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EQUILIBRIUM SELECTION AND PUBLIC-GOOD PROVISION: THE DEVELOPMENT OF OPEN-SOURCE SOFTWARE

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Collective-action problems arise in a variety of situations. Open-source software is a recent and important example. Copyright restrictions on open-source projects stipulate that any user may modify the software so long as any resulting innovation is freely available to all. In economic parlance, the innovation is a public good. The economic theory of public-good provision raises a number of important questions. Who contributes to such a project, and who free rides? How might a social planner exploit the interdependence of project components to encourage contributions? Under what conditions will such actions result in successful provision? Using a simple game-theoretic framework and recent results from the study of equilibrium selection, we attempt to answer these questions. Under reasonable assumptions of asymmetry and less than complete information, the most efficient providers will contribute. Contributions can be elicited by 'integrating' the provision process when providers are sufficiently optimistic about the success of the project. Otherwise, the social planner may be better off 'separating' the components so that individual contributions are independent of each other. The analysis yields recommendations for the leaders of open-source projects and other similar collective-action problems.

I. OPEN-SOURCE SOFTWARE AND PUBLIC GOODS

(i) Community Projects and Collective Action

A variety of community projects experience the problems of collective action.[2] Two aspects are considered here. First, such projects often involve *positive externalities*: Individuals bear the private cost of an activity that serves to benefit others. Second, they often require *coordination*: absent the participation of a sufficient number of individuals, the project as a whole would fail.[3]

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[1] The authors thank Hyun Shin, Margaret Stevens, and Justin Johnson for useful discussions, comments, and criticisms.

[2] A classic analysis of collective-action problems is offered by Olson (1965). The literature which followed is vast and continues to grow in both applied and theoretical directions; see, for example, Bardhan *et al*. (2002) and Marx and Matthews (2000) respectively.

[3] Both these problems have attracted a great deal of interest, and not only from theorists. For a summary of the experimental approach to these problems, see Ledyard (1995).

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Examples from everyday life are commonplace. For instance, local environmental projects benefit everyone, yet only those who voluntarily participate bear any of the costs. In the authors' home city, Oxford Conservation Volunteers (OCV) has been 'carrying out practical work conserving the wildlife and traditional landscape of the Oxford area since 1977'.[4] Its activities range from hedge-laying to the conservation of wildlife habitats via scrub clearance. The volunteers contribute their own time and energy (a private cost) to an activity which generates environmental benefits for all (a positive externality).

Sporting activities provide an example of the need for coordination. Many sports are team-based, and without the presence of all participants, the game will be called off. In May of each year Oxford colleges compete in a rowing tournament known as 'Eights Week'. A team's success in this tournament critically depends upon the ability of its eight members (hence the name) to train on a regular basis. Training sessions are typically conducted in the early morning and require the presence of the entire team—the absence of a single member results in cancellation. A rower will find it optimal to wake early and participate if and only if all the other team members do so. Of course, if seven of the rowers are expected to attend the training session, the eighth team member faces a strong incentive to attend—such attendance will enable the training (and potential success) of the entire team, rather than merely the element contributed by an individual.

Both of these examples involve the provision of a *public good*.[5] According to its classic definition a pure public good is both *non-rival* and *non-excludable*. A good is non-rival if consumption by one individual does not decrease the amount available to others. For instance, everyone may enjoy the pleasures of an improved environment. A good is non-excludable if anyone may consume it once it has been provided. All members of an Oxford college are free to revel in the glory of coming in at the 'Head of the River'.[6]

(ii) Open-source Software as a Pure Public Good

Computer software is a classic non-rival good, since it is almost costless to duplicate. This is true of many other similar products, ranging from popular music to Hollywood movies. Nevertheless, software is excludable. Copyright laws apply, and authors may prevent the unauthorized use of their products. This protection permits software companies to charge for the licensed use of their programs, and thus software is often provided privately. Microsoft, for instance, imposes extensive software licence restrictions that heavily control the use of its ubiquitous operating system and office productivity software.[7]

Intellectual property laws are not the only instrument of exclusion. The technology used to produce software enables the author to hide its inner workings and hence prevent others from copying the design. *Source code* is the sequence of commands written by a human programmer that is the basis of a useful software product. This (human readable) code is then *compiled* to produce machine readable code—the finished product—that cannot easily be read by another programmer. Most commercial software products are *closed source*. This means that users of the software product (and perhaps competing programmers) cannot inspect or change the source code. Hence others are unable to modify or improve the operation of the program, and the exclusionary nature of closed-source software prevents the exploitation of a positive externality.

Some software, however, is non-excludable by design. It is described as 'open source' or 'free'. Users are at liberty to inspect the source code and alter it to suit their own needs. They are also free to make their own improvements, and offer modified products to other computer users. The open-source operating system, Linux, which is popular 'behind

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[4] A description of its activities is available at http://www.ocv.org.uk/

[5] The now standard economic approach to the problem of public-good provision is presented in the central contributions of Bergstrom *et al*. (1986, 1992) and a useful diagrammatic exposition can be found in Ley (1996).

[6] In a rowing regatta the overall winner is said to come in at the 'Head of the River'.

[7] When software is described as excludable, this means that the producer is able to exclude non-purchasing consumers from using the product. It does not refer to any alleged attempts by Microsoft to exclude rival competitors from providing a substitute product to those consumers.

the scenes' throughout the Internet, is based around code written by Linus Torvalds. Other programmers then built upon his original work, offering subsequent innovations that helped to transform it into a real contender to closed-source software operating-system providers, such as Microsoft, Apple, and Sun.[8]

Crucially, programmers who contributed to the Linux project were prevented from taking private control of the source code and turning the software into a closed-source (and hence excludable) product. Open-source software is often copyrighted, and the licence restrictions are designed to protect its non-excludable nature. Richard Stallman, pioneer of the open-source movement and founder of the Free Software Foundation, helped to derive the General Public Licence (GPL) that is applied to many open-source products. This stipulates that, first, the source code for a product must be made freely available to anyone and, second, any new products employing open-source software must be distributed under the same licence. Many of the positive externalities generated by an open-source software product are themselves non-excludable, and the GPL uses a copyright agreement to prevent the exploitation of typical copyright privileges—hence it is often referred to as 'copyleft'.[9] In short, open-source software is a classic example of a pure public good.

Perhaps surprisingly, not only is this pure public good actually provided, but it is also extremely prevalent. The 'closed-source' products offered by commercial giants such as Microsoft, Apple, and Adobe are familiar. Behind the scenes, however, open-source software is extremely important. For instance, electronic mail needs to be transported from a sender to a recipient. This operation is conducted by a piece of software known as an 'Internet mail transfer agent'. Eighty per cent of e-mail traffic is handled by the program Sendmail, which is an open-source product.[10] Similarly, over half of all web servers are powered by Apache, again an open-source project.[11] Moreover, servers often reside on open-source operating systems: Linux has quickly risen to become an extremely popular base for important commercial projects. Even traditionally closed-source software providers are beginning to jump on the open-source bandwagon: Apple has chosen to release the core of its latest operating system under an open-source agreement.[12]

(iii) Integration, Separation, and the Incentive–Coordination Trade-off

Open-source software exhibits both positive externalities and the need for coordination. The entire computing community has access to a finished (open-source) software product, but only its authors spend time and energy on its production. In addition, many open-source projects comprise a range of individual components. These are often interdependent, generating a potential coordination problem for programmers. For instance, Unix-style operating systems (such as Linux) have traditionally involved a large number of modular components. These modules are often produced by individual authors. In use, however, they are sometimes interdependent. Interdependent modules will be of greater use when they are all present—there are additional benefits from coordinated production. Of course, other software modules are independent in use and do not suffer from the same coordination problem.

