Stable Prices and Heterogeneous Buyer Consideration

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February 2025.

Paper: pp. 1–27. Proofs and references: pp. 27–38. Online supplements: pp. 39–60.

Abstract. We study the pricing of homogeneous products sold to customers who consider different sets of suppliers. We identify prices that are stable in the sense that no firm wishes to undercut a rival or to raise its price when rivals are able to respond by offering special deals. We derive predictions for stable and disperse prices across several price-consideration specifications, and we contrast the implications with those of conventional approaches.

Seemingly identical products are regularly offered at different yet persistent prices. We develop the idea that stable pricing positions should be robust to the threats of undercuts from rivals. Our approach predicts stable and dispersed prices, and so it provides a framework for understanding price dispersion in markets where consumers have heterogeneous consideration sets.

For foundational strategic accounts of pricing, such as canonical Bertrand models of competition, the core incentive of a firm is to lower its price sufficiently to "undercut" rivals. The simplest case is typically understood to lead to marginal-cost pricing. Such prices are stable in the sense that no firm would undercut any rival further, but of course they are not dispersed.

An established approach extends this setting to heterogenous price consideration: different buyers evaluate different prices. A price-setting firm faces a trade-off: undercut rivals to sell to buyers who compare many prices, or elevate price to profit from those who do not. Modeling price-setting in the standard way (a non-cooperative single-stage game) results in equilibrium prices that are set randomly via mixed strategies. The realized prices are dispersed, but lack stability: upon seeing cheaper firms' prices, a higher-priced competitor regrets its choice and wishes to undercut a rival.

Our theory delivers prices that are stable and dispersed by recognizing that sellers are often able to slash easily and rapidly prices or to offer special deals in response to rivals' price adjustments. A credible threat disciplines prices: a move upward in a firm's price triggers a rival to undercut it.

Acknowledgements. Some subsections within this paper are based upon elements of our (now superseded) working paper "A Theory of Stable Price Dispersion" (from 2019). We thank Simon Anderson, Mark Armstrong, Dan Bernhardt, Alessandro Bonatti, Tim Cason, Yongmin Chen, Alex de Cornière, Andrea Galeotti, Roman Inderst, Justin Johnson, Jeanine Miklós-Thal, José Moraga-González, Paul Heidhues, Maarten Janssen, Volker Nocke, Martin Pesendorfer, Michael Riordan, Robert Somogyi, Greg Taylor, Juuso Välimäki, Hal Varian, Chris Wilson, Jidong Zhou, and others for helpful comments. Ronayne thanks Deutsche Forschungsgemeinschaft for support (CRC TRR 190 280092119).

The demand side of our model specifies the mass of buyers who consider each subset of firms' prices. On the supply side we seek "stable" prices that satisfy two criteria. Firstly, no firm can profitably undercut a cheaper rival. Secondly, no firm can profit from raising its price given that, were it to do so, other firms can cut prices in response, perhaps by using flash sales or special deals.

Any "undercut proof" profile of prices that satisfies our first criterion is (under a mild regularity condition) entirely dispersed: each firm charges a different price. Such a profile is "maximal" if no price can be raised without violating undercut-proofness. Maximal profiles are candidates for stability. To check our second criterion, we evaluate an upward price deviation and study the equilibrium profits of a price-cutting game in which firms can respond by dropping their prices. Under a general specification for consideration, we find a simple condition under which no firm can gain from a local upward deviation in its price.

For two broad consideration-set specifications we obtain stronger results. In the first we allow for generality in the number of firms considered, while restricting such consideration to be symmetrically distributed across firms. The mass of customers who compare a set of prices depends only on the number of prices and so firms are "exchangeable" across similarly sized consideration sets. We find that a profile of prices is stable if and only if it is the uniquely maximal undercut-proof profile. When firms differ in their captive shares, we provide conditions under which there is a stable price profile in which firms with larger captive bases set higher prices. In a second setting we instead specify full asymmetry of the awareness of firms' prices, while assuming that consideration is independently distributed, as in some popular models of informative advertising. We find that a stable price profile allocates the highest price to the firm with greatest awareness.

The key primitive for our work is the distribution over consideration sets. Amongst our extensions, we use duopoly examples to illustrate how consideration can be influenced by firms and customers. Firstly, we allow firms to influence awareness via advertising. We find that symmetric firms make different advertising decisions, resulting in a marked dispersion of stable prices. Secondly, we allow for buyer search. Search decisions are strategic complements (search by others increases dispersion, raising the benefit of gathering another quotation) which readily generates multiple equilibria. An equilibrium with high search exhibits substantially dispersed stable prices.

Connection to the Established Literature. For the environment of price-setting firms with heterogeneous buyer consideration, the workhorse pricing model is a single-stage non-cooperative game in which all firms simultaneously set prices, where founding contributions including those of Varian (1980), Rosenthal (1980), Narasimhan (1988), and Baye, Kovenock and de Vries (1992). Armstrong and Vickers (2022) made recent and substantial progress under several consideration structures, interpreting the equilibrium strategies as the "patterns of competitive interaction."

The consideration-set demand structure eliminates pure-strategy Nash equilibria in such a game, and so disperse prices are typically understood as the realizations of mixed strategies.[2] Such predictions are unstable in the sense that realized prices are not best replies ex post. A literal interpretation of repeated play suggests rapidly fluctuating prices. In contrast, we harness firms' ability (and credible threat) to cut quickly a price, and arrive at stable outcomes. Stable prices are both undercut-proof and resistant to price increases that can subsequently be undercut. We connect to the traditional approach via an interpretation (Varian, 1980; Baye, Kovenock and de Vries, 1992) of mixed-strategy realizations being special deals or "sales" relative to a higher "regular" price.[3] Viewed that way, we explain how regular prices originate via firms' anticipation of cut-price offers.

Our approach also weakens an implicit commitment assumption within the literature. A firm is fully committed to a price when it cannot adjust it, at all, upwards or downwards. In that case, a mixed-strategy prediction can rationalize persistent prices: firms want to undercut their rivals but cannot. We assume a weaker commitment: firms are free to use special deals at any time.

We discuss further the related literature throughout the paper and in our concluding remarks.

Empirical Considerations. Empirical studies have identified extensive price dispersion. For example, Kaplan and Menzio (2015) used the large Kilts-Nielsen panel of 50,000 households to show that the standard deviation (relative to the mean) of prices at brick-and-mortar stores ranges from 19% (when products are defined narrowly) to 36% (when defined broadly).

Despite both cross-sectional and inter-temporal variation, prices are well-known to be sticky or persistent in many markets.[4] Using US data, Nakamura and Steinsson (2008) estimated the median duration of a (regular) price in the US to be between 8–11 months. The European Central Bank (ECB, 2005) found the "median firm changes its price once a year." Using Norwegian retail data, Wulfsberg (2016) and Moen, Wulfsberg and Aas (2020) found a high persistence of price dispersion: prices on average last 6–16 months depending on the product category and macro environment, and stores charging prices in a particular quartile of the distribution stay there with high probability (0.83–0.93) month-to-month. Remarkably, Gorodnichenko, Sheremirov and Talavera

[2]To illustrate, consider best replies in a duopoly: firms undercut each other to capture "shoppers' and so walk down a staircase of prices; but at a sufficiently low price one firm prefer to elevate its price back up to exploit those who are "captives" to its price. This "Edgeworth cycle" logic (Maskin and Tirole, 1988*a*,*b*) rules out pure-strategy equilibria.

[3]The "regular" and "sales" terminology was also used by Heidhues and Kőszegi (2014). Others have studied combinations of regular and personalized prices. For example, Anderson, Baik and Larson (2023) and Rhodes and Zhou (2024) modeled targeted price discrimination with personalized prices, and Gill and Thanassoulis (2016) studied a duopoly in which some customers see list prices while others have access to discounts.

[4]For example, Kaplan and Menzio (2015) decomposed the variation and found the intertemporal component (their "transaction" component) accounted for a substantial fraction, but less than half. Also with Kilts-Niesen data, Kaplan et al. (2019) reported that "a sizable fraction of the variance of prices for the same good is caused by persistent differences in the price that different stores set for that good [...]."

(2018) examined daily online pricing data and reported (pp. 1764–1766) that "although online prices change more frequently than offline prices, they nevertheless exhibit relatively long spells of fixed prices." Specifically, prices are fixed for long spells of 7–20 weeks and "do not adjust every instant." They concluded that prices tend to vary in the cross section rather than over time.[5]

In principle, long price spells could be because of a paucity of opportunity for firms to change their prices. However, some evidence suggests firms do not change their prices every time they can. For example, 43% of Euro-area firms reviewed prices at least four times a year, but only 14% changed price that often (ECB, 2005, see also ECB, 2019).

These applied considerations motivate a theory of stable price dispersion. We posit that firms are free to undercut rivals at any point, but that it can be hard to raise a price while preventing price-cutting responses, given that firms can offer special deals to customers at short notice.

One stronger price rigidity is that customers may see attempts to raise price as unfair or socially unacceptable. An initial price can set a reference point for loss-aversion arguments (Kahneman and Tversky, 1979), which suggests a higher elasticity above the initial price than below (Heidhues and Kőszegi, 2008).[6] The role of fairness concerns is central to the work of Kahneman, Knetsch and Thaler (1986). Relating their ideas to Okun (1981), they reported "the hostile reaction of customers to price increases that are not justified by increased costs ..."[7] Firms are not ignorant of these concerns: the ECB reports cited above found that a firm's "implicit contract" with their customers (that their prices will not rise) was a primary reason behind the observed price stickiness.[8] Additionally, in some markets legal constraints force a firm to meet any published offer or to limit price rises. For example, in a study of dispersed prices for prescription drugs, Sorensen (2000, p. 837) reported that "price-posting legislation dictates that any posted price must be honored at the request of the consumer."[9] In the gasoline market, Obradovits (2014) documented regulations that prohibited price rises (except once a day at noon), while price cuts were freely permitted.[10]

[5] Many industry-specific studies are also consistent with this summary, including prescription drugs (Sorensen, 2000), illicit drugs (Galenianos, Pacula and Persico, 2012), memory chips (Moraga-González and Wildenbeest, 2008), and textbooks (Hong and Shum, 2010). A contrasting conclusion from Lach (2002) emphasized that (p. 433) "stores move up and down the cross-sectional price distribution." In some industries, gasoline for example, substantial dynamic price movements have been documented (for example, Chandra and Tappata, 2011; Pennerstorfer et al., 2020).

[6] See also Zhou (2011*b*) and Ahrens, Pirschel and Snower (2017). Marketing research documents how "advertised reference prices" set value perceptions and purchase intentions (Urbany, Bearden and Weilbaker, 1988; Lichtenstein, Burton and Karson, 1991; Grewal, Monroe and Krishnan, 1998; Alford and Engelland, 2000; Kan et al., 2013).

[7] The importance of fairness considerations in pricing is central to many marketing studies (for example, Campbell, 1999, 2007; Bolton, Warlop and Alba, 2003; Xia, Monroe and Cox, 2004).

[8] Of course prices sometimes rise, but typically with inflation, unlike cuts (Nakamura and Steinsson, 2008).

[9] Charging "overs" at the point of sale can also fall under definitions of deceptive pricing. For example, the UK's Advertising Standards Authority advises that a product should be available at its listed price.

[10] Price-control and profit-control laws (which are often temporary measures) also typically impose frictions, explicitly or implicitly, on upwards but not downwards price movements.

Plan of the Paper. For our pricing solution concept (Section 2) we state a general characterization (Section 3) and then obtain stronger results for two classes of consideration: exchangeability (Section 4) and independent awareness (Section 5). We provide extensions and applications including endogenous consideration (Section 6) before concluding (Section 7). Formal proofs and several other extensions appear in main (Appendix A) and supporting (Appendices B and C) supplements.

2. A Model of Heterogeneous Price Consideration

The Economic Environment. On the supply side, $n > 1$ firms indexed by $i \in \{1, \dots, n\}$ produce a homogeneous product with the same constant marginal cost which, without (further) loss of generality, we set to zero. A firm's profit is its price multiplied by its sales.

On the demand side, each customer is willing to pay at most $v > 0$ for a single unit. A customer's *consideration set* lists those firms from whom they may buy. Each customer buys from the cheapest firm in their consideration set. Ties can be broken in any interior way.

We write $\lambda(B)$ for the mass of customers who consider the prices of firms within $B \subseteq \{1, \dots, n\}$, we use $B_i \in \{0, 1\}$ to indicate whether firm $i \in B$, and $\lambda_i = \lambda(\{i\})$ for the mass who are "captive" to firm i. To set aside uninteresting cases, we assume that each firm i has some captive customers ($\lambda_i > 0$) and that a positive mass consider i together with at least one other firm $j \neq i$.[11]

In a classic "captive and shopper" model a mass of $\lambda_S \equiv \lambda(\{1, \dots, n\})$ customers are "shoppers" who consider every firm and all other non-singleton consideration sets have zero mass.[12] An important property of consideration sets for pricing analysis is "twoness" meaning that all consideration pairs have positive mass: $\lambda(\{i, j\}) > 0$ for $i \neq j$.[13] For example, in a setting where consideration sets are formed randomly and symmetrically, Johnen and Ronayne (2021) showed the single-stage pricing game has a unique equilibrium if and only if this holds. Our exposition simplifies appreciably if we maintain this property as an assumption, and so (for convenience, and for most of the paper) we do so. Our key claims hold without it, but exposition becomes tedious.

