c_i}{\Delta q_i}\right) > \frac{1}{\lambda_{i+1}}\left(\frac{1}{2} - \frac{\Delta c_{i+1}}{\Delta q_{i+1}}\right). \tag{25}$$

A special case of this (for two qualities, L and H) is Equation (21) in the main text. Rearranging this inequality yields the condition stated in the proposition.

The proposition restricts to the case where there are increasing returns to quality, and when $c_L/q_L < \frac{1}{2}$. In the text, we noted that if

$$\frac{c_L}{q_L} > \frac{1}{2} > \frac{c_H - c_L}{q_H - q_L},$$

then firms sell only the high-quality product. This follows straightforwardly: the left-hand side of the inequality in (25) is negative, while the right-hand side is positive, and so the inequality must fail. Maintaining increasing returns to quality, the remaining case is when

$$\frac{c_L}{q_L} > \frac{c_H - c_L}{q_H - q_L} > \frac{1}{2}.$$

In essence, the low-quality product and the upgrade both have only niche appeal. The required inequality (in the statement of the proposition) is satisfied only when λ_H is small relative to λ_L. Hence, for product qualities that all have only niche appeal, and in the presence of increasing returns to quality, if both qualities are to be sold then the demand for the upgrade needs to be more elastic when evaluated at $Z = \frac{1}{2}$. □

Proof of Proposition 6. We continue to retain a full set of product qualities. Examining the Cournot outputs in a standalone market for upgrade i, the first-order condition for firm r is

$$P_i(Z_i^\dagger) + Z_{ir}^\dagger P_i'(Z_i^\dagger) = \Delta c_{ir} \quad \Leftrightarrow \quad \frac{1}{2} + \lambda_i\left(\frac{1}{2} - Z_i^\dagger\right) - \lambda_i Z_{ir}^\dagger = \gamma_r \frac{\Delta c_i}{\Delta q_i}.$$

Noting that $Z_i^\dagger = [R/(1+R)]\bar{Z}_i$ where $\bar{Z}_i$ is the competitive output, and noting the expression that we obtained for $\bar{Z}_i$ from the proof of Proposition 5,

$$\begin{aligned} Z_{ir}^\dagger &= \frac{1}{\lambda_i}\left[\frac{1}{2} + \lambda_i\left(\frac{1}{2} - Z_i^\dagger\right) - \gamma_r \frac{\Delta c_i}{\Delta q_i}\right] \\ &= \frac{1}{\lambda_i}\left[\frac{1}{2} + \lambda_i\left(\frac{1}{2} - \frac{R}{R+1}\left[\frac{1}{2} + \frac{1}{\lambda_i}\left(\frac{1}{2} - \frac{\Delta c_i}{\Delta q_i}\right)\right]\right) - \gamma_r \frac{\Delta c_i}{\Delta q_i}\right] \\ &= \frac{1}{\lambda_i(R+1)}\left[\frac{1+\lambda_i}{2} - [(R+1)\gamma_r - R]\frac{\Delta c_i}{\Delta q_i}\right]. \end{aligned}$$

If all firms sell complete product lines then $Z_{ir}^\star = Z_{ir}^\dagger$ for each i and r, and the standalone Cournot suppliers must satisfy $Z_{ir}^\dagger > Z_{(i+1)r}^\dagger$. That is,

$$\frac{1}{\lambda_i}\left[\frac{1+\lambda_i}{2} - [(R+1)\gamma_r - R]\frac{\Delta c_i}{\Delta q_i}\right] > \frac{1}{\lambda_{i+1}}\left[\frac{1+\lambda_{i+1}}{2} - [(R+1)\gamma_r - R]\frac{\Delta c_{i+1}}{\Delta q_{i+1}}\right].$$

Upon re-arrangement, this inequality readily becomes

$$\frac{1}{2}\left[\frac{1}{\lambda_i} - \frac{1}{\lambda_{i+1}}\right] > [(R+1)\gamma_r - R]\left(\frac{\Delta c_i/\Delta q_i}{\lambda_i} - \frac{\Delta c_{i+1}/\Delta q_{i+1}}{\lambda_{i+1}}\right).$$