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[8] Raymond (1998) studies the way in which the operating system Linux has developed via the 'part time hacking by several thousand developers scattered all over the planet'. This is a prime example of a community project involving collective action and has already received the attention of economists. Lerner and Tirole (2002) describe some of the economics of open-source software provision. The notion of open-source software as the private provision of a public good is studied by Johnson (2002).

[9] Richard Stallman is very careful to emphasize that 'free' does not mean without cost. For instance, the producer of an open-source product may charge a fee. The key feature is that a subsequent user is able to use the source code in any way. Thus the open-source use of the word 'free' is in many ways close to the economist's concept of non-excludability.

[10] These mail-handling operations are also handled by commercial products, such as Microsoft's Exchange. In this and many other 'behind the scenes' cases, such commercial products have a small share of the user base.

[11] Such statistics are obtained by interrogating public web servers. Private web servers are protected by 'firewalls' and hidden from such inspection, hence Apache's penetration of the 'intranet' market may be rather different. For a more extensive discussion of open-source software's importance see Kogut and Metiu (2001).

[12] Apple's new Mac OS X is built upon the Unix variant FreeBSD. The underlying operation system (known as Darwin) is distributed under an open-source agreement. The graphical interface (known as Aqua) is closed source, however, and must be purchased from Apple. This is an interesting example of a case where the project leader (Apple) chose to split its product into open-source and closed-source components.

Two examples help to illustrate the implications of interdependence. A driver is a small piece of software that allows an operating system to communicate with a piece of hardware. Typically, drivers do not require the presence of other software in order to work. In contrast, the X windows graphical front-end to Unix-style operating systems is a large and complex collection of many integrated components. A single component is only of use if all others are functioning. Its programmer will find it worthwhile to contribute only if the other complementary components are also provided. But when this is true, the marginal value of the final component is large as it enables the whole software project to function. This is similar to the example of the rowing team, where the eighth and final team member faces a strong incentive to participate. Of course, in the case of an independent piece of software (such as the driver), the incentive to participate is perhaps smaller, and yet the coordination problem is less severe.

This discussion suggests that small projects are less valuable, and hence potential contributors face relatively low incentives. Nevertheless, they are often independent. On the other hand, large projects require coordinated effort, but the incentives to contribute are enhanced. Thus, if the size or degree of integration of a particular project are choice variables, a project leader may face an incentive–coordination trade-off.[13]

Open-source projects may provide a real example of this trade-off. Such projects (for example, Linux) are instigated by a leader (in the case of Linux, Linus Torvalds) who specifies the parameters of the problem. Everyday examples are also available. Rowing-squad coaches select the type of training session for the team. Rather than attempt to get a full eight on to the river early in the morning they might choose to use smaller boats. By doing so they are reducing the severity of the coordination problem (if one team member oversleeps, not all is lost) but at the same time decreasing the incentive to each rower (oversleeping no longer results in the complete collapse of the training session).

The discussion suggests that the integration and interdependence of public goods may generate a coordination issue. Even when each public good (in our leading examples, a software component) is entirely independent, there remains a residual coordination issue. An individual will only provide a public good if others do not, and if the private benefit of provision exceeds its private cost. In contrast, when others are expected to provide, then an individual is happy to free ride. The question of who provides the public good (if at all) has not been answered. The good may be provided by an inefficient provider (inefficient provision) or contributors may fail to coordinate on an individual provider, leading to either the absence of the good or wasteful duplication of effort.

(iv) Overview

Three separate questions arise from this discussion. First, when free riding is an issue, who provides and who free rides? Second, can the interdependence of provision (as in the case of open-source software) be exploited by a social planner to enhance the incentives to contribute? Third, when will such integration result in a coordination failure, and hence when might it be better to separate a project into independent components?

To answer these questions, public-good provision is modelled as a simple binary-action game.[14] Such games will exhibit multiple equilibria. The techniques of global games (Carlsson and van Damme, 1993; Morris and Shin, 2002*a*) are used to select equilibria. This leads to policy conclusions on the likely success of different collective-action problems and an assessment of when to integrate or separate the components of a public good.

II. COLLECTIVE-ACTION GAMES

(i) A Binary-action Contribution Game

An individual's decision to contribute to a public good will depend upon the expected contribution

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[13] This distinction is reminiscent of the concepts 'cathedral' and 'bazaar' introduced in Raymond (1998). The cathedral corresponds (loosely) to integration of components and the bazaar to separation. Johnson (2002) uses the term 'modularity' in place of separation.

[14] The use of game-theoretic concepts has become standard in the public-goods literature. Textbooks commonly employ such language to introduce the problem; see, for example, Cornes and Sandler (1996).

decisions of others. Formally, therefore, the collective-action problem may be modelled as a game. The players are the potential contributors to the public good—for example, a community of open-source programmers.[15] For simplicity the case of two players will be considered, and they will be indexed by $i \in \{1, 2\}$.[16] Players simultaneously choose the size of their contributions. Again for simplicity, the analysis here restricts to the binary-action case: a player either contributes (C) or does not (D).

Payoffs involve both costs and benefits. If Player i contributes (C) then a private cost c_i is incurred. Costs may vary ($c_1 \neq c_2$): a talented programmer finds it less onerous to write a software component successfully. Costs are entirely private, and do not depend on the decisions of others. In contrast, the benefits do depend on the actions of others—there are positive externalities. Each unit of the public good (where successful production may depend on all action choices) generates a benefit of v for each player. Again for simplicity, this benefit does not vary across the different players.[17] Some possible payoff configurations are displayed in Figure 1, and are discussed in sections II(ii)–(iv).

A final consideration will be the knowledge of the players. Players are only partially informed about the payoffs involved. Before writing a piece of software, a programmer can only make an educated guess as to the time it will take, based upon any relevant and available information. Of course, this information reveals something about the payoffs faced by others, and hence their likely actions. Furthermore, different players may have different information, and hence may not agree on the precise nature of the game they are playing. For the moment, however, common knowledge of payoffs will be assumed. Perhaps surprisingly, it is the relaxation of this assumption in section III that will help to answer the questions posed in section I.

(ii) A Single Public Good: The Free-riding Problem

Suppose that a single public good may be produced. It requires a contribution from only one player. If both contribute, then no additional benefit is generated—there is a duplication of effort—and the contributions of the two players are *substitutes*. Two possible configurations are Games (*a*) and (*b*) in Figure 1.