Solution Concept. Our (novel) pricing concept captures the idea that it can be relatively easy for a firm to use a promotion or special deal to respond to any price movements by competitors. Prices are "stable" if no firm can gain by offering such a deal to undercut a cheaper competitor, and if no firm wishes to raise its price given that others may subsequently respond by offering special deals.

[11] If i is not considered with another firm, then i's audience is entirely captive and it sets the monopoly price. If there are no captive customers, then Bertrand competition forces all profits down to zero.

[12] The latter feature guarantees that a single-stage pricing game with symmetric firms has (infinitely) many equilibria. Under asymmetry, two (smallest) firms mix, while others charge v (Baye, Kovenock and de Vries, 1992).

[13] It is Assumption 3 of Armstrong and Vickers (2022).

Definition (Stable Prices). *Fix a profile of prices where* $p_i \in [0, v]$ *is the regular price of firm* i*. We define a* **price-cutting game** *as a simultaneous-move game in which risk-neutral profit-maximizing firms simultaneously choose to offer special deals where a price* $\tilde{p}_i \in [0, p_i]$ *is the offer of firm* i*.*

Given this definition, we say that the profile of prices is **stable** *if it satisfies these two criteria.*

(1) No firm gains from a special deal that undercuts a cheaper opponent. Formally, the associated price-cutting game has a Nash equilibrium in which all firms set $\tilde{p}_i = p_i$*.*

(2) No firm gains from a price rise, given that all firms have a subsequent opportunity to offer special deals. Formally, following an adjustment in the price of a firm i*, there is a Nash equilibrium of the price-cutting game that gives a weakly lower expected profit to firm* i*.*

A profile is **weakly stable** *if we weaken criterion (2) by requiring that no firm gains from a sufficiently small price rise, given that others can respond with special deals.*

A profile is **strongly stable** *if we strengthen criterion (2) by requiring that all Nash equilibria of price-cutting games that follow a price adjustment result in a weakly lower profit for firm* i*.*

Condition (1) (seeking "undercut proof" prices) also holds for any pure-strategy Nash solution to a simultaneous-move pricing game. A Nash profile of prices (if it were to exist) has the property that there is no profitable upward movement in price. If a firm were to make such a move, then in the follow-on price-cutting game there would be an equilibrium in which the deviant firm reverses its deviation. That means any Nash prices satisfy (2) and so would be stable (if they existed).

Condition (2) modifies the "no profitable deviation" property of (pure) Nash prices by giving competitors the opportunity to respond to upward (or, indeed, to any) movements in a firm's price by offering special deals.[14] We seek not only to eliminate profitable undercutting opportunities (our first condition) but also anything that creates such opportunities for others that ultimately hurts the deviating firm (our second condition).[15] The three variations of our second condition are nested. The looser "weak stability" criterion allows us to study situations in which there may be other constraints (such as those documented in our introductory remarks) to larger upward price rises. More pragmatically, we are able to prove results for a broader range of consideration-set specifications and so demonstrate the reach of our approach when we seek only weak stability. We find the more demanding "strong stability" criterion is met under more structured classes of consideration, including when firms are suitably symmetric in the sense of "exchangeability" (which we define and study in Section 4) and under "independent awareness" (studied in Section 5).

[14]In a previous version we called the "weak" requirement (no firm gains from a small price rise) "creep resistance."

[15]We could instead wrap the first condition into the second so that a deviation either up or down from a price triggers a price-cutting game. However, we find the separation into two provides a clearer guide for deriving stable prices.

3. Characterizing Stable Prices

In this section we characterize the price profiles that can satisfy our stability criteria by identifying a unique such profile for each possible ordering of firms' prices. We derive a simple sufficient condition for the existence of a weakly stable price profile, followed by sharper results for a duopoly.

Maximal Undercut-Proof Prices. Our first stability criterion is to seek "undercut proof" prices. One such profile is the trivial profile of zero prices. On the other hand, some profiles are never undercut-proof: if there are ties of positive prices then they are pairwise compared (given the maintained "twoness" assumption) which generates an incentive to undercut.[16] We conclude that any profile of strictly positive stable (and so undercut-proof) prices must be entirely distinct.

We now (without loss of generality) label firms in decreasing order of price, so that $p_1 \geq \cdots \geq p_n$, where (as an implication of the argument above) the inequality $p_i \geq p_{i+1}$ is strict if $p_i > 0$.

If firm j charges $p_j > 0$ then it wins all price comparisons that involve only firms from the j most expensive. These are the consideration sets $B \subseteq \{1, \ldots, j\}$. We need to include only those in which firm j is considered, which is achieved via the indicator variable $B_j \in \{0, 1\}$. Hence firm j earns $p_j \sum_{B \subseteq \{1,\ldots,j\}} B_j \lambda(B)$. If firm j undercuts a cheaper firm $i > j$ then it wins all price comparisons which involve the i most expensive firms. To avoid a profitable undercut we need $p_j \sum_{B \subseteq \{1,\ldots,j\}} B_j \lambda(B) \geq p_i \sum_{B \subseteq \{1,\ldots,i\}} B_j \lambda(B)$, or equivalently

$$p_i \leq \frac{p_j \sum_{B \subseteq \{1,\ldots,j\}} B_j \lambda(B)}{\sum_{B \subseteq \{1,\ldots,i\}} B_j \lambda(B)}. \tag{1}$$

This must hold for every $j < i$, which gives the characterization of eq. (2) in Lemma 1 below.

Lemma 1 (Undercut-Proof Prices). *Label firms in price order, so that* $p_1 \geq \cdots \geq p_n$.

(i) Any profile of strictly positive undercut-proof prices is strictly ordered: $p_1 > \cdots > p_n > 0$.

(ii) A profile of prices is undercut-proof if and only if

$$p_i \leq \min_{j \in \{1,\ldots,i-1\}} \left\{ \frac{p_j \sum_{B \subseteq \{1,\ldots,j\}} B_j \lambda(B)}{\sum_{B \subseteq \{1,\ldots,i\}} B_j \lambda(B)} \right\} \quad \textit{for all } i \in \{2, \ldots, n\}, \tag{2}$$

(iii) For any such profile of undercut-proof prices, the associated price-cutting game is dominance solvable, and the unique Nash equilibrium of such a game satisfies $\tilde{p}_i = p_i$ *for all* i.

(Any proof of formal result statements beyond the argument in the text is reported in Appendix A.)

[16] More generally, if a "k-ness" property holds, so that any consideration set containing $k > 1$ firms has positive mass (formally: $|B| = k \Rightarrow \lambda(B) > 0$) then at most $k - 1$ strictly positive undercut-proof prices can be tied.

Of the many undercut-proof profiles, a focal profile is that to which would firms collectively agree. Presumably, they wish prices to be as high as possible, and among such maximal profiles, would prefer those with superior industry profits. Maximal undercut-proof prices are readily identified. Positive undercut-proof profiles remain so if we raise the highest price to $p_1 = v$. We then iteratively raise each successively lower price so that eq. (2) binds. Each price rise is an improvement for the industry because it does not alter the allocation of sales to firms. Lemma 2 summarizes.

Lemma 2 (Maximal Prices). *We define the* **maximal undercut-proof prices** *for an ordering of firms to be those that are higher than all other undercut-proof profiles for that firm order. They are*

$$p_1 = v, \text{ and iteratively, } p_i = \min_{j \in \{1,\dots,i-1\}} \left\{ \frac{p_j \sum_{B \subseteq \{1,\dots,j\}} B_j \, \lambda(B)}{\sum_{B \subseteq \{1,\dots,i\}} B_j \, \lambda(B)} \right\} \text{ for all } i \in \{2,\dots,n\}. \quad (3)$$

An undercut-proof profile is **industry optimal** *if there is no other (Pareto) superior (for profits) undercut-proof profile. Such profiles are a (non-empty) subset of the* $n!$ *profiles defined via* (3)*.*

We will connect maximal and industry-optimal price profiles to our notions of price stability.

Stable Prices. Weakly stable prices (and so those meeting stricter criteria) are undercut-proof, and so in the associated price-cutting game there is (Lemma 1) a unique Nash equilibrium without special-deal offers. Mirroring our discussion of industry profitability, if prices are not maximal then a firm can nudge upward its price while maintaining a "no special deals" equilibrium. This generates a strict gain for the deviating firm. This focuses our attention on maximal prices.

There are (at most) $n!$ maximal undercut-proof price profiles. Fixing a profile (and so ordering of firms) the maximal prices satisfy the binding no-undercutting constraints of eq. (3). Each constraint checks which firm $j < i$ is most tempted to undercut firm i. If one of the binding "temptation" constraints is generated by the firm immediately above, so that firm $i-1$ is (one of the) most tempted to undercut firm i, then we are able to show resistance to a small upward price movement.[17]

Proposition 1 (Necessary and Sufficient Conditions for Weakly Stable Prices).

(i) **(Necessity)** *A weakly stable price profile comprises maximal undercut-proof prices.*

(ii) **(Sufficiency)** *Fix an order of firms. Consider the unique maximal undercut-proof prices. If*

$$i-1 \in \arg\min_{j \in \{1,\dots,i-1\}} \left\{ \frac{p_j \sum_{B \subseteq \{1,\dots,j\}} B_j \, \lambda(B)}{\sum_{B \subseteq \{1,\dots,i\}} B_j \, \lambda(B)} \right\} \quad \text{for all } i \in \{2,\dots,n\}, \quad (4)$$

so that $i-1$ *is one of the most tempted to undercut* i*, then the profile is weakly stable.*

[17] The proof constructs a price-cutting equilibrium following a price rise by some firm $i > 1$ which gives it a profit equal to what it earns under the original prices. In this equilibrium firms i and $i-1$ mix over an interval including p_i.

Duopoly. We now fully develop Proposition 1 for a duopoly: $\lambda_i > 0$ customers are "captive" to i while $\lambda_S \equiv \lambda(\{1,2\}) > 0$ "shoppers" compare prices. There are two possible rankings of the firms' prices. Choosing labels so that $p_1 \geq p_2$ the maximal undercut-proof prices are

$$p_1 = v \text{ and } p_2 = \frac{v\lambda_1}{\lambda_1 + \lambda_S} \text{ which yield profits of } \pi_1 = v\lambda_1 \text{ and } \pi_2 = v\lambda_1 \frac{\lambda_2 + \lambda_S}{\lambda_1 + \lambda_S}, \tag{5}$$

so that the high-price firm earns its "captive only" profit while the low-price firm earns (strictly) more or less than its captive-only profit according to whether it has a (strictly) smaller or larger captive audience. If $\lambda_1 > \lambda_2$ then eq. (5) offers strictly more profit for both firms than if we flipped their order: industry-optimal prices allocate the firm with more captives to the high-price position.

The condition of eq. (4) is satisfied for a duopoly, given that there is only one firm that can be tempted to undercut a rival, and so both maximal undercut-proof price profiles are weakly stable. Specifically, consider an increase $\Delta > 0$ in the second (and cheaper) firm's price. If $\Delta > 0$ is not too large then the price-cutting game has a unique Nash equilibrium in which both firms mix continuously over the interval $[p_2, p_2 + \Delta)$ with residual atoms at v and $p_2 + \Delta$ respectively. Firms earn expected profits equal to those in eq. (5). Their mixed strategies are

$$F_1(p) = 1 - \frac{\lambda_S(\lambda_1 - \lambda_2)v + \lambda_2(\lambda_1 + \lambda_S)(v - p)}{(\lambda_1 + \lambda_S)\lambda_S p} \text{ and } F_2(p) = 1 - \frac{\lambda_1(v - p)}{\lambda_S p}. \tag{6}$$

If $\lambda_1 \geq \lambda_2$ then these distributions characterize the unique mixed-strategy Nash equilibrium for any $\Delta \leq v - p_2$: it is straightforward to check that $F_1(v) = (\lambda_2 + \lambda_S)/(\lambda_1 + \lambda_S) \leq 1$ and $F_2(v) = 1$. This is a stronger result: the "industry optimal" prices are fully (in fact, strongly) stable.

Notably, this is not true if $\lambda_1 < \lambda_2$. In that case, the mixed-strategy solution (above) fails to satisfy $F_1(p) \leq 1$ if p is sufficiently close to v, which is relevant if Δ is sufficiently large. In this (industry-suboptimal) case the second (and cheaper) firm with the larger captive audience would prefer to raise its price to v, even if both firms then play a price-cutting game.

Proposition 2 (Weakly and Strongly Stable Prices in a Duopoly). *Consider a duopoly.*

(i) Both maximal undercut-proof price profiles are weakly stable.

(ii) The (unique) industry-optimal undercut-proof price profile is (uniquely and) strongly stable.

Claim (ii) predicts a unique pair of stable prices. However, in our construction the low price firm is indifferent to a price rise. Here we discuss briefly two possible sources of strict preference.