Setting $R = 2$, $i = L$, and $i + 1 = H$ yields the inequality stated in the proposition. □

Proof of Proposition 7. Suppose $\bar{q}_r > \bar{q}_s$. The first step is showing that $Z^\star_{ir} \geq Z^\star_{is}$ for all i. Suppose this is false. Take the lowest j for which $Z^\star_{jr} < Z^\star_{js}$. Since this is the lowest such j, then either $j = 1$ or $j > 1$ and $Z^\star_{(j-1)r} \geq Z^\star_{(j-1)s}$, which implies that $Z^\star_{(j-1)r} > Z^\star_{jr}$. For both of these cases, firm r faces no upward monotonicity constraint in market j. Next, take the highest k for which $Z^\star_{ks} = Z^\star_{js}$. Since this is the highest k with this property, and since $Z^\star_{js} > Z^\star_{jr} \geq 0$, firm s faces no downward monotonicity constraint in market k. Hence firm s has a weak incentive to raise simultaneously the supply of all upgrades $i \in \{j, \ldots, k\}$. Since $Z^\star_{is} > Z^\star_{ir}$ for all $i \in \{j, \ldots, k\}$, and firm r has the same costs, firm r must have a strict incentive to raise simultaneously all of these outputs. Since this is feasible by construction, we have a contradiction.

Now suppose there is a product k that both firms are capable of producing that is sold by r but not by s, so that $Z^\star_{kr} > Z^\star_{(k+1)r}$ and $Z^\star_{ks} = Z^\star_{(k+1)s}$, implying that $Z^\star_{kr} > Z^\star_{sr}$. Observe that firm r faces no downward monotonicity constraint in market k, and hence is able to simultaneously lower its supplies for all $i \in \{1, \ldots, k\}$. It does not, and so must have at least a weak incentive to raise them. Firm s faces the same costs in these markets, and produces weakly less. It produces strictly less in market k. It must have, therefore, a strict incentive to simultaneously expand for all $i \in \{1, \ldots, k\}$. This is a contradiction, and so it follows that it is more capable producers who omit products from their product lines (amongst the firms capable of producing that product).

We now show that in fact $Z^\star_{jr} = Z^\star_{js}$ for all products $j \leq k$ where k is a product that both firms sell. To prove this claim, consider the highest product j that both can produce and for which $Z^\star_{jr} > Z^\star_{js}$. firm r could slightly lower its output of all upgrades $i \in \{1, \ldots, j\}$, but does not and so must have a weak incentive to raise them. Because s offers less output in upgrade market j, s has a strict incentive to raise them. $\square$

Proof of Proposition 8. We begin with this claim: if firm r's product line has a gap from j to k that does not include the maximum feasible quality of another firm, then industry output for $i \in \{j, \ldots, k+1\}$ is the same. To see this, note that if firm r has such a gap then $Z^\star_{jr} = \cdots = Z^\star_{(k+1)r} > Z^\star_{(k+2)r} \geq 0$. Also note that firm r produces product $k+1$ and hence $\bar{q}_r \geq q_{k+1}$. We will show that any other firm s satisfies $Z^\star_{js} = \cdots = Z^\star_{(k+1)s}$. By assumption, either $\bar{q}_s < q_j$ or $\bar{q}_s > q_k$. If $\bar{q}_s < q_j$, then $Z^\star_{is} = 0$ for all $i \in \{j, \ldots, k+1\}$, and hence $Z^\star_{js} = \cdots = Z^\star_{(k+1)s}$. If $q_k < \bar{q}_s \leq \bar{q}_r$ then, since firm r produces product $k+1$, so must firm s (from the proof of Proposition 7). This means (again from the proof of Proposition 7) that $Z^\star_{ir} = Z^\star_{is}$ for all $i \leq k+1$, and once again $Z^\star_{js} = \cdots = Z^\star_{(k+1)s}$. Finally, suppose that $\bar{q}_s > \bar{q}_r$. If firm s sells product $i \in \{j, \ldots, k\}$, then firm r would also do so, following Proposition 7. This would be a contradiction. Hence it must be, once again, that $Z^\star_{js} = \cdots = Z^\star_{(k+1)s}$. We have considered all cases, and conclude that $Z^\star_j = \cdots = Z^\star_{k+1}$.