In Game (*a*) the private cost of provision outweighs the private benefit ($v < \min\{c_1, c_2\}$), and so each agent has a dominant strategy not to contribute. Nevertheless, it may well be the case that $2v > \min\{c_1, c_2\}$, so that it would be socially optimal for the more efficient agent (that is, the agent with the lowest cost of provision) to provide the good. For $2v > \min\{c_1, c_2\} > v$ there is a unique Nash equilibrium and yet inefficient provision—a classic collective-action problem.

In Game (*b*), the private benefit outweighs the private cost for both players ($v > \max\{c_1, c_2\}$). Both {C,D} and {D,C} are (pure-strategy Nash) equilibria. In equilibrium, one player enjoys a benefit without cost and 'free rides' on the other. This raises two potential problems. First, players may coordinate on an inefficient equilibrium where the least-efficient player provides the public good: they may play {C,D} even though $c_1 > c_2$. Second, the multiplicity of equilibria may lead to a coordination failure, with either no provision (if they play {D,D}) or the wasteful duplication of effort (if they play {C,C}). Game (*b*) generates the following question.

Question 1: for the production of a single public good, who provides and who free rides?

Of course, there are cases where no problem arises. For instance, if $c_1 < v < c_2$, then Player 2 will never contribute. Anticipating this, Player 1 will then find it optimal to do so. Hence the good is provided efficiently.

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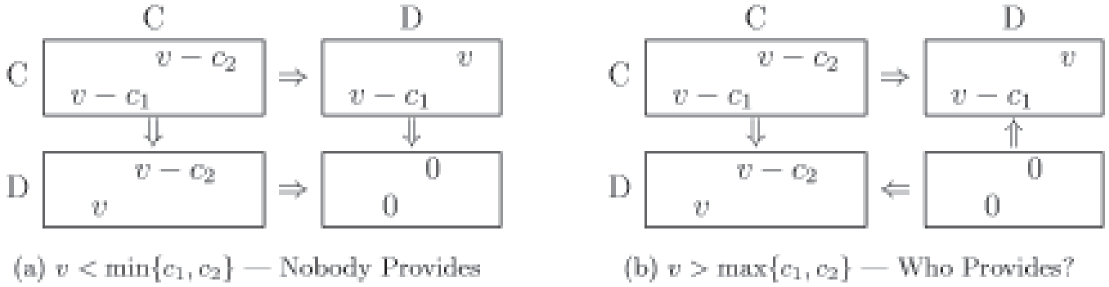
[15] Of course, other individuals (such as the end users of software programs) will benefit from the provision of the public good, but will be unable (perhaps owing to a lack of programming skill) to contribute.

[16] The results presented here (and proven formally in the Appendix) can be readily applied to the case of many players. This generates additional technicality and some new and important insights, but is not crucial for the key ideas raised here.

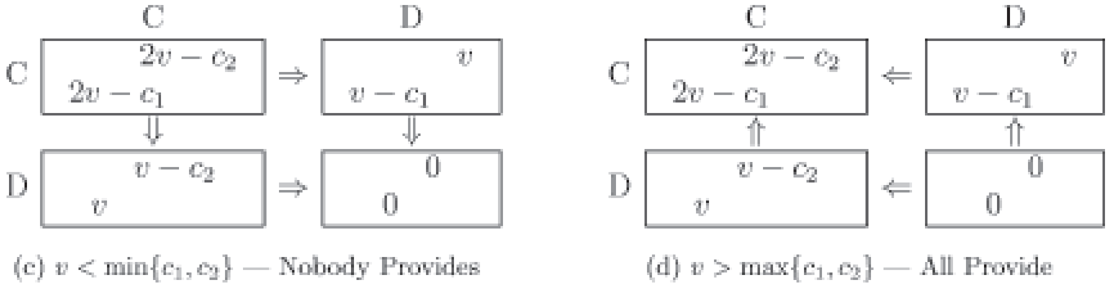
[17] Simplifying assumptions such as these are without loss of some generality.

Figure 1
Public-good Contribution Games



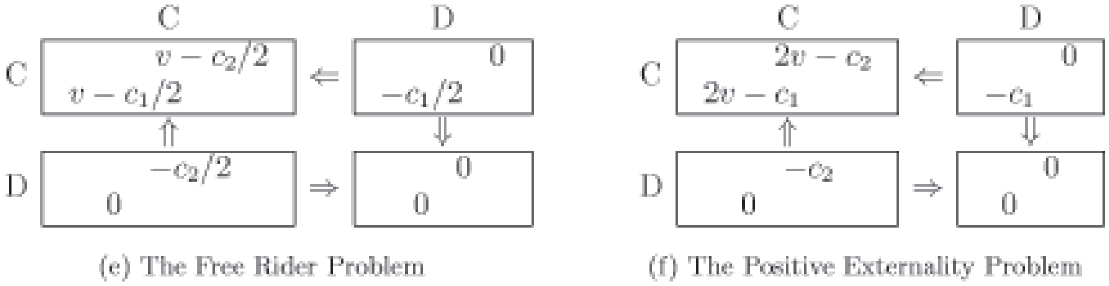
(a) $v < \min\{c_1, c_2\}$ — Nobody Provides (b) $v > \max\{c_1, c_2\}$ — Who Provides?

Games (a) and (b): Free Rider Problems



(c) $v < \min\{c_1, c_2\}$ — Nobody Provides (d) $v > \max\{c_1, c_2\}$ — All Provide

Games (c) and (d): Positive Externality Problems



(e) The Free Rider Problem (f) The Positive Externality Problem

Games (e) and (f): The Effect of Integration

Notes: These six diagrams illustrate the different games that are considered. For each game, 'C' and 'D' correspond to the actions 'contribute' and 'do not contribute' respectively. Each cell represents a strategy profile. The bottom-left payoff accrues to the row player (Player 1) while the top-right payoff accrues to the column player (Player 2). For each game the arrows represent the direction of better response. Thus a cell with two arrows pointing inward represents a pure-strategy Nash equilibrium.

For Games (*a*) and (*b*) it may be possible for a social planner to change the relationship between the contributions, and hence the structure of the game. Suppose that the project may be *integrated*, so that the successful production of the public good requires contributions from *both* players. In doing so, the costs are assumed to halve to $c_1/2$ and $c_2/2$.[18] This generates Game (*e*) in Figure 1. The contributions are now *complements*, since neither is of use without the presence of the other. Doubling the payoffs yields the (strategically equivalent) Game (*f*). For $2v > \max\{c_1, c_2\}$ this has two symmetric pure-strategy Nash equilibria {C,C} and {D,D}—this game is discussed in section II(iv) below.

(iii) Two Public Goods: The Positive-externality Problem

Suppose instead that two independent public goods may be produced. Each good requires the contribution of a single player. Contributions from two players yield twice the benefit of a single contribution. Furthermore, these contributions are neither complements nor substitutes. Two possible configurations are Games (*c*) and (*d*) in Figure 1.