Firstly, the indifference holds only if both firms can offer special deals in response to a price rise by the cheaper firm. If (instead of our all-play price-cutting game) the rival were uniquely able to respond in the triggered price-cutting game, then it would undercut the deviant, which would then lose its sales to shoppers.[18] More generally, if there is a small chance that the deviant firm is unable to participate in special-offer responses, then the stable price pair is strictly resistant to upward price movements.[19] As such, our choice of price-cutting game (all-play and single-stage) is a conservative one that makes the profit from a deviation from a stable price as high as possible.

Secondly, stable prices offer certainty. The entry into a price-cutting game generates the same expected profits, but those profits are risky. Specifying risk aversion when a firm contemplates a price rise can readily generate a strict preference for maintaining stable price positions.[20]

Discussion. To satisfy (the three variations of) our criteria for stability we may focus on the maximal prices from the $n!$ possible ordering of firms. The duopoly case illustrates that not all such profiles are industry optimal and yet the maximal prices from a sub-optimal order of firms' prices satisfy our weak stability definition. Happily, however, full (or even strong) stability (or a requirement for industry optimality) pins down a unique pair of prices. Ideally, we would offer a similar result for a general oligopoly with an arbitrary specification of consideration.

For this we would need to consider fully the properties of general n-firm price-cutting games in which the participating firms offer special deals relative to possibly different regular prices. Such games include the conventional models of price competition with heterogenous buyer consideration (that is, extensions of the classic model of sales) in which firms begin with the same regular price. We inherit, therefore, the substantial analytic complexity found in that established approach, in which the key primitive is a distribution over 2^n possible consideration sets.

The literature does not contain a solution of the classic pricing game under this level of generality. Narasimhan (1988) solved the duopoly case, but only recently have we seen a full analysis of triopoly from Armstrong and Vickers (2022); the step from $n = 2$ to $n = 3$ involves substantial intricacies. For asymmetric consideration distributions they considered $n > 3$ only for some special cases such as the nested consideration of firms, but studied (in their Section 3) general symmetric consideration specifications. For asymmetric cases, other papers have made progress by specifying that buyers are independently aware of each firm (Ireland, 1993; McAfee, 1994) or by considering a pure captive-and-shopper specification (Baye, Kovenock and de Vries, 1992).

[18] The unique best reply is to match the deviant price with the tie broken in the responder's favor. The endogenous settlement of the tie-break à la Simon and Zame (1990) solves a standard best-reply existence issue.

[19] In contrast, if there is a chance that the deviant firm's rival (in this case, the high-price firm) is unable to engage in a special-offer response, then there is an incentive for the low-price firm to raise its price. In Appendix B.1 we develop a brief extension to our model which allows for different probabilities that firms are able to use special offers.

[20] We develop this idea explicitly in Appendix B.2. Elsewhere (Myatt and Ronayne, 2025) we discuss briefly a risk-averse industry association that chooses collusive prices to maximize industry profit while anticipating possible secret price cuts by its members. Such an association strictly prefers the industry-optimal undercut-proof price profile.

Our first finding (Proposition 1) imposes little structure on the distribution over consideration sets. However, it offers a result only for weak stability, and it does not offer further guidance on when the sufficient condition for weak stability is likely to be satisfied. We now (over the next two sections) impose more structure across two broad classes of specification in order to develop such guidance to obtain conditions (just as we did for our duopoly analysis) for full price stability.

Our first class of consideration distributions (in Section 4) allows for complete flexibility in the distribution over how many prices buyers compare, while assuming firms are symmetrically considered in the aggregate. A generalized version allows additionally for asymmetric captive bases.[21] The second broad class (in Section 5) allows for firms to be considered by asymmetric shares of buyers, but buyers' considerations of different sellers are independent. Overall, our generality broadly aligns with that seen in the bulk of the literature on simultaneous-move pricing games.

4. Exchangeability

Here we allow differently sized consideration sets to have arbitrary masses, but all (non singleton) consideration sets of the same size have the same mass. We call this property *exchangeability*.

Exchangeable Consideration Sets. In addition to captive customers, $\lambda_i > 0$ for each i, $I_m \geq 0$ customers consider $m \in \{2, \dots, n\}$ prices. Such consideration is random and symmetric across firms so that the mass I_m comprises equal shares of every combination of m firms: consideration sets of the same size have equal mass. For $B \subseteq \{1, \dots, n\}$ with $|B| \geq 2$ members,

$$\lambda(B) = I_{|B|} \Big/ \binom{n}{|B|} \tag{7}$$

Under this specification, firms can differ by the size of their captive audiences, but are otherwise *exchangeable*. We use the term *fully exchangeable* when $\lambda_i = \lambda_j$ for all $i \neq j$.[22]

An interpretation is that non-singleton consideration sets arise from shoppers who obtain quotations via a search technology that does not bias toward any firm. On the other hand, the singleton consideration sets include some (possibly different in mass) local, loyal, or non-shopper customers who are exogenously locked in to a specific supplier. We retrieve the classic captive-shopper setting with $\lambda_S \equiv I_n$ and $I_m = 0$ for $1 < m < n$. However exposition is smoother if we abstract (for now) from zero masses of comparison shoppers and set $I_m > 0$ for all m.[23]

[21] This also allows us to study the (general, asymmetric version of the) classic captive-shopper configuration.

[22] The full exchangeability setting was analysed with a single-stage game by many (for example, Burdett and Judd, 1983; Johnen and Ronayne, 2021; Nermuth et al., 2013).

[23] Those zero-mass cases are cumbersome to carry, but our results extend naturally (continuously) to them. We return to cover the pure captive-shopper specification at the end of this section.

Maximal and Industry-Optimal Undercut-Proof Prices. Because $I_2 > 0$, the "twoness" property holds. We know, therefore, that any profile of maximal undercut-proof prices contains n distinct (and positive, because $\lambda_i > 0$ for all i) prices. We label the firms so that $p_1 > \cdots > p_n > 0$. It is convenient to denote by X_i, the mass of customers (excluding captives) buying from $i \geq 2$:

$$X_i \equiv \sum_{m=2}^{i} I_m \left[\binom{i-1}{m-1} \Big/ \binom{n}{m} \right] \quad \text{and so using this notation} \quad \sum_{B \subseteq \{1,\ldots,i\}} B_i \lambda(B) = \lambda_i + X_i. \tag{8}$$

The term X_i sums over the relevant consideration-set sizes (no sale is made if $m > i$ because then m is cheaper than i). For each m, there are $\binom{n}{m}$ equally-sized consideration sets. Firm m makes a sale only if compared to $m-1$ others from the $i-1$ competitors with higher prices. There are $\binom{i-1}{m-1}$ such sets. We define $X_1 = 0$ for completeness.

To find the maximal undercut-proof prices for this ordering of firms, we can apply Lemma 2:

$$p_1 = v \quad \text{and} \quad p_i = \min_{j<i} \left\{ p_j \frac{\lambda_j + X_j}{\lambda_j + X_i} \right\} \quad \text{for } i > 1. \tag{9}$$

Because cheaper firms have more sales ($X_j < X_i$), the term $(\lambda + X_j)/(\lambda + X_i)$ increases in λ. This means that a firm with fewer captives has a greater incentive to undercut. To keep prices high, therefore, it is helpful to place larger firms (with more captives) higher in the ladder of prices. This also pushes captive customers to higher prices. This suggests that industry optimality will order firms so that $\lambda_1 \geq \cdots \geq \lambda_n$: firms with more captives are more expensive.

Our proof of this claim (in Appendix A) works by contradiction. We proceed down the list of firms until we find the lowest k where $\lambda_k < \lambda_{k+1}$. We can then show that switching the positions of those two firms results in a superior profile for the industry. Once this is established, we can also show that the binding "no undercutting" constraint for each price p_i is the one corresponding to firm $i-1$ undercutting firm i. We can then use eq. (9) to solve recursively for prices.

Lemma 3 (Industry-Optimal Prices under Exchangeability). *For the exchangeability setting:*

(i) An industry-optimal undercut-proof price profile orders firms so that higher prices are charged by firms with more captive customers. Given the order $\lambda_1 \geq \cdots \geq \lambda_n$, those prices are

$$p_i = \begin{cases} v & \text{if } i = 1 \\ v \displaystyle\prod_{j=2}^{i} \frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j} & \text{if } i \geq 2. \end{cases} \tag{10}$$

(ii) This profile is unique if firms have differently sized captive audiences so that $\lambda_1 > \cdots > \lambda_n$.

In Appendix C we supplement this with the more general Lemma A2, which shows that the prices identified by Lemma 3 are upper bounds for any maximal prices, and so for any stable prices.

Stable Prices. Suppose that captive masses are distinct and label firms so that $\lambda_1 > \cdots > \lambda_n$. Lemma 3 maps firms to prices, uniquely: firms with more captives are more expensive. If there are ties, then the map is unique subject to changes in order for pairwise symmetric firms. In either case, the conditions that we need to apply Proposition 1 are met.

Specifically, suppose that firm $k > 1$ increases its price to $p_k + \Delta \in (p_k, p_{k-1}]$. For the associated price-cutting game, we construct the following (asymmetric) mixed-strategy profile:

$$F_{k-1}(p) = \frac{(p - p_k)(\lambda_k + X_k)}{p(X_k - X_{k-1})} \quad \text{and} \quad F_k(p) = \frac{(p - p_k)(\lambda_{k-1} + X_k)}{p(X_k - X_{k-1})}, \tag{11}$$

which generates pre-deviation expected profits. The firms place any remaining mass at their initial prices. Each firm $i < k - 1$ has more captive customers than $k - 1$ and k, and does not have a profitable deviation into the interval in which $k - 1$ and k mix. This leads us to Proposition 3.

Proposition 3 (Weakly Stable Prices under Exchangeability). *The (unique, if captive-audience sizes differ) industry-optimal undercut-proof profile of prices is weakly stable.*

We can say more when firms are fully exchangeable, so that $\lambda_i \equiv \lambda > 0$ for all i. Lemma 3 applies, and, because firms are symmetric, there is only one maximal profile (albeit we cannot predict which firm charges which price). This industry-optimal profile is weakly stable. However, we can say more by allowing for a large increase in price (up to the monopoly price v) and by considering the expected profit that a firm can achieve in any equilibrium of the associated price-cutting game.

Lemma 4 (Profits in Price-Cutting Games: Full Exchangeability). *Under full exchangeability, so that every firm has a captive audience of size λ. For prices $p_1 \geq \ldots \geq p_n$, consider the price-cutting game in which each firm i chooses a price $\tilde{p}_i \in [0, p_i]$. In equilibrium, the expected profit of each firm is bounded above by $\lambda p_1 \leq \lambda v$, and so no firm earns more than its captive-only profit.*

We sketch the proof here. In an equilibrium consider a firm that charges (using an atom, if an atom-playing firm exists) the highest price from the joint support of all firms. Such a firm setting this price sells only to "captive" customers and so earns at most $p_1 \lambda$. This is weakly more than any other competitor, owing to a "profit stealing" argument: given the symmetric specification, the highest-price firm can earn (at least) the profit of a competitor by undercutting the lower bound of the mixed-strategy support of that rival. Whereas this argument cannot be used for all consideration specifications, it does extend (in Lemma 6) to the asymmetric cases we exhibit in Section 5.

Under full exchangeability, consider the unique (again, subject to firms' labels) profile of prices described in Lemma 3. Such prices generate the captive-only profit λv for each firm. From Lemma 4, this profit is the highest that a firm can ever achieve in a Nash equilibrium of a relevant price-cutting game. This immediately implies that no firm can gain from raising its price.

Proposition 4 (Strongly Stable Prices under Full Exchangeability). *Under full exchangeability, the unique maximal undercut-proof profile of prices is strongly stable.*

Lemma 4 places an upper bound on the profit of a deviant firm, and from this the stated result follows straightforwardly. We can, however, construct an equilibrium of the price-cutting game in which a deviant firm achieves that upper bound following an increase in its price.[24] This implies (just as in the duopoly case from Section 3) that a firm has only a weak incentive to avoid an increase in its price. One possibility, therefore, is that we might see higher prices and subsequent randomized special deals. Another possibility is that the factors discussed in Section 3 apply: if there is either risk aversion of if there is the possibility that a deviant firm cannot participate in special deals, then the incentive to maintain the (strongly) stable prices becomes strict.

Limits to Stability under Exchangeability. We might ask whether we can extend Proposition 4 to establish strong stability of maximal undercut-proof prices when firms have differently sized captive audiences. This is sometimes true, but we can also find a counter-example.

Consider a triopoly with $\lambda_1 > \lambda_2 > \lambda_3 > 0$ and so industry-optimal undercut-proof prices

$$p_1 = v, \quad p_2 = \frac{v\lambda_1}{\lambda_1 + X_2}, \quad \text{and} \quad p_3 = \frac{v\lambda_1}{\lambda_1 + X_2}\frac{\lambda_2 + X_2}{\lambda_2 + X_3}. \tag{13}$$

We can handle any upward deviation in the price of firm 2 (where only the first two firms mix in a price-cutting game). Similarly, if firm 3 deviates upward to $\hat{p}_3 \in (p_3, p_2]$ then, similar to the approach used to show weak stability, we construct the following mixed-strategy profile,

$$F_2(p) = \frac{(\lambda_3 + X_3)(p - p_3)}{p(X_3 - X_2)} \quad \text{and} \quad F_3(p) = \frac{(\lambda_2 + X_3)(p - p_3)}{p(X_3 - X_2)}, \tag{14}$$

which generates the required expected profits for both firms across this range. These functions are increasing from $F_2(p_3) = F_3(p_3) = 0$, and for higher prices they satisfy satisfy $F_2(p) < F_3(p) \leq F_3(p_2) = 1$. The distributions are completed by placing remaining mass at firms' regular prices.