We can now prove the result. If firm r has a gap in its product line from j to k then either $j = 1$ or $Z^\star_{(j-1)r} > Z^\star_{jr}$. For both of these cases, there is no upward constraint on $Z^\star_{jr}$. From the definition of a gap, firm r must sell product $k+1$, so that either $q_{k+1} = \bar{q}_r$ and $Z^\star_{(k+1)r} > 0$, or $q_{k+1} < \bar{q}_r$ and $Z^\star_{(k+1)r} > Z^\star_{(k+2)r}$. Since its downward monotonicity constraint in market $k+1$ does not bind, firm r must have a weak incentive to raise $Z_{(k+1)r}$. Hence,

$$0 \leq P_{(k+1)r}(Z^\star_{k+1}) + Z^\star_{(k+1)r} P'_{k+1}(Z^\star_{k+1}) \Delta c_i \quad \Leftrightarrow$$
$$0 \leq 1 - \frac{\Delta c_{k+1}}{P_{k+1}(Z^\star_{k+1})} + \frac{Z^\star_{(k+1)r}}{Z^\star_{k+1}} \frac{Z^\star_{k+1} P'_{k+1}(Z^\star_{k+1})}{P_{k+1}(Z^\star_{k+1})} = 1 - \frac{\Delta c_{k+1}}{P_{k+1}(Z^\star_{k+1})} - \frac{Z^\star_{(k+1)r}}{Z^\star_{k+1}} \frac{1}{\epsilon_{k+1}(Z^\star_{k+1})}. \tag{26}$$

Now, because of the maintained assumptions that $\epsilon_i(Z)$ and $\Delta c_i / P_i(Z)$ are decreasing in i, with at least one being strictly decreasing, the fact that r has weakly positive incentives to expand $Z^\star_{(k+1)r}$ implies that it has strictly positive incentives to expand $Z^\star_{jr}$. In particular,

$$0 < 1 - \frac{\Delta c_j}{P_j(Z^\star_j)} - \frac{Z^\star_{jr}}{Z^\star_j} \frac{1}{\epsilon_j(Z^\star_j)}, \tag{27}$$

where this uses the fact that $Z^\star_{jr} = Z^\star_{(k+1)r}$ (because this is a gap of firm r) and $Z^\star_j = Z^\star_{k+1}$ (because as shown at the start of this proof, aggregate outputs must be equal throughout a gap that does not contain the maximum feasible quality of a rival). But this means that r has a strict incentive to increase its output of upgrade j, and this is feasible because the relevant upgrade constraint does not bind. Hence, we have arrived at a contradiction, and so any gap of r must contain the maximum feasible quality of a rival.

The final claim is similar to claim (d) of Proposition 4. Consider quality q_i that is the maximum for some firm. Suppose further than that its upgrade supplies exceed some $\varepsilon > 0$. Hence, going from upgrade market to $i+1$ other firms face a drop in competition of at least ε, which means that other firms would wish to supply strictly more if upgrade markets i and $i+1$ were otherwise the same. Now, if the quality increments Δq_i and Δq_{i+1} are sufficiently small then (noting that $c(q)$ and $v(\theta, q)$ are continuous) these neighboring markets are sufficiently similar. The desire to raise output in upgrade market $i+1$, owing to the ε drop in output from limited competitors, is sufficient to cause the upgrade monotonicity constraints to bind for more able firms. □

Proof of Proposition 9. Claim (a) is straightforward. Claim (b) follows from the argument in the text. Specifically, suppose that r and s choose the same maximum quality q_k for some k. Each earns a profit which exceeds the fixed cost of operating in that market. firm r could deviate by stepping into upgrade market $k+1$ and offer the combine output of both r and s from market k. If the quality increments become small, then the profit gain (per unit of quality) approaches the combined variable profit in market k. However, the associated fixed cost (per unit of quality) approaches the fixed cost of market k, which is below the profit earned (again per unit of quality) by r alone. Hence, if quality increments are small then this deviation would be profitable. We conclude that (for small quality increments) no two firms choose the same maximum quality. □

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