Game (*c*) has the same structure as Game (*a*). The private cost outweighs the private benefit ($v < \min\{c_1, c_2\}$), and so neither player contributes. Nevertheless, it may well be the case that $2v > \max\{c_1, c_2\}$, so that it would be socially optimal for both to contribute. Each player fails to take into account the positive externality exerted on the other player. In contrast, when $v > \max\{c_1, c_2\}$ (as in Game (*d*)) both players are willing to provide. There is a unique pure-strategy Nash equilibrium in both cases.

Once again, it may be possible for a social planner to change the notion of interaction. Suppose that the two public goods are integrated, so that the contributions are complementary. Successful production requires the participation of both players. This results once again in Game (*f*), with two symmetric pure-strategy Nash equilibria.

(iv) The Effects of Integration

The analysis raises the possibility that a social planner may attempt to change the nature of the interaction between different components of a collective-action project.[19] Returning to the example of open-source software helps to illustrate this idea. In this context, two *separated* products might be a printer driver and a mouse driver. Both are independently useful, and may be provided by independent computer programmers—as in Games (*c*) and (*d*). An open-source project leader (the social planner) might change the interaction in the following way. Production of the drivers might be divided into two different types of task. The first might be the production of the code that interacts with the computer hardware. The second might be the user interface. Neither will be useful without the other—they become *integrated* public goods. This leads to the following question.

Question 2: can the interdependence of public-good provision be exploited by a social planner in order to enhance the incentives of individuals to contribute?

Integration will turn Game (*c*) into Game (*f*), or Game (*a*) into Game (*e*). In both Games (*a*) and (*c*), there is no provision by either player. In Games (*e*) and (*f*), however, there are equilibria where both players choose to contribute. It seems possible that integration may solve a collective-action problem.

Unfortunately, this is not the whole story. Games (*e*) and (*f*) exhibit multiple equilibria. In addition to the 'nice' equilibrium {C,C}, there is a 'nasty' equilibrium {D,D}, in which neither player contributes. Notice that integration turns Game (*d*) into Game (*f*). In Game (*d*), both players provide. In Game (*f*), however, there is the possibility that this outcome will be overturned. Integration of public-good provision is potentially dangerous: it enhances incentives in the 'nice' equilibrium, but runs the risk of pushing players into the 'nasty' equilibrium. According to

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[18] The results would change in an obvious way if the cost of the project were to be split in some other way. The case of most interest might be if only the lower of the two costs were to be paid when both agents contribute.

[19] The production technology of a public good has long been known to be critical. For example, Varian (1994) compares provision when contributions are made sequentially as opposed to simultaneously.

which equilibrium is played, either integration or separation of provision may be optimal. This leads to a third question.

Question 3: when will integration result in coordination failure, and hence when might it be better to separate a project's components?

To answer this question, the equilibrium selection problem must be addressed. So far, it has been assumed that the payoffs of the game are common knowledge. Relaxing this assumption will provide a solution to this problem.

III. EQUILIBRIUM SELECTION

Consider the following symmetric version of Game (*f*) from Figure 1:

	C	D
C	$2v - c$ $2v - c$	0 $-c$
D	$-c$ 0	0 0

Both players face the same cost of contribution, c. When $2v > c > 0$ this game has two symmetric pure-strategy Nash equilibria. Which one (if any) will be played?

(i) Allowing for Uncertain Costs

Suppose the assumption that the payoffs are common knowledge is relaxed. Specifically, suppose that the cost of provision c is uncertain, whereas the benefit v remains commonly known. Player 1, for instance, will be uncertain of both the contribution cost and the behaviour of Player 2. Examining the game, and assuming without loss of generality that Player 1 contributes whenever indifferent, optimal play is to:

$$\begin{aligned}\text{Contribute} &\Leftrightarrow 2v\,\Pr[\text{Player 2 contributes}] \geq \mathrm{E}[c] \\ &\Leftrightarrow \Pr[\text{Player 2 contributes}] \geq \frac{\mathrm{E}[c]}{2v}.\end{aligned}$$

Player 1 will contribute when confident that Player 2 will also do so. Of course, the probability that Player 2 contributes will always lie between 0 and 1. Hence:

$$\mathrm{E}[c] < 0 \Rightarrow \text{Play C and } \mathrm{E}[c] > 2v \Rightarrow \text{Play D}.$$

For certain expectations of the contribution cost, a player has a dominant strategy either to contribute (C) or not (D). The expectation of the contribution cost may be negative when a player derives some positive private benefit from the act of contribution—and although this may be unlikely, it will be retained as a possibility.

In order to form expectations over c, a player must have beliefs, which must stem from any available information. Consider the following simple specification. Both players begin with no knowledge of the contribution cost c—formally they have a *diffuse prior belief* over c. They are given an unbiased signal c_i of this true cost. Since they are unbiased, Player *i* expects the contribution cost to be equal to the signal: $\mathrm{E}[c \mid c_i] = c_i$. Crucially, it is supposed that Players 1 and 2 receive *different* signals. A straightforward example is one where the signals differ by some small amount $\varepsilon > 0$, so that $c_2 = c_1 \pm \varepsilon$.

This specification captures two features. First, players are initially unaware of the game being played and must use their information sources to form expectations. For instance, a programmer must draw upon experience and an examination of the problem in hand in order to estimate the likely cost. Second, players possess different information, and hence may hold different beliefs. Notice that these opinions may be almost perfectly accurate and may differ only slightly—so that c_i will be close to the true c and ε may be arbitrarily small.

(ii) The Infection Argument

Perhaps surprisingly, the specification above yields a *unique* equilibrium. This claim is backed by the use of an *infection* argument.[20] Such an argument begins with this observation: for certain signal realizations players have a dominant strategy. When a player receives a signal $c_i < 0$, then $\mathrm{E}[c] < 0$ and thus the player will always contribute (play C). Similarly,

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[20] Infection arguments are used by Morris and Shin (1998) in their analysis of self-fulfilling exchange-rate attacks.

when a player receives a signal $c_i > 2v$ then D is a dominant strategy.

The second step in the infection argument is a little more subtle. Suppose that Player 1 receives a signal c_1 satisfying $0 \le c_1 < \varepsilon$. The expected contribution cost is small, yet positive. Player 1 will think about the signal, and hence likely behaviour, of Player 2.

Remembering that $c_2 = c_1 \pm \varepsilon$:

$$\begin{aligned}\Pr[\text{Player 2 contributes} \mid 0 \le c_1 < \varepsilon] & \\ &\ge \Pr[c_2 < 0 \mid 0 \le c_1 < \varepsilon] \\ &\ge \Pr[c_2 = c_1 - \varepsilon \mid 0 \le c_1 < \varepsilon] \\ &= \frac{1}{2}.\end{aligned}$$

Thus, for a signal c_1 between 0 and ε, Player 1 believes it is more likely than not that Player 2 will contribute. Now, so long as $\varepsilon < v$:

$$\begin{aligned}E[c] = c_1 < \varepsilon \Rightarrow \frac{E[c]}{2v} < \frac{1}{2} & \\ \Rightarrow \Pr[\text{Player 2 contributes}] &\ge \frac{E[c]}{2v}\end{aligned}$$

and so Player 1 finds it optimal to contribute. This argument can applied in exactly the same way to Player 2, and hence Player *i* will always play C if $c_i < \varepsilon$. An identical argument establishes that Player *i* will always play D if $c_i > 2v - \varepsilon$.