[24] Consider an increase by firm $k > 1$ (there is no possible upward deviation by the highest price firm) to $\hat{p}_k > p_k$. The most straightforward case is when $\hat{p}_k \in (p_k, p_{k-1}]$ so that the ordering of prices is maintained. We can construct an equilibrium in which firms k and $k-1$ continuously (and symmetrically) mix over the interval $[p_k, \hat{p}_k)$ via

$$F_k(p) = F_{k-1}(p) = \frac{(\lambda + X_k)(p - p_k)}{p(X_k - X_{k-1})}, \tag{12}$$

which (by construction) gives the firms the expected profit λv. If $\hat{p}_k = p_{k-1}$, the solution is continuous up to the common upper bound, and satisfies $F_k(p_{k-1}) = F_{k-1}(p_{k-1}) = 1$. If $\hat{p}_k < p_{k-1}$ then the firms place residual mass at their regular prices. A more complex case is if k deviates further upward. We discuss an example here: $\hat{p}_k = p_{k-2}$. We can construct an equilibrium in which the three firms $k-2$, $k-1$, and k all mix (symmetrically) over the interval $[p_k, p_{k-1})$. Firm $k-1$ then places an atom with remaining mass at its constraining initial price p_{k-1}. Firms k and $k-2$ then begin mixing again at some price $p^\dagger \in (p_{k-1}, p_{k-2})$, and the construction continues. We can repeat this process similarly for higher deviations. (There are complexities here. For example, if k deviates to just above a higher price then we construct an equilibrium in which one firm stops mixing partway through the support of mixing prices.)

Now suppose that firm 3 raises its price to $\hat{p}_3 \in (p_2, p_1]$, and so "leapfrogs" the price of firm 2. We can build an equilibrium of the associated price-cutting game in which firms 2 and 3 mix over $[p_3, p_2)$ according to the distribution functions reported in eq. (14) above and where firm 2 places remaining mass at p_2. We have noted, however, that firm 3's distribution satisfies $F_3(p_2) = 1$. Nevertheless, its higher regular price means that firm 3 is able to price above p_2. Indeed, if it chooses $p > p_2$ (sacrificing the capture of the atom played by firm 2) then it will optimally move all the way up $p = \hat{p}_3$. It earns this price on λ_3 captive customers and X_2 customers with the consideration set $\{1, 3\}$, and so receives an expected profit of $\hat{p}_3(\lambda_3 + X_2)$. For our equilibrium construction to work, this must be less than $p_3(\lambda_3 + X_3)$. The deviant price can be as high as $p_1 = v$, and so for our equilibrium construction we need

$$v(\lambda_3 + X_2) \leq p_3(\lambda_3 + X_3) \quad \Leftrightarrow \quad \frac{\lambda_3 + X_2}{\lambda_3 + X_3} \leq \frac{\lambda_1}{\lambda_1 + X_2} \frac{\lambda_2 + X_2}{\lambda_2 + X_3}. \tag{15}$$

This holds if X_2 is sufficiently small. If X_2 approaches zero then this exchangeable triopoly becomes a model of sales: if we are close to a captive-shopper model then industry-optimal prices are stable. However, eq. (15) fails if λ_2 and λ_3 are close. (It strictly fails if $\lambda_2 = \lambda_3$.)

This discussion suggests that we might find circumstances (for example, $\lambda_1 > \lambda_2 \approx \lambda_3$) in which industry-optimal prices are not strongly stable. The procedure above relies upon a "tango" of pairwise mixed strategies danced by firms 2 and 3. For our fully exchangeable specification, we can deal with higher price deviations (of the "leapfrog" kind documented here) by constructing equilibria in which firms with higher regular prices join in. (Our result did not require us to perform the equilibrium construction, but in Appendices A and C we do so for completeness.) Here, this procedure does not work: firm 1 (with the largest captive audience) is strictly unwilling to mix down to p_3. The proof of the result that follows confirms that if eq. (15) fails then we cannot construct a suitable equilibrium of the relevant price-cutting game, and in fact we construct another equilibrium of that game which is strictly better for the deviant.

Proposition 5 (An Exchangeable Triopoly). *In an exchangeable triopoly, the industry-optimal undercut-proof prices are (strongly) stable if and only if*

$$\frac{\lambda_3 + X_2}{\lambda_3 + X_3} \leq \frac{\lambda_1}{\lambda_1 + X_2} \frac{\lambda_2 + X_2}{\lambda_2 + X_3}. \tag{16}$$

This holds if the mass of customers who conduct pairwise comparisons is sufficiently small, but it fails if the masses of captive customers for the second and third firms are sufficiently similar.

Models of Sales. The exchangeable specification can encompass the model of sales (Varian, 1980). However, we simplified exposition by setting $I_m > 0$ for all m. Nevertheless, we can apply our results to specifications that are arbitrarily close to the classic model.

Fix a model of sales with asymmetric captive audiences: $\lambda_1 > \cdots > \lambda_n$. We specify exchangeable models indexed by $\varepsilon > 0$ where, using obvious notation, (i) $\lambda_i^\varepsilon = \lambda_i$; (ii) $\lambda_S^\varepsilon = \lambda_S$; and (iii) $0 < I_m^\varepsilon \leq \varepsilon$ for $m \in \{2, \ldots, n-1\}$. This converges to a model of sales as $\varepsilon \to 0$.

Recall from eq. (8) that X_i is the mass of non-captive customers who buy from firm i. Inspecting that expression, notice that $\lim_{\varepsilon \to 0} X_i^\varepsilon = 0$ for all $i \in \{2, \ldots, n-1\}$, and so in the model-of-sales limit ($\varepsilon \to 0$) only firm n serves non-captive customers. Similarly, an inspection of eq. (10) from Lemma 3 shows that for industry-optimal undercut-proof prices

$$\lim_{\varepsilon \to 0} p_i^\varepsilon = v \quad \text{for} \quad i < n \quad \text{and} \quad \lim_{\varepsilon \to 0} p_n^\varepsilon = p_{n-1}^\dagger \quad \text{where} \quad p_i^\dagger \equiv \frac{v\lambda_i}{\lambda_i + \lambda_s}. \tag{17}$$

Here $p_i^\dagger$ is the lowest undominated price for firm i: by setting this price and serving all shoppers, it earns the same as it does from exploiting its captive audience at the monopoly price v. Equation (17) says that the n distinct prices collapse to two points: the smallest firm (in terms of captive audience) charges a distinct lowest price, while all other firms serve only their captives.

Our key results extend (continuously) to an exact ($\varepsilon = 0$) model-of-sales specification.

Proposition 6 (Stable Prices for a Model of Sales). *Consider an asymmetric model of sales.*

(i) There is a unique industry-optimal undercut-proof profile in which firm n, with fewest captives, sets $p_n = p_{n-1}^\dagger$ and others price at v. Each firm $i < n$ earns its "captive only" profit $v\lambda_i$. Firm n earns more: $p_{n-1}^\dagger(\lambda_n + \lambda_S) > v\lambda_n$. This profile is uniquely and strongly stable.

(ii) Any other profile of maximal undercut-proof prices is weakly stable.

The mixed strategies of Varian (1980) were intended to capture "sales." Realized prices may be either close or far apart, and the identity of the cheapest firm is uncertain.[25] In contrast, we predict $n-1$ "regular" prices alongside one starkly-lower "bargain" price from the firm with fewest captive customers. Of course, over time there can be changes in which firm has the fewest captives, which changes the identity of the bargain firm. Such shifts can be more frequent if the sizes of captive audiences are relatively close, and so their order can change more easily. In that sense, we retain the spirit of sales, while providing new insights and predictions.

[25] With asymmetric captive shares, this is true for the two firms who "tango" while $n-2$ others maintain the "regular" monopoly price (Baye, Kovenock and de Vries, 1992, Section V). In particular, $i \in \{1, \ldots, n-2\}$ charge $p_i = v$ while n and $n-1$ mix over $[p_{n-1}^\dagger, v)$ with firm $n-1$ placing an atom at v. When marginal costs and captive shares are asymmetric, the analysis is substantially complicated: there is (generically) a unique equilibrium in which the (cost and captive) parameters dictate how many firms mix, which can be any number, up to n (Myatt and Ronayne, 2023).

5. Independent Awareness

Exchangeability allows generality in the mass of customers who consider a particular number of firms. It also allows consideration to be correlated: in the captive-shopper case, a customer who sees more than one firm sees them all. However, it does impose substantial symmetry. Here we restrict the correlation of firms in consideration sets, but allow for much greater generality in asymmetries. We build upon prior and classic work including Butters (1977), Grossman and Shapiro (1984), Ireland (1993), McAfee (1994), and Eaton, MacDonald and Meriluoto (2010): each price is exposed to an independent (but asymmetric) fraction of potential customers.

Consideration Sets. On the demand side, a fraction $\alpha_i \in (0,1)$ of customers is *independently aware* of firm i.[26] The mass of customers who consider firms $B \subseteq \{1,\dots,n\}$ is

$$\lambda(B) = \left(\prod\nolimits_{i\in B}\alpha_i\right)\left(\prod\nolimits_{i\notin B}(1-\alpha_i)\right) = \prod_{i=1}^{n}\alpha_i^{B_i}(1-\alpha_i)^{1-B_i}. \tag{18}$$

We say that firm i is larger than firm j if it enjoys greater awareness, so that $\alpha_i \geq \alpha_j$.[27]

Maximal and Industry-Optimal Undercut-Proof Prices. The "twoness" property holds and so maximal undercut-proof prices are again distinct (and positive): $p_1 > \cdots > p_n > 0$. Applying Lemma 2 to obtain those prices, we find that all no-undercutting constraints bind simultaneously. To see why, note that firm j receives p_j from customers aware of it, true with probability α_j, and who are aware of no cheaper firm, true with probability $\prod_{k>j}(1-\alpha_k)$, giving profit $p_j\alpha_j\prod_{k>j}(1-\alpha_k)$. If j undercuts $i>j$ then it gets p_i (or close to it) from customers who consider j and no firm cheaper than i. That gives j a profit of (or arbitrarily close to) $p_i\alpha_j\prod_{k>i}(1-\alpha_k)$. We need:

$$p_i\alpha_j\prod_{k>i}(1-\alpha_k) \leq p_j\alpha_j\prod_{k>j}(1-\alpha_k) \quad\Leftrightarrow\quad p_i \leq p_j\prod_{k=j+1}^{i}(1-\alpha_k). \tag{19}$$

This inequality does not involve α_j, and so "no undercutting" constraints do not depend on the firm considering the undercut, but instead the awarenesses of firms being undercut are relevant.[28]

A special case is the local constraint: $p_i \leq p_{i-1}(1-\alpha_i)$. Raising prices as high as possible, we obtain $p_1 = v$ and iteratively $p_i = p_{i-1}(1-\alpha_i)$ for $i>1$. This generates claim (i) of Lemma 5.

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[26] If two or more firms enjoy complete awareness, $\alpha_i = 1$, then the Bertrand (zero profit) outcome follows. Allowing (at most) one firm to be known to all customers does not affect our results (and is relevant to some results under endogenous advertising). But for smoother exposition, we carry $\alpha_i \in (0,1)\,\forall i$ forward in the text.

[27] The special case of symmetry ($\alpha_i = \alpha_j$ for all i,j) falls within the full exchangeability specification of Section 4.

[28] This contrasts with our exchangeability specification under which it is the type of the firm contemplating the undercut that matters: local no-undercutting constraints take the form $p_i \leq p_{i-1}(\lambda_{i-1}+X_{i-1})/(\lambda_{i-1}+X_i)$. Note that this depends on the type (λ_{i-1}) of firm $i-1$, which is the firm that contemplates the undercut.

Lemma 5 (Industry-Optimal Undercut-Proof Prices for Independent Awareness).

(i) *Maximal undercut-proof prices are* $p_1 = v$ *and* $p_i = v\prod_{j=2}^{i}(1-\alpha_j)$ *for* $i > 1$*. Every firm* j *is indifferent to undercutting any other firm* $i > j$*. The profit of firm* i *is* $\pi_i = v\alpha_i \prod_{j=2}^{n}(1-\alpha_j)$*.*

(ii) *The industry-optimal undercut-proof profiles order firms' prices so that the largest firm charges the monopoly price* v*. All orders satisfying this generate the same profits for every firm.*

The profit expressions reported in Lemma 5 are of particular interest. Note that

$$\pi_i = v\alpha_i \prod_{j=2}^{n}(1-\alpha_j) = \frac{v\alpha_i}{1-\alpha_1}\prod_{j=1}^{n}(1-\alpha_j). \tag{20}$$

A firm's profit depends on its own awareness α_i in a natural way. The product term in the second expression ranges over all firms, and does not depend on the order of them. That order influences firm i's profit only via the denominator term $1-\alpha_1$, which depends upon the awareness of the firm at the top of the pricing ladder. The profit of firm i (and of every firm) is increasing in α_1, and this is the only way in which the order of firms influences the profits obtained from maximal undercut-proof prices, giving claim (ii) of Lemma 5.