The intuition is as follows. When a player believes that the contribution cost is very low, then the player becomes suspicious that the other player may believe the contribution cost to be negative—in which case the other player will contribute. This is enough to persuade the initial player to contribute. In other words, a player may suspect that the other player believes that a different game (one with a dominant strategy) is being played.

Of course, this infection argument may be iterated. Suppose that Player 1 receives a signal c_1 satisfying $\varepsilon \le c_1 < 2\varepsilon$. Remembering, once again, that $c_2 = c_1 \pm \varepsilon$:

$$\begin{aligned}\Pr[\text{Player 2 contributes} \mid \varepsilon \le c_1 < 2\varepsilon] & \\ &\ge \Pr[c_2 < \varepsilon \mid \varepsilon \le c_1 < 2\varepsilon] \\ &\ge \Pr[c_2 = c_1 - \varepsilon \mid \varepsilon \le c_1 < 2\varepsilon] \\ &= \frac{1}{2}.\end{aligned}$$

In fact, the argument may continue to step *n*, so long as $n\varepsilon < v$. The unique equilibrium obtained by this procedure becomes:

$$\text{Contribute} \Leftrightarrow c_i \le v.$$

This result is remarkable for a number of reasons. The assumptions have been changed only slightly—players have slightly different information about the payoffs of the game. Lifting the common knowledge assumption in this way is doubtless a step towards realism.

Nevertheless, the equilibrium is unique. The strategies used are easy to understand—a player uses a *threshold rule*, contributing if and only if the contribution cost is perceived to be sufficiently low.[21] Furthermore, the equilibrium strategies correspond to the optimal action taken by a player who places 50:50 odds on the likely actions of the other player—such a player has *Laplacian beliefs* over the other player's actions. Against such beliefs, it is optimal to contribute if and only if $E[c] < v$.

(iii) Equilibrium Selection via Risk Dominance

In the context of the symmetric coordination game analysed in section III(ii), the best response to Laplacian beliefs over the other player's actions (50:50 odds of C versus D) is a *risk-dominant* strategy. In a game of complete information, with common knowledge of the payoffs, contribute (C) is risk dominant whenever $c < v$, and do not contribute (D) is risk dominant whenever $c > v$. The infection argument offered above reveals that players will play what appears to be (from their signals) the risk-dominant strategy.

Risk dominance was introduced to game theory by Harsanyi and Selten (1988),[22] and may be applied to

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[21] Other applications, such as those found in Morris and Shin (1998, 2002b) and the paper by Myatt and Fisher in this issue also exhibit threshold-rule equilibria.

[22] The global-game literature is not alone in its support of risk dominance. The evolutionary stochastic adjustment dynamics literature, typified by Kandori *et al.* (1993) and Young (1993), also provides independent theoretical justification for the selection of risk-dominant equilibria.

a wider class of 2 × 2 games. A pure-strategy Nash equilibrium is risk dominant if it is relatively robust to potential deviations. A measure of robustness is as follows: fix a pure-strategy Nash equilibrium. Consider the payoff lost by a player when that player deviates from the prescribed equilibrium. Take the product of such payoff losses for each player. The equilibrium for which this measure (similar to the Nash product of bargaining theory) is highest is the risk-dominant equilibrium.

This procedure may be applied to the symmetric version of Game (*f*) considered above. Take the pure Nash equilibrium {C,C}, where both players receive a payoff of $2v - c$. A deviation by the either player results in the loss of this payoff. Taking the product yields $(2v-c)^2$. Turning to {D,D}, deviation results in a loss of c. Taking the product for each player yields c^2. Thus, {C,C} risk dominates {D,D} if and only if:

$$(2v - c)^2 > c^2 \Leftrightarrow c < v.$$

This corresponds exactly to the result from the infection argument given previously—when players are unsure of the exact game being played, they plump for the risk-dominant equilibrium. In fact, this argument can be made more general. A result due to Carlsson and van Damme (1993) establishes that when players privately observe very accurate (but potentially different) signals of the game's payoffs, then they will almost always play strategies which correspond to the risk-dominant equilibrium in a game where the payoffs were commonly known.[23] Thus, risk dominance will be used as an equilibrium selection criterion to answer Questions 1–3.

IV. APPLYING RISK DOMINANCE

It has been argued that the relaxation of the common-knowledge assumption enables the selection of a unique equilibrium: the risk-dominance criterion may be used to highlight the strategy profile that will almost always be played when players are almost (but not quite) perfectly informed. The criterion is now applied to the games in Figure 1.

(i) Symmetric Contribution Costs

First, consider symmetric versions of all the games ($c_1 = c_2 = c$). In Games (*a*), (*c*), and (*d*) there is a dominant strategy for each player, and hence there is no selection problem. Specifically, in Games (*a*) and (*c*) it is a dominant strategy for neither player to contribute. Similarly, in Game (*d*) it is a dominant strategy for both players to contribute.

In the remaining Games (*b*), (*e*), and (*f*) there are multiple pure-strategy Nash equilibria. Under the symmetric specification considered here, risk dominance cannot help select an equilibrium in Game (*b*). The product of the equilibrium payoff minus the deviation payoff for each player is $c(v - c)$ for both equilibria. They cannot be ranked, therefore, in the sense of risk dominance.

Games (*e*) and (*f*) exhibit the same strategic properties. Focusing on Game (*f*), consider first the strategy profile {C,C}. The product of the equilibrium payoffs minus the deviation payoffs is $(2v - c)^2$. For the strategy profile {D,D} the equivalent product is c^2. The equilibrium in which both players contribute will be risk dominant if and only if:

$$(2v - c)^2 \geq c^2 \Leftrightarrow v > c.$$

Games (*e*) and (*f*) arose when the activities of the two players were integrated in order to generate sufficient incentives for the provision of the public good. This was necessary when $c > v$, since it was only in this case that no provision took place in a world of separated technologies. When integration is desirable it is ineffective. The 'wrong' equilibrium will be selected. So far, the application of risk dominance has been of limited use. It was unable to select an equilibrium in the case of Game (*b*). For the 'integrated' Games (*e*) and (*f*) it revealed that integration fails to work in exactly the circumstances for which it was designed. These rather negative results are alleviated by a return to an asymmetric specification.

(ii) Asymmetric Contribution Costs

Asymmetry assists most directly with the problem of free riding. Examining Game (*b*) once more, the

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[23] Appendix sub-section (v) provides a proof of this informal argument for a version of the model considered here.

two pure-strategy equilibria {C,D} and {D,C} exhibit different deviation payoffs. In fact, {C,D} risk dominates {D,C} if and only if:

$$c_2\ (v - c_1) > c_1\ (v - c_2) \Leftrightarrow c_2 > c_1.$$

That is, if Player 1 is more efficient, the equilibrium involving Player 1 contributing to the project is risk dominant and hence selected. This yields an answer to Question 1.

Result 1: when public-good provision involves a free-riding problem, and at least one agent obtains a value greater than their private cost, the most efficient agent will bear the cost of provision.

Now consider the possibility of integration, as in Game (*e*). As long as $v > c_1/2$ and $v > c_2/2$, there is an equilibrium where both contribute. This will be risk dominant if:

$$\left(v - \frac{c_1}{2}\right)\left(v - \frac{c_2}{2}\right) > \frac{c_1}{2}\frac{c_2}{2} \Leftrightarrow v > \frac{c_1 + c_2}{2}.$$

This simply restates the earlier observation that when integration is desirable, it is ineffective. Integration is only desirable if both $v < c_1$ and $v < c_2$, but the social optimum involves a positive level of contribution. This rules out the above condition for risk dominance—{C,C} might be an equilibrium under integration but it is always risk dominated by {D,D}. Nothing is gained by integration. It is in this case that the informational assumptions need to be relaxed (see section V).

In the case of positive externalities it is possible that only one agent contributes when it is socially optimal for both to do so. Here, relaxing the symmetry result can be of value. Integration yields Game (*f*). The risk-dominance condition for {C,C} is:

$$(2v - c_1)(2v - c_2) > c_1 c_2 \Leftrightarrow v > \frac{c_1 + c_2}{2}.$$

Given that one agent found it optimal to contribute in the separated case, it is quite possible that equilibrium {C,C} is risk dominant and thus selected. Hence integration can be effective when it is desirable. The next result provides an answer to Question 2.

Result 2: when there is a positive externality involved in the provision of a good, and at least one agent has a private value which exceeds their private cost, then integration can result in the socially optimal level of provision.

Of course, this is only a possibility. Social optimality of dual provision corresponds to $v > (c_1 + c_2)/4$, whereas actual dual provision only takes place if $v > (c_1 + c_2)/2$. Hence there is a range of values v for which the good is not provided to the socially optimal level. Integration then results in no provision at all, whereas separation would at least result in partial provision if $v > c_i$ for some *i*.

V. OPTIMISM AND INTEGRATION

(i) Public versus Private Information

So far, players have been privately informed of the costs associated with contributing to the project. An alternative scenario is one in which players have access to both public and private information sources. Their beliefs about *c*, therefore, will combine both public and private information.

A concrete example helps to illustrate this idea. Suppose that Player *i* observes a private signal c_i and a public signal $\bar{c}$ of the contribution cost *c*. The expected cost might then be a weighted average of these two signals.[24] For some λ:

$$\mathrm{E}[c \mid c_i] = \lambda\bar{c} + (1 - \lambda)c_i.$$

Thus a player's opinion draws upon both public and private information. But what about the player's opinions about the opinions of the other player? For instance, Player 1 might consider Player 2's expectation of the contribution cost:

$$\begin{aligned}\mathrm{E}[\mathrm{E}[c \mid c_2] \mid c_1] &= \lambda\bar{c} + (1 - \lambda)\mathrm{E}[c_2 \mid c_1] \\ &= \lambda\bar{c} + (1 - \lambda)\mathrm{E}[c \mid c_1].\end{aligned}$$

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[24] In the Appendix, a model is presented in which both players receive a common public signal of the cost of the project and in addition a private signal that they alone observe. When the signals are normally distributed, then the posterior expectation of *c* is a simple weighted average of the public and private signals.

Thus Player 1 does *not* expect Player 2 to hold the same opinion. In fact, Player 1 expects that Player 2's expectation of the contribution cost will be biased toward the public signal. As Amato, Morris, and Shin explain in their article in this issue, there is an overreaction of higher-order beliefs to public information. Thus, for a fixed expectation of the contribution cost, an *optimistic* public signal (a low value for $\bar{c}$) may make Player 1 optimistic about the beliefs of Player 2. A *pessimistic* public signal (a high value for $\bar{c}$) has the opposite effect.

(ii) Optimism, Pessimism, and the Effect of Public Information

When players were purely privately informed, they played the risk-dominant strategy. Thus, in the context of the symmetric version of Game (*f*) analysed in section III, Player 2 would employ the following strategy:

$$\text{Contribute} \Leftrightarrow \mathrm{E}[c \mid c_2] \leq v.$$

Is this still an equilibrium in the presence of a public signal? Suppose that Player 2 adopts such a strategy, and consider the behaviour of Player 1. Suppose that Player 1 expects that the cost is just equal to v. Then:

$$\mathrm{E}[c \mid c_1] = v \Rightarrow \mathrm{E}[\mathrm{E}[c \mid c_2] \mid c_1] = \lambda\bar{c} + (1 - \lambda)v.$$

Hence, if $\bar{c} < v$, then Player 1 expects that Player 2 has a cost expectation strictly below v. But this means that Player 1 will expect Player 2 to contribute with probability strictly greater than 1/2, giving Player 1 a strict incentive to contribute. Continuing this argument, Player 1 is more likely to contribute, hence Player 2 is more likely to contribute, and so on.

This argument is, of course, heuristic. The formal analysis in the Appendix establishes that, in equilibrium, Player i will contribute if and only if $\mathrm{E}[c \mid c_i] \leq c^*$.[25] Interestingly, the threshold c^* moves up and down with the public signal $\bar{c}$.[26] Furthermore:

$$c^* > v \Leftrightarrow \bar{c} < v.$$

Recall that in the pure private-information case $c^* = v$. Intuitively, when $\bar{c} < v$, the public signal generates *optimism*. For $\bar{c} > v$, it generates *pessimism*. Optimistic public signals bias players towards the play of {C,C}, and pessimistic signals bias players towards the play of {D,D}. This helps to generate an answer to Question 3. Suppose that a social planner is able to observe a public signal prior to specifying the integration or separation of a collective action. Then:[27]

Result 3: when information is imperfect, if the public signal indicates that private costs lie below (above) private valuations, integration yields higher (lower) levels of public-good provision than separation.

Quite simply, an optimistic environment runs less risk of coordination failure, and hence the social planner will be keen to take advantage of the increased incentives that it offers.

Returning to the leading example of open-source software, the results offer the following lessons. First, we might expect less able programmers to free ride on more able individuals (Result 1). Second, any asymmetries between programmers might be exploited to generate *integrated* public-good provision (Result 2). Finally, the success of integrated versus separated public goods may depend upon an air of optimism or pessimism (Result 3). Of course, according to Raymond (1998), the success of the Linux project was due to the 'bazaar' (or separated) nature of the production technology. Was there then substantial pessimism over the potential benefits of the project at the outset? Raymond (1998) appears to think so, beginning his article with the rhetorical question 'Who would have thought . . . that a world-class operating system could coalesce as if by magic?'

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[25] In a symmetric game the equilibrium cut-off values (unsurprisingly) are the same for both players (see Appendix, Proposition 2). In the asymmetric case, the agent with the cost advantage (the more efficient) has a higher cut-off value, and hence contributes with higher probability (see Proposition 1).