We can contrast this result with Lemma 3 from our exchangeability specification. There, we found (at least for strictly asymmetric firms) a unique industry-optimal undercut-proof profile of prices. Here, however, we identify $(n-1)!$ such profiles (all of which are profit equivalent).

Stable Prices. For profiles identified by Lemma 5 and for $i > 1$, there is a binding no-undercutting constraint from $i-1$ to i. Proposition 1 applies: a maximal undercut-proof profile is weakly stable.

It is instructive to look at equilibrium strategies in a price-cutting game following an adjustment. Suppose that firm $i > 1$ nudges upward its price by $\Delta < p_{i-1} - p_i$. We construct an equilibrium in which firms $j \notin \{i-1, i\}$ charge their regular prices while firms $i-1$ and i continuously mix over $[p_i, p_i + \Delta)$ with distributions $F_j(p) = (1/\alpha_j)(1-(p_i/p))$ for $j \in \{i-1, i\}$, and place remaining mass at p_{i-1} and $p_i + \Delta$ respectively, earning their regular profits.

For larger deviations, this "tango" between firms $i-1$ and i can fail. To see why, suppose that firm i raises its price to match that of firm $i-1$. For our mixing distributions to be valid, we need

$$\max_{j\in\{i-1,i\}} F_j(p_{i-1}) = \frac{1}{\min\{\alpha_{i-1},\alpha_i\}}\left(1-\frac{p_i}{p_{i-1}}\right) = \frac{\alpha_i}{\min\{\alpha_{i-1},\alpha_i\}} \le 1 \quad\Leftrightarrow\quad \alpha_i \le \alpha_{i-1}. \tag{21}$$

This says that the two firms must be in awareness order, with the larger firm at the higher price position. If they are out of order, so that $\alpha_i > \alpha_{i-1}$, then this construction fails.

To resolve the problem of deviations to higher prices, we bring another firm on to the "dance floor." For example, in this case, and if $i > 2$, we can construct an equilibrium in which firms $i-2$, $i-1$, and i mix. The construction is quite complex, and becomes more so for higher prices by the deviant firm i that leapfrogs the prices of other (lower indexed) firms. What is crucial for the construction is to find some more expensive firm that is larger (in terms of awareness) than the deviant firm. For example, if $i = 2$ then we must have $\alpha_1 \geq \alpha_2$ if the construction of the equilibrium is to work. This holds for all possible deviations by all possible deviants if we place the largest firm at the top of the price sequence, so that $\alpha_1 \geq \alpha_i$ for all $i \in \{2, \ldots, n\}$.

More generally, if we do not place the largest firm at the highest price position then this firm can gain from a price rise. To see why, suppose that $\alpha_i > \alpha_1$ so that firm i is strictly larger than the highest-priced firm. Using Lemma 5, i's profit is

$$\frac{v\alpha_i \prod_{j=1}^{n}(1-\alpha_j)}{1-\alpha_1} < \frac{v\alpha_i \prod_{j=1}^{n}(1-\alpha_j)}{1-\alpha_i} = v\alpha_i \prod_{j\neq i}(1-\alpha_j). \tag{22}$$

This last expression is the profit that i achieves by setting $p_i = v$ and selling only to captive customers. From this, we conclude that if we are to construct a profile of fully stable prices then we must order firms with the largest awareness at the top. Equivalently, the only prices that can satisfy our stability criteria are (applying Lemma 5) industry-optimal undercut-proof prices.

Given this necessary condition, we restrict attention to these industry-optimal prices, so that the highest price firm is the largest. Given the labeling of firms in price order, this means that the most expensive firm satisfies $p_1 = v$ and $\alpha_1 \geq \alpha_i$ for all other firms.

We can now apply a "strategy stealing" technique similar to the "profit stealing" approach that we used to obtain Lemma 4, and use that technique to bound profits of firms in price-cutting games.

Lemma 6 (Profits in Price-Cutting Games: Independent Awareness). *Suppose that buyers are independently aware of firm each firm i with probability α_i, and label those firms so that $\alpha_1 \geq \alpha_i$ for all i. Consider the price-cutting game in which each firm i chooses a price $\tilde{p}_i \in [0, p_i]$. In equilibrium, the expected profit of each firm i is bounded above by $v\alpha_i \prod_{j=2}^{n}(1-\alpha_j)$, and so no firm earns more than it does from a profile of industry-optimal undercut-proof prices.*

To prove this result (which we find to be instructive) we label firms so that the first firm is (one of) the largest, and we write $\tilde{\pi}_j$ for the expected profit of a firm j in an equilibrium of a price-cutting game. We write $\underline{p}_j$ for the lower bound of the support of this firm's mixed strategy in this equilibrium. We know that no other firm places an atom at $\underline{p}_j$ (otherwise j would choose to undercut it) and so we can write the expected profit of firm j as $\tilde{\pi}_j = \underline{p}_j \alpha_j \prod_{k\neq j}(1-\alpha_k F_k(\underline{p}_j))$.

Now consider a firm i that charges (using an atom, if an atom-playing firm exists; in which case it is the only atom from standard atom-undercutting arguments) the highest price from the joint support of all firms. This firm sells only to "captive" customers, and so cannot earn more than $v\alpha_i \prod_{k\neq i}(1-\alpha_k)$. Of course, an option open to firm i is for it to "steal" the strategy of some firm j by undercutting $\underline{p}_j$. Doing so it earns $\underline{p}_j \alpha_i \prod_{k\notin\{i,j\}}(1-\alpha_k F_k(\underline{p}_j))$. We can conclude that

$$v\alpha_i \prod_{k=2}^{n}(1-\alpha_k) \geq v\alpha_i \prod_{k\neq i}(1-\alpha_k) \geq \tilde{\pi}_i \geq \underline{p}_j \alpha_i \prod_{k\notin\{i,j\}}(1-\alpha_k F_k(\underline{p}_j)) \geq$$

$$\underline{p}_j \alpha_i \prod_{k\neq j}(1-\alpha_k F_k(\underline{p}_j)) = \frac{\alpha_i}{\alpha_j}\tilde{\pi}_j \quad \Rightarrow \quad \underbrace{\tilde{\pi}_j \leq v\alpha_j \prod\nolimits_{k=2}^{n}(1-\alpha_k) = \pi_j}_{\text{firm earns at most its industry-optimal undercut-proof profit}} \quad . \quad (23)$$

Given that a firm cannot earn, in the equilibrium of a price-cutting game, an expected profit that exceeds that from a profile of industry-optimal undercut-proof prices, it follows immediately that a firm cannot gain from a shift in its price away from that charged in such a profile.

Proposition 7 (Stable Prices under Independent Awareness). *If buyers are independently aware of each firm, then a profile of prices is strongly stable if and only if it is an industry-optimal undercut-proof profile. All such profiles generate the same profits for all firms, and allocate the highest (monopoly) price v to a firm that enjoys the greatest awareness.*

Similarly to our use of Lemma 4 in Section 4, Lemma 6 places an upper bound on the profit of a deviant firm which leads to Proposition 7. Here again we can construct an equilibrium of the price-cutting game in which a deviant firm achieves that upper bound following an increase in its price. We repeat an earlier conclusion: a firm has only a weak incentive to avoid an increase in its price. To obtain a strict incentive we can include (just as we discussed for our duopoly analysis) either risk aversion or the lack of a special-deal response opportunity from the deviating firm.

6. Extensions: Prominence, Advertising, and Search

Here we extend our core results in three directions. Firstly, we explore a consideration specification in which one firm is "prominent" in the sense of enjoying awareness from all potential buyers. In that setting, we show that the unique stable price profile does not place firms in a natural "size" order. Secondly, we expand our independent awareness specification to allow firms to influence buyer consideration via advertising activities. There we show that a commonality of expected payoffs with established pricing solutions (that is, mixed strategy equilibria of pricing games) results in no change for the predictions of a deeper model. Thirdly, we expand our duopoly specification to allow for endogenous buyer search. Here, the difference in pricing patterns (relative to mixed-strategy solutions) does influence substantially search behavior in equilibrium.

A Prominence Setting. Here we work with a triopoly example that does not fit within either of our leading specifications of exchangeability (Section 4) or independent awareness (Section 5).

Amongst $n = 3$ competitors, firm $i = 1$ is "prominent" and so is known to all customers. Customers consider at most one of firms $i \in \{2, 3\}$, but never all three. Summarizing, the three positive-mass consideration sets are $\{1\}$, $\{1, 2\}$, and $\{1, 3\}$. We use the following notation:

$$\phi_1 = \lambda(\{1\}),\ \phi_2 = \lambda(\{1, 2\}),\ \text{and } \phi_3 = \lambda(\{1, 3\}). \tag{24}$$

An interpretation is that the prominent firm is a national sales channel, whereas other firms are local suppliers. Each local firm $i \in \{2, 3\}$ has access customers who see i's price. Additionally, all such customers are informed of firm 1's price.[29] We let $\phi_2 \geq \phi_3$.

Only a customer with consideration set $\{1\}$ is truly captive: $\lambda_1 = \phi_1 > 0$ but $\lambda_2 = \lambda_3 = 0$. This does not fit the regularity condition (in Section 2) which says that all firms have captive customers. Similarly, "twoness" fails: there are no pairwise comparisons of firms 2 and 3. Nevertheless, we are able to characterize undercut-proof, maximal, and stable prices.

If prices are undercut-proof and strictly positive, then they must place the prominent firm at the top.[30] (If the prominent firm charges strictly less than a local firm, then that local firm would undercut the prominent firm.) If prices are maximal, then of course $p_1 = v$.

Turning to no-undercutting constraints, we need only to check that the prominent firm does not wish to undercut the local firms. If local firms are ordered so that $p_2 \geq p_3$, then the relevant constraints are $v\phi_1 \geq p_2(\phi_1 + \phi_2)$ and $v\phi_1 \geq p_3(\phi_1 + \phi_2 + \phi_3)$. For prices to be maximal, these bind. Let $j > 1$ set the lowest price, p_j. It is undercut-proof if $p_j \leq v\phi_1/(\phi_1 + \phi_2 + \phi_3)$, which binds to be maximal, so that the lowest price is independent of which firm sets it. Notice also that if both local firms set this price, then either could raise it slightly without provoking an undercut by firm 1, so such prices are not maximal and firms must set distinct prices.

Let $i \neq j$ be the local firm that sets the higher price. Firm 1 does not undercut p_i if $p_i \leq v\phi_1/(\phi_1 + \phi_i)$, which binds to be maximal: p_i depends on how many customers consider i's price, unlike p_j. Given $p_1 = v$, each local supplier prefers that they charge p_i and the other charges p_j, rather than vice versa. Thus, there are two industry-optimal undercut-proof profiles: one in which $i = 2$ and $j = 3$, and one in which $i = 3$ and $j = 2$ (these coincide if $\phi_2 = \phi_3$).

[29] Inderst (2002) considered a related single-stage model, but did not fully characterize equilibrium; we characterize an equilibrium for one of his cases in Appendix B.3. Armstrong and Vickers (2022, Section 4) solved the single-stage game in a closely related setting, interpreting firms as a chain store with local rivals. In their setting any comparison involving a local firm involves the chain store, but local rivals also have captive audiences.

[30] The claim holds for the ordered-search model of Arbatskaya (2007) and in the search-and-prominence duopoly model of Moraga-González, Sándor and Wildenbeest (2021). In contrast, Armstrong, Vickers and Zhou (2009) used a sequential-search model to predict that a prominent firm offers the lowest price (see also Rhodes, 2011).

We proceed to investigate deviations in regular prices. We can construct an equilibrium in the price-cutting game following a deviation upward by a non-prominent firm that preserves the order of firms' regular prices: the deviator and the prominent firm mix in the interval up to the deviant initial price, while the other firm maintains its initial price. The deviator's profit is unchanged.

It remains to consider a deviation by the cheapest firm j to an initial price $\hat{p}_j \in (p_i, p_1]$. In Appendix C we show that when $\phi_2 > \phi_3$ and $j = 3$, such a deviation leads to a price-cutting game in which any Nash equilibrium gives firm 3 strictly greater profit. The reason is that the larger non-prominent firm 2 charges a low intermediate price to prevent the prominent firm undercutting it ($p_2 = v\phi_1/(\phi_1+\phi_2)$). This leaves an interval of prices, $(p_2, v\phi_1/(\phi_1+\phi_3))$, which are dominated for the prominent firm (by v), and so are safe for firm 3 to deviate to and yield it a profit strictly greater than $p_3\phi_3$. (If $j = 2$, then a deviation of this sort is unavailable.)

Proposition 8 (Stable Prices in a Prominence Setting). *In the prominence triopoly:*

(i) There are two industry-optimal undercut-proof profiles, described by:

$$p_1 = v,\ p_i = \frac{v\phi_1}{\phi_1 + \phi_i},\ \textit{and}\ p_j = \frac{v\phi_1}{\phi_1 + \phi_2 + \phi_3}\ \textit{for}\ i, j \in \{2, 3\}\ \textit{and}\ i \neq j, \tag{25}$$

Each firm $k \in \{1, 2, 3\}$ makes a profit equal to $p_k\phi_k$. Both of these profiles are weakly stable.

(ii) There is a unique stable profile in which the larger local firm is the cheapest: $i = 3$, $j = 2$.

(iii) With n symmetrically-sized firms ($\phi_i = \phi$ for all $i \in \{1, \ldots, n\}$), there is a unique stable profile of prices in which a firm's price declines inversely to its position in the sequence:

$$p_i = \frac{v}{i} \quad \textit{for all} \quad i \in \{1, \ldots, n\}. \tag{26}$$

A (non-prominent) firm with a larger audience is cheaper. It remains the case that the profit of a non-prominent firm is increasing in its own size. However, the larger non-prominent firm can make a smaller profit than the other. This is true whenever their sizes are sufficiently close. As the prominent firm's position strengthens (greater ϕ_1) price cuts hurt it more and so its rivals can set higher prices without being undercut. This implies non-prominent firms' prices and profits are increasing in ϕ_1 and that customers are worse off with a larger prominent firm.[31]

Endogenous Consideration (i): Advertising. Consideration sets themselves might respond to the actions by firms (such as advertising choices) and customers (such as price discovery). Naturally, a full study is beyond the scope of (at least the main body of) this paper. However, here we use duopoly analyses to illustrate the likely impact of firms' and customers' actions.

[31] In Appendix B.3 we develop this model of prominence by considering the incentives of a "prominence provider" which brings one of many local firms to national prominence. We find that this provider makes a prominence offer (which is accepted) to the firm with the largest local customer base. This is the worst choice for customers.

For a duopoly, recall that the profits from industry-optimal undercut-proof prices are

$$\pi_1 = v\lambda_1 \quad \text{and} \quad \pi_2 = v\lambda_1 \frac{\lambda_2 + \lambda_S}{\lambda_1 + \lambda_S}. \tag{27}$$

The expected profits match those from single-stage pricing models. Notice that (the larger, in terms of awareness) firm 1 cares solely about expanding its captive audience. The incentives of (the smaller) firm 2 are nuanced, for example, firm 2 benefits from an expansion in firm 1's captives.

We now build upon the independent awareness model of Section 5, where the awareness α_i of a firm is a consequence of its advertising activities. This maps to the general duopoly model via $\lambda_1 = \alpha_1(1-\alpha_2)$, $\lambda_2 = \alpha_2(1-\alpha_1)$, and finally $\lambda_S = \alpha_1\alpha_2$. For $\alpha_1 > \alpha_2$,

$$\pi_1 = v\alpha_1(1-\alpha_2) \quad \text{and} \quad \pi_2 = v\alpha_2(1-\alpha_2). \tag{28}$$

We see that the two firms have very different incentives. The larger and so more expensive firm sets a price of $p_1 = v$ that is not limited by any "no undercutting" constraint. For a given $\alpha_2 < 1$, its profits are linearly increasing in its advertising intensity. The smaller and cheaper firm, however, sets $p_2 = v(1-\alpha_2)$, to prevent an undercut by firm 1. The more it advertises, the more attractive such an undercut becomes, and so the lower its price must be to keep firm 1 at bay. This leads to a trade off for firm 2 when choosing how much to advertise: its profit is non-monotonic in α_2. In particular (and regardless of advertising costs) firm 2 (if it is smaller) always prefers $\alpha_2 \leq \frac{1}{2}$.

We can embed the profits of eq. (28) into a simultaenous-move advertising-choice game in which risk-neutral firms maximize expected profits via choices of advertising. For example, if advertising is free and awareness is chosen from $\alpha_i \in [0, \bar{\alpha}]$ for some $\bar{\alpha} \in \left(\frac{1}{2}, 1\right)$, then a pure-strategy Nash equilibrium of an advertising game takes the form $\alpha_1 = \bar{\alpha}$ and $\alpha_2 = \frac{1}{2}$. The stable prices

$$p_1 = v \quad \text{and} \quad p_2 = v\lambda_1/(\lambda_1 + \lambda_S) = v(1-\alpha_2) = v/2 \tag{29}$$

are dispersed. One firm maximizes its exposure to customers and sets the monopoly price, while the other limits its exposure to a minority of customers and charges half the monopoly price.

In Appendix B.4 we provide a full treatment with n firms and asymmetric advertising cost functions. We find one firm charges v and advertises distinctly more than all the others, who each advertise to a minority of customers and set mutually distinct lower prices. When advertising is costless, adding extra competitors adds additional lower prices (while retaining existing price positions) and increases the range of dispersed prices. With costly adverting, a fall in costs increases the awareness of each firm and the dispersed prices of the firms become further apart.

A conclusion here is that our pricing approach maintains some established predictions for advertising (Ireland, 1993; McAfee, 1994) which use conventional pricing games, owing to the fact that firms' expected profits are the same. Our actual pricing predictions differ, of course.

Endogenous Consideration (ii): Search. Retaining the duopoly framework, we now allow consideration sets to be determined endogenously by the actions of buyers rather than firms.[32]

Suppose that a potential buyer uses fixed-sample search technology à la Burdett and Judd (1983): this customer requires (without replacement) either zero, one, or two price quotations, and does while anticipating the selection of stable prices by firms. The buyer is interesting in purchasing a single unit of the product, and is willing to pay at most v for that unit.

Searching once finds each firm with equal probability. If the quotation is from the high-price firm that charges the monopoly price v then there is no benefit; but if the low-price firm is found (with probability $\frac{1}{2}$) then the customer gains $v - p_2$. A second search is guaranteed to find the cheaper firm, but this is beneficial only if the first search did not already do so. As such, the second search also generates a gain of $v - p_2$ with probability $\frac{1}{2}$. Summarizing,

$$\text{E[benefit of 1st search]} = \text{E[benefit of 2nd search]} = \frac{v - p_2}{2} = \frac{v\lambda_S}{2(\lambda_1 + \lambda_S)}. \tag{30}$$

Now adopt the classic constant-returns search technology so that gathering each quotation costs $\kappa \in \left(0, \frac{v}{2}\right)$. A customer finds it strictly optimal to obtain two quotations if and only if

$$\kappa < \frac{v\lambda_S}{2(\lambda_1 + \lambda_S)} \quad \Leftrightarrow \quad \lambda_S > \frac{2\kappa\lambda_1}{v - 2\kappa}, \tag{31}$$

will not search at all if the opposite strict inequality holds, and will be indifferent between all search strategies if there is an equality. This inequality reveals a strategic complementarity: if many others seek out both price quotations (so that the mass of shoppers λ_S is large) then there is greater price dispersion, and this increases the incentive of a customer to search.

This strategic complementarity suggests there may be multiple equilibria with endogenous search. To sketch a model of this, let us suppose that a mass $\bar{\lambda}_i$ of customers are exogenously captive to firm i, a mass $\bar{\lambda}_S$ are exogenously shoppers, and mass μ decide whether to search once, twice, or not at all. Writing μ_L and μ_H for the masses of buyers searching once and twice (we choose these subscripts to avoid confusion with firm labels) we have $\mu_L + \mu_H \leq \mu$, and

$$\lambda_i = \bar{\lambda}_i + \mu_L \quad \text{for } i \in \{1, 2\} \text{ and } \quad \lambda_S = \bar{\lambda}_S + \mu_H. \tag{32}$$

[32] We provide a full oligopoly treatment of the model sketched here in related work (Myatt and Ronayne, 2025). There and here we study fixed-sample search, while acknowledging that elsewhere sequential-search models à la Wolinsky (1983, 1986) have been successfully used and extended to include elements such as additional captives (Chen and Zhang, 2011), shoppers (Stahl, 1989), multiple products (Zhou, 2014), prominence (Armstrong, Vickers and Zhou, 2009), targeted search (De Cornière, 2016), and ordered search (Zhou, 2011*a*). Our oligopoly analysis allows a study of entry (following Janssen and Moraga-González, 2004) and we find that such entry lowers endogenously search and so raises industry profit. In other related models (Zhou, 2014; Rhodes and Zhou, 2019; Chen, Li and Zhang, 2022) larger search costs can raise rather than lower consumer surplus. Non-standard entry effects also occur in sequential-search settings (Janssen, Pichler and Weidenholzer, 2011; Chen and Zhang, 2018).

There are multiple equilibria if the various parameters here satisfy

$$\bar{\lambda}_S + \mu > \frac{2\kappa\bar{\lambda}_1}{v - 2\kappa} > \bar{\lambda}_S. \tag{33}$$

We can construct a "high search" equilibrium (of a fully defined game, with an appropriate solution concept) in which all endogenous searchers seek out two quotations ($\mu_L = \mu$) and become shoppers. In this equilibrium prices are more dispersed:

$$p_1 = v \quad \text{and} \quad p_2 = \frac{v\bar{\lambda}_1}{\bar{\lambda}_1 + \bar{\lambda}_S + \mu}. \tag{34}$$

In a second "low search" equilibrium endogenous searchers stay home ($\mu_L = \mu_H = 0$). In this equilibrium the two price points are closer together and total search is limited to $\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_S$.[33] In a model with a conventional pricing game (Burdett and Judd, 1983) the "high search" equilibrium has incomplete search, and so our approach is consequential for search behavior.

7. Concluding Discussion

An interest in price competition with heterogeneous consideration arises in contexts with comparison websites and more general platforms (Baye and Morgan, 2001, 2009; Ronayne, 2021; Bergemann and Bonatti, 2024; Hagiu and Wright, 2024), price discrimination (Armstrong and Vickers, 2019; Fabra and Reguant, 2020), product substitutability (Inderst, 2002), consumer search (Stahl, 1989), and boundedly rational consumers (Carlin, 2009; Chioveanu and Zhou, 2013; Heidhues, Johnen and Kőszegi, 2021; Inderst and Obradovits, 2020; Piccione and Spiegler, 2012). Our work shows that stable and dispersed prices can be recovered within these environments.[34]

Our construction of stable dispersed prices has the feature that a cheaper firm is indifferent to raising its undercut-proof price. This makes the prices weak, rather than strict, best replies. This feature follows from our specification that price-cutting games are both all-play and single-stage.

[33] There can be an interior equilibrium satisfying $\bar{\lambda}_S + \mu_H = \kappa(2\bar{\lambda}_1 + \mu_L)/(v - 2\kappa)$. This is unstable in the sense of Fershtman and Fishman (1992): shifting extra customers to search twice (and letting firms' prices adjust) the benefit of a second search increases and so all customers wish to search twice. In contrast, a single-stage duopoly analysis yields an equilibrium with search with (using present notation) $\mu_L + \mu_H = \mu$ and $\mu_L, \mu_H > 0$. Another alternative specification, explored further in Myatt and Ronayne (2025), is to allow a first search for free. Such a situation is also readily obtained when buyers wish to obtain a quotation even if they expect to face a monopoly price, as they do (from obtaining positive consumer surplus) un under the downward-sloping demand of Baye and Morgan (2001) or from a surplus-generating additional purchase (Ellison, 2005; Johnson, 2017; Rhodes, 2015; Rhodes, Watanabe and Zhou, 2021). In this setting a "low search" equilibrium can be one in which all searchers obtain a single quotation.

[34] Other (very different) environments can generate dispersed pure-strategy prices. Reinganum (1979) offered a version of Diamond (1971) in which firms have different costs and set different monopoly prices. Anderson and De Palma (2005) studied customers who (exogenously) consider firms in a random order, but without any meaningful price comparison. Arnold (2000) studied capacity-constrained firms and single-search customers who see prices but not whether a firm is stocked out. Firms trade off price for the (endogenous) number of buyers that buy from them. For csome valuations, there is an equilibrium in which firms choose different prices.

This provides both the toughest test for stable prices and a clear connection to the literature's standard approach to pricing. Nevertheless, different specifications readily make best replies strict.

We have noted (in Section 3) and explore more fully (in Appendix B.1) that one approach is to consider the asymmetric ability of firms to respond with special deals. If a deviant firm is unable to offer offer such deals that a price increase can be strictly harmful. Another natural case (also discussed in Section 3, and explored in Appendix B.2) is when a decision-maker is averse to risk. A price rise enters a firm into a price-cutting game in which its profit is (undesirably) uncertain.

We conclude by comparing our results and predictions to those from conventional approaches. We identify three advantages. Firstly, we predict that disperse prices can be stable rather than randomized. Secondly, we provide a clear process to solve for those stable prices. Thirdly, those prices are expressed via relatively simple closed-form analytic solutions.

Turning to predictions, we can identify (in some, but not all, circumstances) consequences for firms and aspects of their behavior that coincide with the conventional approach. For buyers, however, we identify distinctly different implications for their incentives to search.