[26] This key result is discussed in Proposition 3 for the symmetric case and Proposition 4 for the asymmetric case.

[27] This intuition and result is formalized in the Appendix. A similar result and intuition holds with asymmetry of cost contributions, and is also formalized in the Appendix.

APPENDIX: THE GLOBAL-GAME FRAMEWORK

(i) The Game

This appendix presents the formal model underlying the previous discussions. Consider the following asymmetric two-player (integrated) 'positive externality' public-good contribution game:[28]

	C	D
C	$2v - c$ $2v - c + \gamma$	0 $-c + \gamma$
D	$-c$ 0	0 0

The interesting case is when $2v > c > \gamma$, which is assumed throughout. Notice that {C,C} is risk dominant if and only if $v \geq c - \gamma/2$. Suppose that c is unknown, but that each player receives a public signal $\bar{c}$ and a private signal c_i such that:

$$\bar{c} = c + \varepsilon \text{ and } c_i = c + \varepsilon_i \text{ where } \varepsilon \sim \mathrm{N}(0, 1/\alpha)$$
$$\text{and } \varepsilon_i \sim \mathrm{N}(0, 1/\beta).$$

(ii) The Players' Beliefs

Calculating Player 2's posterior $\bar{c}_2$ yields:

$$\bar{c}_2 = \frac{\alpha\bar{c} + \beta c_2}{\alpha + \beta} = \frac{\alpha\bar{c}}{\alpha + \beta} + \frac{\beta c}{\alpha + \beta} + \frac{\beta\varepsilon_2}{\alpha + \beta}. \qquad (1)$$

Suppose Player 2 will play C whenever $\bar{c} \leq c^*_2$. Now, from Player 1's perspective, $c \sim \mathrm{N}(\bar{c}_1, 1/(\alpha + \beta))$. From equation (1), $(\alpha + \beta)\bar{c}_2 - \alpha\bar{c} = \beta c + \varepsilon_2$ and hence:

$$(\alpha + \beta)\bar{c}_2 - \alpha\bar{c} \sim \mathrm{N}\left(\beta\bar{c}_1, \frac{\beta^2}{\alpha + \beta} + \beta\right).$$

Finally, from Player 1's perspective:

$$\bar{c}_2 \sim \mathrm{N}\left(\frac{\beta\bar{c}_1 + \alpha\bar{c}}{\alpha + \beta}, \frac{\beta^2}{(\alpha + \beta)^3} + \frac{\beta}{(\alpha + \beta)^2}\right).$$

Player 1 can now calculate the probability that Player 2 will contribute to the project:

$$\Pr(\bar{c}_2 \leq c_2^* \mid \bar{c}_1) = \Phi\left[\frac{c_2^* - (\beta\bar{c}_1 + \alpha\bar{c})/(\alpha + \beta)}{(1/(\alpha + \beta) + 1/\beta)^{1/2}\beta/(\alpha + \beta)}\right].$$

Now suppose Player 1 will choose to contribute if and only if $\bar{c}_1 \leq c_1^*$. Evaluating the above when Player 1 is exactly indifferent between the two strategies obtains:

$$\Pr(\bar{c}_2 \leq c_2^* \mid \bar{c}_1 = c_1^*) = \Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\bar{c} - c_1^*}{\sqrt{\frac{1}{\alpha + \beta} + \frac{1}{\beta}}}\right].$$

The following two equations then solve to give the cut-off values c_1^* and c_2^*.

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\bar{c} - c_1^*}{\sqrt{\frac{1}{\alpha + \beta} + \frac{1}{\beta}}}\right] = c_1^* - \gamma. \qquad (2)$$

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_1^* + c_1^* - \frac{\alpha}{\beta}\bar{c} - c_2^*}{\sqrt{\frac{1}{\alpha + \beta} + \frac{1}{\beta}}}\right] = c_2^*. \qquad (3)$$

Proposition 1: the cut-off values are such that: $\gamma > c_1^* - c_2^* > 0$.

Proof: suppose $c_1^* \leq c_2^*$. Then from equations (2) and (3):

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[28] Notice that the asymmetric general costs c_1 and c_2 have been replaced by $c - \lambda$ and c respectively. Hence Player 1 has a cost advantage throughout, and this is known. The uncertainty revolves around the actual value of c. This makes the analysis easier to follow, without affecting the main results.

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\bar{c} - c_1^*}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]+\gamma$$

$$\leq 2v\Phi\left[\frac{\frac{\alpha}{\beta}c_1^* + c_1^* - \frac{\alpha}{\beta}\bar{c} - c_2^*}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right].$$

Since $\gamma > 0$ it follows that $c_1^* > c_2^*$—a contradiction. Subtracting the left-hand side of equation (3) from the left-hand side of equation (2) yields a negative number. The right-hand side is $c_1^* - c_2^* - \gamma$. So $\gamma > c_1^* - c_2^* > 0$ as required.

(iii) The Symmetric Case

Consider $\gamma = 0$. The following result is immediate.

Proposition 2: if $\gamma = 0$ then $c_1^* = c_2^* = c^*$ where:

$$c^* = 2v\Phi\left[\frac{\frac{\alpha}{\beta}(c^* - \bar{c})}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]. \quad (4)$$

Proof: again subtracting equation (3) from equation (2) yields $c_1^* - c_2^*$. Suppose $c_1^* > c_2^*$. The left-hand side is negative, a contradiction. Likewise for $c_1^* < c_2^*$—which implies $c_1^* = c_2^*$. Substitute into either equation to yield the value above.

Notice that if the public information is just equal to the individual value, $\bar{c} = v$, then a solution is $c^* = v$. A sufficient condition for equation (4) to have a unique solution is that the right-hand side has a slope less than 1 at c^*. This leads to the following condition.

Condition 1: there is a unique solution to equation (4) if:

$$\frac{2\beta^2 + \alpha\beta}{\alpha^2(\alpha+\beta)} > \frac{2v^2}{\pi}.$$

Proof: differentiate the right-hand side of equation (4) to yield:

$$2v\phi\left[\frac{\frac{\alpha}{\beta}(c^* - \bar{c})}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]\frac{\frac{\alpha}{\beta}}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}.$$

This must be less than one. Notice that $\phi(\cdot)$ is maximized at $1/\sqrt{2\pi}$. After some manipulation, the above condition is obtained.

This condition will be satisfied for small α or large β— which are the cases of interest—and is assumed to hold in the remainder. Hence the following key proposition in the symmetric case is available.

Proposition 3: if Condition 1 is met, $\bar{c} > v \Rightarrow c^* < v$ and $\bar{c} < v \Rightarrow c^* > v$.

Proof: notice that $\Phi(\cdot)$ is an increasing function. A fall in $\bar{c}$ results in a rise in c^*. Combining that with the fact that if $\bar{c} = v$ then $c^* = v$ yields the result.