For firms, and for the major settings that we covered, expected profits match those earned in the equilibrium of a single-stage game.[35] This means that researchers analyzing settings with a single-stage model in a subgame (for example, the platform analysis by Hagiu and Wright, 2024) can supplement or replace it with our approach.[36] The profit equivalence (in many cases) means there is no disruption to earlier stages (at least with risk-neutral players) and so we do not expect substantial changes in firm-related actions such as advertising. An illustrative exception is the case of the prominence triopoly in Section 6. Firms' profits stable prices strictly exceed, for one firm, the expected profit earned from an equilibrium of a conventional single-stage pricing game.[37]

From the perspective of buyers, however, things are markedly different. Consider the duopoly search model (of Section 6) where "search" refers to buyers gathering both prices. Stable prices diverge as buyers search more, revealing strategic complementarities to search. In contrast, single-stage mixed-strategy pricing predicts that firms use prices drawn randomly from the same distribution. Crucially, there is strategic substitutability as search becomes sufficiently strong: the incentive for an individual buyer to search falls if search amongst others is higher. As search becomes complete (so that almost all buyers obtain two price quotations) the firms' (symmetric)

[35]In the captive-shopper setting, the profits from Proposition 6 (i) match those of Baye, Kovenock and de Vries (1992); for full exchangeability, profits from the profile identified by Proposition 4 match those of Johnen and Ronayne (2021); for independent awareness, the profits from Lemma 5 match those of Ireland (1993) and McAfee (1994).

[36]The analysis of Hagiu and Wright (2024) includes a fee-setting plaform, requiring them to consider a captive-and-shopper subgame with asymmetric marginal costs, and for that they use results from our analysis of asymmetric models of sales (Myatt and Ronayne, 2023). Stable prices also naturally extend to asymmetric marginal cost models of sales.

[37]For completeness, we characterize that equilibrium in Appendix B.3.

mixing distribution collapses to marginal cost. This, of course, makes a second search redundant; and so this rules out an equilibrium in which search is complete.

We pursue a full oligopoly analysis of stable prices with costly buyer search elsewhere (Myatt and Ronayne, 2025). We identify novel effects: (in a focal equilibrium with positive search) search is higher than predicted by the conventional approach; an increase in the number of firms lowers the intensity of search; and such entry raises (rather than lowers) aggregate profit in the industry. The duopoly sketches this paper demonstrate how stable prices can generate novel applied insights.

Appendix A. Omitted Proofs

Proof of Lemma 1. Claims (i) and (ii) follow from the main text. (See also Appendix C.)

For claim (iii) it is without loss to focus on strictly positive prices. For undercut-proof prices $p_1 > \cdots > p_n > 0$ write $\pi_i = p_i \sum_{B \subseteq \{1,\ldots,i\}} B_i \, \lambda(B)$ for the profit of firm i.

By charging $\tilde{p}_1 = p_1$, firm 1 achieves the profit π_1, independent of the choices of other firms. For any other price it may charge, $\tilde{p}_1 < p_1$, its profit is highest if all others maintain their initial prices. The profile of initial prices is undercut-proof, and firm 1 earns strictly less than π_1 by strictly undercutting any other firm. If it matches another firm, then (given that ties are broken in an interior way) it also earns strictly less. Therefore, all $\tilde{p}_1 < p_1$ are strictly dominated for firm 1. We conclude (as an induction basis) that firm 1 must charge $\tilde{p}_1 = p_1$.

For $i > 1$, suppose that $\tilde{p}_j = p_j$ for all $j < i$. Firm i guarantees a profit π_i by charging p_i. Recycling the argument above, even if others maintain their prices then firm i earns strictly less from $\tilde{p}_i < p_i$. We conclude that $\tilde{p}_i = p_i$. By the principle of induction, this holds for all $i \in \{1, \ldots, n\}$. ☐

Proof of Lemma 2. This follows from the arguments in the main text. ☐

The following lemma is used in the proof of Proposition 1 that follows.

Lemma A1. *Consider a strategy profile in a pricing game in which firms i and j mix (continuously) over an interval $[p_L, p_H]$ not intersecting the support of any other firm, and where i and j are both indifferent (as they are in mixed-strategy Nash equilibrium) across that interval. The expected profit of any other firm from deviating to a price $p \in [p_L, p_H]$ is convex in p.*

Proof of Lemma A1. We note that $F_l(p)$ is constant for $p \in [p_L, p_H]$ and $l \notin \{i, j\}$; we write F_l for this constant. Varying the prices of i and j within the interval $[p_L, p_H]$ has no effect on their sales when there is no comparison between them. We write Y_i and Y_j for such sales:

$$Y_i = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_i(1-B_j) \prod_{l\notin\{i,j\}} (1-B_lF_l) \tag{A1}$$

$$Y_j = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_j(1-B_i) \prod_{l\notin\{i,j\}} (1-B_lF_l) \tag{A2}$$

We also write Z for the sales made by the cheaper of i and j when they are compared:

$$Z = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_iB_j \prod_{l\notin\{i,j\}} (1-B_lF_l). \tag{A3}$$

With this notation in hand, the firms' expected profits from any price $p \in [p_L, p_H]$ are

$$\pi_i(p) = p\left(Y_i + Z(1-F_j(p))\right) \quad \text{and} \quad \pi_j(p) = p\left(Y_j + Z(1-F_i(p))\right). \tag{A4}$$

These profits are constant across this interval and so

$$1 - F_j(p) = \frac{\pi_i - pY_i}{pZ} \quad \text{and} \quad 1 - F_i(p) = \frac{\pi_j - pY_j}{pZ}. \tag{A5}$$

Now consider the profit of some firm $k \notin \{i,j\}$ deviating to a price in this interval. We write Y_k for the sales made when there is no comparison between k and either (or both) of i and j:

$$Y_k = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_k(1-B_i)(1-B_j) \prod_{l\notin\{i,j,k\}} (1-B_lF_l), \tag{A6}$$

where these expected sales are guaranteed for any price in $[p_L, p_H]$. Other possible sales involve comparisons of k with i, with j, or with both i and j. Possible sales for these three cases are

$$Z_{ik} = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_kB_i(1-B_j) \prod_{l\notin\{i,j,k\}} (1-B_lF_l), \tag{A7}$$

$$Z_{jk} = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_k(1-B_i)B_j \prod_{l\notin\{i,j,k\}} (1-B_lF_l), \tag{A8}$$

$$Z_{ijk} = \sum_{B\subseteq\{1,\ldots,n\}} \lambda(B)B_k(1-B_i)(1-B_j) \prod_{l\notin\{i,j,k\}} (1-B_lF_l). \tag{A9}$$

The expected profit of firm k from charging price $p \in [p_L, p_H]$ is

$$\begin{aligned} \pi_k(p) &= p\left[Y_k + Z_{ik}(1-F_i(p)) + Z_{jk}(1-F_j(p)) + Z_{ijk}(1-F_i(p))(1-F_j(p))\right] \\ &= p\left[Y_k + Z_{ik}\frac{\pi_j - pY_j}{pZ} + Z_{jk}\frac{\pi_i - pY_i}{pZ} + Z_{ijk}\frac{\pi_j - pY_j}{pZ}\frac{\pi_i - pY_i}{pZ}\right] \\ &= pY_k + Z_{ik}\frac{\pi_j - pY_j}{Z} + Z_{jk}\frac{\pi_i - pY_i}{Z} + \frac{Z_{ijk}}{Z^2}\left[\frac{\pi_i\pi_j}{p} + Y_iY_jp - (\pi_iY_j + \pi_jY_i)\right], \end{aligned} \tag{A10}$$

which by inspection is convex in p, and strictly so if $Z_{ijk} > 0$. □

Proof of Proposition 1. We begin with the first claim, which identifies a necessary condition for stable prices. Suppose that the no-undercutting constraint of firm i is slack, so that either $i = 1$ and $p_1 < v$ or $i > 1$ and equation (2) holds as a strict inequality. Firm i can strictly raise p_i while maintaining undercut-proofness, and enter a price-cutting game with (by claim (iii) of Lemma 1) a unique Nash equilibrium which gives firm i a strictly higher expected profit.

For the sufficient condition, fix the candidate price profile. Given (4), this satisfies $p_1 = v$ and

$$p_i = p_{i-1} \frac{\sum_{B\subseteq\{1,\dots,i-1\}} B_{i-1}\,\lambda(B)}{\sum_{B\subseteq\{1,\dots,i\}} B_{i-1}\,\lambda(B)} \quad \text{for all } i \in \{2,\dots,n\}. \tag{A11}$$

These prices are undercut-proof, and so $\tilde{p}_i = p_i$ for all i is the unique Nash outcome of a price-cutting game, by claim (iii) of Lemma 1. Firm i earns a profit $p_i \sum_{B\subseteq\{1,\dots,i\}} B_i\lambda(B)$.

Consider a nudge upward in price by $i > 1$ of $\Delta > 0$ sufficiently small such that $p_i + \Delta \le p_{i-1}$, and that each firm $j < i-1$ that strictly prefers p_j to undercutting p_i also strictly prefers p_j to undercutting $p_i + \Delta$. In the price-cutting game construct a strategy profile in which $j \notin \{i-1, i\}$ choose $\tilde{p}_j = p_j$, while $j \in \{i-1, i\}$ mix over $[p_i, p_i + \Delta)$ with distributions

$$F_j(p) = \frac{(p - p_i)\sum_{B\subseteq\{1,\dots,i\}} B_k\,\lambda(B)}{p\sum_{B\subseteq\{1,\dots,i\}} B_i B_{i-1}\,\lambda(B)} \quad \text{for } j,k \in \{i-1,i\}, j \neq k, \tag{A12}$$

and then place remaining mass at p_{i-1} and $p_i + \Delta$ respectively. These are valid CDFs which continuously increase from $F_j(p_i) = 0$ and satisfy $F_j(p_i + \Delta) \le 1$ for $j \in \{i-1, i\}$ if Δ is sufficiently small. Moreover, prices within this interval give the firms $j \in \{i-1, i\}$ their pre-deviation expected profits. To see why, note that for $j \in \{i-1, i\}$ and $k \in \{i-1, i\}$ for $k \neq j$,

$$\underbrace{(p - p_i)\sum_{B\subseteq\{1,\dots,i\}} B_j\,\lambda(B)}_{\text{gain from lifting price}} = \underbrace{pF_k(p)\sum_{B\subseteq\{1,\dots,i\}} B_i B_{i-1}\,\lambda(B)}_{\text{lost sales to } k\neq j}. \tag{A13}$$

The left-hand side is the gain to j from charging a price higher than p_i. (The summation represents sales from being the cheapest of $\{1,\dots,i\}$.) The right-hand side is then the value of sales lost to the competitor k, which incorporates the probability that k prices below p.

The condition (4) says that $i-1$ is a firm that is indifferent to undercutting i. We chose Δ such that any firm that strictly prefers not to undercut p_i also strictly prefers not to undercut $p_i + \Delta$ and so prefers not to join the "tango" between $i-1$ and i. It remains to check that any firm $j < i-1$ that is indifferent to undercutting p_i is unwilling to join the dance (i.e., set some $p \in [p_i, p_i + \Delta)$). By Lemma A1 we only need to check that j does at least as well with p_j than both (i) p_i, and (ii) (undercutting) $p_i + \Delta$. As for (i), we know j is indifferent between p_j and p_i. For (ii), recall that Δ is sufficiently small such that $F_j(p_i + \Delta) \le 1$ for $j \in \{i-1, i\}$. In fact, using (A12) we find $F_i^{-1}(1) = p_{i-1}$ (i.e., the function F_i reaches 1 at exactly p_{i-1}). Therefore, j gets a strictly lower profit from undercutting p_{i-1} than charging p_j (i has no mass at p_{i-1}, so no matter the mass $i-1$ places there, j would not undercut p_{i-1} because the initial price profile is undercut-proof). It follows that undercutting $p_i + \Delta$ (which is $\leq p_{i-1}$) gets j an even lower expected profit than if it were to undercut p_{i-1}, and so j prefers p_j. □

Proof of Proposition 2. This follows from the arguments in the main text. □

Proof of Lemma 3. Fix a profile of maximal undercut-proof prices. Find the first $k \in \{1, \ldots, n-1\}$ such that $\lambda_k < \lambda_{k+1}$. We claim that for all $i \in \{2, \ldots, k+1\}$

$$p_i = p_{i-1}\frac{\lambda_{i-1} + X_{i-1}}{\lambda_{i-1} + X_i} \quad \text{and so} \quad p_i \equiv v\prod_{j=2}^{i}\frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j}. \tag{A14}$$

The first equality says the local no-undercutting constraints bind; setting $p_1 = v$, repeated substitution gives the second equality. To prove this, note it must be true for $i = 2$ (forming an induction basis) as there is only one no-undercutting constraint, which binds because prices are maximal. Now suppose the claim holds (as an induction hypothesis) for all $j \in \{2, \ldots, i-1\}$. For i,

$$\begin{aligned}
p_i &= \min_{j<i}\left\{p_j\frac{\lambda_j + X_j}{\lambda_j + X_i}\right\} = \min_{j<i}\left\{p_j\frac{\lambda_j + X_j}{\lambda_j + X_i}\left(\prod_{k=j+1}^{i}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}}\right)\left(\prod_{k=j+1}^{i}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k}\right)\right\} \\
&= v\prod_{j=2}^{i}\frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j}\min_{j<i}\left\{\frac{\lambda_j + X_j}{\lambda_j + X_i}\left(\prod_{k=j+1}^{i}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}}\right)\right\} \\
&= v\prod_{j=2}^{i}\frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j}\left\{1, \min_{j<i-1}\left\{\frac{\lambda_j + X_j}{\lambda_j + X_i}\left(\prod_{k=j+1}^{i}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}}\right)\right\}\right\} \\
&= v\prod_{j=2}^{i}\frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j} = p_{i-1}\frac{\lambda_{i-1} + X_{i-1}}{\lambda_{i-1} + X_i}.
\end{aligned} \tag{A15}$$

The first four lines use algebraic re-arrangement. The final line holds because for each $j < i-1$,

$$\frac{\lambda_j + X_j}{\lambda_j + X_i}\prod_{k=j+1}^{i}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}} \leq \frac{\lambda_j + X_j}{\lambda_j + X_i}\prod_{k=j+1}^{i}\frac{\lambda_j + X_k}{\lambda_j + X_{k-1}} = \frac{\lambda_j + X_j}{\lambda_j + X_i}\frac{\lambda_j + X_i}{\lambda_j + X_j} = 1. \tag{A16}$$

The inequality in the chain holds because $X_k \geq X_{k-1}$ in each of the ratio terms, which means that such terms are each decreasing in λ_{k-1}. An upper bound for each term is obtained by replacing λ_{k-1} with $\lambda_j \leq \lambda_{k-1}$, where this inequality holds because $j \leq k-1 \leq i-1$ and (by assumption) firms below i are in size order. The claim holds by the principle of induction.