This is precisely the optimism result discussed in section V. When public information is 'bad' ($\bar{c} > v$), agents adopt a lower cut-off value $c^* < v$ and hence are *less* likely to contribute in the integrated case than they would be in the separated case.[29] A social planner would be better selecting a separated public-good technology. The reverse is true when public information is 'good'—in an optimistic world integration of public-good technology is advantageous.

(iv) The Asymmetric Case

The following lemma is a first step toward a similar optimism argument to that given in the symmetric case.

Lemma 1: the cut-off values for both agents are decreasing in public information:

$$\frac{dc_1^*}{d\bar{c}} < 0 \text{ and } \frac{dc_2^*}{d\bar{c}} < 0.$$

Proof: begin by totally differentiating equations (2) and (3). This gives:

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[29] In the separated case agents simply contribute when their information suggests cost is less than value—their cut-off level is always v.

$$\frac{2v\phi_1}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\left\{\frac{\alpha}{\beta}\frac{dc_2^*}{d\bar{c}}+\frac{dc_2^*}{d\bar{c}}-\frac{dc_1^*}{d\bar{c}}-\frac{\alpha}{\beta}\right\}=\frac{dc_1^*}{d\bar{c}} \quad (5)$$

$$\frac{2v\phi_2}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\left\{\frac{\alpha}{\beta}\frac{dc_1^*}{d\bar{c}}+\frac{dc_1^*}{d\bar{c}}-\frac{dc_2^*}{d\bar{c}}-\frac{\alpha}{\beta}\right\}=\frac{dc_2^*}{d\bar{c}} \quad (6)$$

where

$$\phi_i=\phi\left[\frac{\frac{\alpha}{\beta}(c_j^*+c_j^*-\frac{\alpha}{\beta}\bar{c}-c_i^*)}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]>0 \text{ for } i\neq j\in\{1,2\}.$$

Label $D_i = dc_i^*/d\bar{c}$. If $D_i > 0$ then $(1 + \alpha/\beta)D_j - D_i - \alpha/\beta > 0$ and hence $D_j > 0$ for all i and j. Hence, without loss of generality, suppose $D_1 \geq D_2$ and $D_2 > 0$. This implies that $(1 + \alpha/\beta)D_2 - D_1 - \alpha/\beta > 0$ and hence $(1 + \alpha/\beta)D_1 - D_1 - \alpha/\beta > 0$. So $D_1 > 1$. By equation (5) and $D_1 \geq D_2$:

$$\frac{2v\phi_1}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\left\{\frac{\alpha}{\beta}D_1+D_1-D_1-\frac{\alpha}{\beta}\right\}\geq D_1.$$

Which implies (since $D_1 > 1$) that:

$$\frac{\alpha 2v\phi_1/\beta}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\geq\frac{D_1}{D_1-1}>D_1.$$

Recall that $\phi(\cdot)$ is maximized at $1/\sqrt{2\pi}$ and hence the above equation implies:

$$\frac{\alpha 2v/\beta\sqrt{2\pi}}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}>D_1>1.$$

This is ruled out by Condition 1 yielding a contradiction. So $D_1 < 0$ and $D_2 < 0$.

The cut-off values are strictly decreasing in public information, guaranteeing uniqueness, and the first part of the 'optimism' result of section V in the asymmetric case. Proposition 4 then follows.

Proposition 4: assume Condition 1 holds. Then (i) $\bar{c} \leq v - \gamma\beta/\alpha \Rightarrow c_1^* > v + \gamma$ and $c_2^* > v$, and (ii) $\bar{c} \geq v + \gamma + \gamma\beta/\alpha \Rightarrow c_1^* < v + \gamma$ and $c_2^* < v$.

Proof: consider $c_2^* = v$. By equation (2), $\alpha(c_1^* - \bar{c})/\beta + c_1^* - c_2^* = 0$. Hence $c_1^* = (\beta v + \alpha\bar{c})/(\alpha + \beta)$. Now from Proposition 1, $v + \gamma > c_1^* > v$. Substituting in for c_1^*:

$$v+\gamma+\frac{\beta}{\alpha}\gamma>\bar{c}>v.$$

By Lemma 1, if $\bar{c} \geq v + \gamma + \gamma\beta/\alpha$ then $c_2^* < v$ and therefore via another use of Proposition 1, $c_1^* < c + \gamma$. This gives part (ii). For part (i) consider $c_1^* = v + \gamma$ and use equation (3) in a similar way.

Again this reveals a very similar story to the symmetric case of Proposition 3. If public information is sufficiently 'good' ($\bar{c} \leq v - \gamma\beta/\alpha$), the cut-off values are respectively above $v + \gamma$ and v. In the game with separated contributions, the optimal cut-off values for the two agents are exactly $v + \gamma$ and v respectively. Hence, integration will increase the likelihood of contribution. If public information is sufficiently 'bad', the reverse is true. The difference with asymmetry is that, unlike the symmetric case, there is now a range of ambiguity. The midpoint of this range is where public information is exactly equal to the critical risk-dominant value ($v + \gamma/2$). Even for very precise public information ($\alpha \rightarrow \infty$) the range is bounded away from this point, on $[v, v + \gamma]$. It remains to generate the standard selection results.

(v) Equilibrium Selection

Fix $\alpha > 0$ and consider $\beta \rightarrow \infty$. This represents increasingly precise private information and hence an increasingly good approximation to the complete information game. In the complete information game, there are multiple equilibria. In the incomplete information game, uniqueness results (when Condition 1 is satisfied) and note that $\beta \rightarrow \infty$ will guarantee this eventually.

Proposition 5: equilibrium {C,C} is played if and only if $c \le v + \gamma/2$ as $\beta \to \infty$.

Proof: as $\beta \to \infty$ suppose $c_1^* - c_2^* \not\to 0$. Then, from equations (2) and (3), $2v\Phi(\infty) = c_2^*$ and $2v\Phi(-\infty) = c_1^* - \gamma$ so that $c_2^* = 2v$ and $c_1^* = \gamma$. But $2v > \gamma$ by assumption.

The contradiction implies that $c_1^* - c_2^* \to 0$ as $\beta \to \infty$. Moreover, (informally) writing equations (2) and (3) as:

$$2v\Phi\left[0 + \frac{c_1^* - c_2^*}{\to 0}\right] = c_2^*$$

$$2v\Phi\left[0 + \frac{c_1^* - c_2^*}{\to 0}\right] = c_1^* - \gamma$$

note that $\Phi(-x) = 1 - \Phi(x)$ and hence:

$$2v = c_1^* + c_2^* - \gamma \text{ as } \beta \to \infty.$$

Since (in the limit) $c_1^* = c_2^* = c^*$, then $c^* = v + \gamma/2$. Hence both agents will play strategy C if their posterior $\bar{c}_i$ is less than this value. But the posterior becomes their private information precisely as $\beta \to \infty$ (see equation (1)). Moreover $c_i \to c$ as $\beta \to \infty$, which yields the required result.

The risk-dominant equilibrium is selected as discussed in section III. Similar arguments can be used to obtain all of the risk-dominance selection results referred to throughout the current paper.

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