Now consider firms k and $k+1$. This is the first out-of-order pair.

$$\frac{p_{k+1}}{p_k} = \frac{\lambda_k + X_k}{\lambda_k + X_{k+1}} < \frac{\lambda_{k+1} + X_k}{\lambda_{k+1} + X_{k+1}}. \tag{A17}$$

The equality is the (binding) no undercutting constraint. The inequality holds because $X_k < X_{k+1}$ and $\lambda_{k+1} > \lambda_k$. This means that if we switch the positions of k and $k+1$ in the profile of prices (while maintaining the actual prices) then the no-undercutting holds strictly. Firm k is now lower in the price order than before, but with the same profit, and so (as before) does not want to undercut any lower-priced firms. Firms $j < k$ face the same no-undercutting opportunities as before, and so their no-undercutting constraints still hold. Firm $k+1$ is now strictly better off. We have constructed an undercut-proof profile that increases industry profit. □

As noted in the main text, we can also state a more general version of Lemma 3.

Lemma A2. *For firms placed in size order, $\lambda_1 \geq \cdots \geq \lambda_n$, define the following prices:*

$$p_1^\ddagger = v \quad \textit{and} \quad p_i^\ddagger \equiv v \prod_{j=2}^{i} \frac{\lambda_{j-1} + X_{j-1}}{\lambda_{j-1} + X_j}. \tag{A18}$$

Next, for any *given order of firms, consider the set of maximal undercut-proof prices:*

(1) These prices satisfy $p_i \leq p_i^\ddagger$ for all i.

(2) If $\lambda_1 \geq \cdots \geq \lambda_{i-1}$ (so that firms indexed below i are in size order) then $p_i = p_i^\ddagger$.

(3) The i^{th} highest price is highest when firms are in size order, for all i.

(4) Placing firms in size order maximizes the industry profit.

(5) All firms would unanimously prefer to be placed in size order.

Proof of Lemma A2. The proof is relegated to Appendix C. □

Proof of Lemma 4. The proof is sketched in the main text. Further details of the proof approach are contained within the proof of Lemma 6, which is also within the main text. □

Proof of Proposition 4. The proof of the proposition follows directly from the argument preceding its statement in the main text: any equilibrium of a price cutting game results in weakly lower profit for each firm than achieved from the industry-optimal prices. After the proposition, we also say that we can "construct an equilibrium of the price-cutting game in which a deviant firm achieves that upper bound following an increase in its price." A formal statement is as follows.

Lemma A3 (An Equilibrium of a Price-Cutting Game under Full Exchangeability). *In the full exchangeability setting, with equally sized captive audiences, consider the price-cutting game following the prices reported in Lemma 3, with the exception of firm $k > 1$, which deviates to $\hat{p}_k > p_k$. There is a Nash equilibrium in which each firm i earns a profit of $p_i(\lambda + X_i) = v\lambda$.*

This construction involves lengthy intricacies, and so is relegated to Appendix C. □

Proof of Proposition 5. A sketch is given in the text; formalities are relegated to Appendix C. □

Proof of Proposition 6. This proposition is concerned with an exact (rather than approximate) captive-and-shopper model of sales: $I_m = 0$ for $m \in \{2, \ldots, n-1\}$. Pairwise consideration sets are empty and so the "twoness" property does not hold. This means that claim (i) of Lemma 1 does not apply: strictly positive undercut-proof prices are not necessarily distinct.[38]

The lowest price in a maximal undercut-proof profile cannot be zero: such a firm (even if there is more than one) could raise its price locally without violating no-undercutting constraints.[39] Given that the lowest price is strictly positive it must be charged by only a single firm (else there would be a profitable undercut). Firms who are not the cheapest and so sell only to their captives, can raise their prices to v while maintaining undercut-proofness. We conclude there are only two price points: a lowest (strictly positive) price and v. Consider, then, profiles in which one firm i sets $p_i < v$ while others $j \neq i$ set $p_j = v$. All such firms j earn $v\lambda_j$, and so to dissuade undercutting we need $p_i \leq p_j^\dagger$, where $p_j^\dagger$ is from eq. (17). Thus, the maximal profile when i is cheapest must satisfy

$$p_i = \min_{j \neq i} p_j^\dagger = \begin{cases} p_{n-1}^\dagger & \text{if } i = n, \text{ or} \\ p_n^\dagger & \text{if } i \in \{1, \ldots, n-1\}. \end{cases} \tag{A19}$$

We have found n maximal undercut-proof price profiles, which vary according to the identity of the shopper-capturing cheapest firm. All firms $j \neq i$ earn $v\lambda_j$. If $i < n$, firm i earns strictly less than $v\lambda_i$; but if $i = n$ then firm i earns strictly more. From this we conclude that $i = n$ (the firm with fewest captives is cheapest) pins down the unique industry-optimal undercut-proof profile.

For the next statement (concerning the stability of the industry-optimal prices) note that the only possible upward deviation in price is by firm n to a price $\hat{p}_n \in (p_{n-1}^\dagger, v]$.

Claim. *Consider the price-cutting game following $\hat{p}_n > p_{n-1}^\dagger$ and $p_i = v$ for $i < n$. There is a unique Nash equilibrium in which firm n earns $p_{n-1}^\dagger(\lambda_n + \lambda_S)$ and each firm i earns profit $v\lambda_i$.*

The present model specification (a model of sales with asymmetric captive shares across firms and symmetric marginal costs) is effectively nested within our more general analysis of asymmetric models of sales (Myatt and Ronayne, 2023). Formally, the present specification is slightly different when $\hat{p}_n < v$ so that firm n may only price in $[0, \hat{p}_n]$. Nevertheless, our analysis from the otherwise more general setting applies and generates only a small (and profit-inconsequential) change in

[38] We noted earlier that if a "k-ness" property holds, so that all consideration sets comprising $k > 1$ firms have positive mass, then there can be at most $k-1$ tied prices. A model of sales has this property only for $k = n$, which leaves open the possibility of $n-1$ tied prices. Ultimately, this is what we predict. Claim (ii) of Lemma 1 does not hold as stated. However, this is simply because we need to adjust our notation to deal with cases of tied prices. Finally, claim (iii) of Lemma 1 continues to hold more generally even without the twoness property.

[39] Any higher-priced firm earns strictly positive profits, and would earn less by pricing close enough to zero.

the equilibrium strategies such that firm n places its residual mass at $\hat{p}_n$ rather than extending its support all the way up to v. Formally, to construct the unique equilibrium we can follow the algorithm we laid out there (Myatt and Ronayne, 2023, Appendix B; summarized on p. 42). We state the unique equilibrium strategies below here for the current claim for completeness.

Let firm n set a regular price $\hat{p}_n \in (p^\dagger_{n-1}, p^\dagger_{n-2})$, then in the price-cutting game Nash equilibrium each $i < n-1$ sets its price equal to v, while firms $n-1$ and n mix continuously over $[p^\dagger_{n-1}, \hat{p}_n)$ with distributions $F_{n-1}(p) = (p - p^\dagger_{n-1})(\lambda_S + \lambda_n)/p\lambda_S$ and $F_n(p) = (p - p^\dagger_{n-1})(\lambda_S + \lambda_{n-1})/p\lambda_S$ with n and $n-1$ placing remaining mass at $\hat{p}_n$ and v, respectively. The profit of firm n equals that in the statement of the proposition, making the upward deviation in its price non-profitable. We conclude the profile with $p_n = p^\dagger_{n-1}$ and $p_i = v$ for $i < n$ is strongly stable.

We next consider any of the other $n-1$ profiles of maximal undercut-proof prices, in which some firm $i \in \{1, \ldots, n-1\}$ chooses $p_i = p^\dagger_n$ while each $j \neq i$ chooses $p_j = v$ so that i's profit is $p^\dagger_n(\lambda_S + \lambda_i)$. Suppose i deviates to v. From its captives alone, it makes $v\lambda_i > p^\dagger_n(\lambda_S + \lambda_i)$ and so the profile is not stable. Thus the strongly stable profile identified above is the unique such profile.

Now suppose i deviates by a little, to $p^\dagger_n + \Delta < p^\dagger_{n-1}$. Over $[p^\dagger_n, p^\dagger_n + \Delta)$, let i and n mix via distributions $F_i(p) = (p - p^\dagger_n)(\lambda_S + \lambda_n)/p\lambda_S$ and $F_n(p) = (p - p^\dagger_n)(\lambda_S + \lambda_i)/p\lambda_S$, and place remaining mass at their $p^\dagger_n + \Delta$ and v, respectively. The distributions are continuously increasing from $F_i(p^\dagger_n) = F_n(p^\dagger_n) = 0$, and satisfy $F_i(p) < F_n(p) \leq 1$ if p is not too large (guaranteed by Δ sufficiently small). By construction, firms i and n earn their original expected profits. We conclude that this price profile is weakly stable, giving claim (ii) of the proposition. □

Proofs of Lemmas 5 and 6. These results follow from the arguments in the main text. □

Proof of Proposition 7. The proof of follows from the same argument used to prove Proposition 4: any price-cutting equilibrium results in weakly lower profit for each firm than from the industry-optimal prices. Just as before, we can find an equilibrium in which this bound is achieved.

Lemma A4 (An Equilibrium of a Price-Cutting Game with Independent Awareness). *In the independent-awareness setting, consider the price-cutting game following the prices reported in Lemma 5 when firms are ordered by size but where firm $k > 1$ deviates to $\hat{p}_k > p_k$. There is a Nash equilibrium in which each firm i earns its industry-optimal undercut-proof profit.*

Similarly to Lemma A3, this involves lengthy intricacies, and so is relegated to Appendix C. □

Proof of Proposition 8. This is relegated to Appendix C. □

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Supplements for "Stable Prices and Heterogeneous Buyer Consideration"

David P. Myatt and David Ronayne · February 2025

Appendix B. Extensions

In this appendix we supplement and extend various results or points in the main text. We:

- B.1 study firms that have asymmetric abilities to offer special deals;
- B.2 consider the impact of risk-aversion making equilibrium strict;
- B.3 detail the model of prominence outlined in Section 6; and
- B.4 describe the n-firm model of costly advertising outlined for duopoly in Section 6.

B.1. **Ability to Respond.** In Section 3 we noted that if a rival is (with some probability) uniquely able to respond to a price increase then their can be a strict incentive to maintain stable prices.

We illustrate that idea in a duopoly context, using prices and profits from eq. (5). Suppose that, following a price rise by the cheaper firm 2, firm $i \in \{1, 2\}$ is uniquely able to respond (with a special deal) with probability ρ_i, that both are able to respond (the case studied in the paper) with probability ρ_{12}, and that neither can (this is the implicit assumption of a classic pricing game) with probability $\rho_0 = 1 - \rho_1 - \rho_2 - \rho_{12}$. If firm 2 does raise its price, then optimally it chooses the monopoly price v, given that it can fully exploit that price to earn $(\lambda_2 + \lambda_S)v$ with probability $\rho_2 + \rho_0$. With probability ρ_{12} this generates the same expected profit as earned from the stable prices, π_2, but with probability ρ_1 it will be undercut by the (uniquely responding) higher-price firm and so earn only $\lambda_2 v$. There is a strict incentive to maintain the regular price if

$$\pi_2 > (\rho_0 + \rho_2)(\lambda_2 + \lambda_S)v + \rho_{12}\pi_2 + \rho_1\lambda_2 v \;\Leftrightarrow\; \frac{\rho_0 + \rho_2}{\rho_0 + \rho_1 + \rho_2} < \frac{\pi_2 - \lambda_2 v}{\lambda_S v} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \quad \text{(B1)}$$

This is satisfied if (for example) $\rho_1 > 0$ but $\rho_0 + \rho_2 = 0$.

B.2. **Risk Aversion.** We observe that under out stable-pricing solution concept firms are typically indifferent to raising (at least locally) their prices. If a firm deviates by doing so, then (for each setting—see the proofs of Lemmas 6 and A3 and Propositions 5 and 8) we were able to construct a mixed-strategy equilibrium of the associated price-cutting game which generates the stable-price profit for the deviator. This means that there is only a weak incentive to maintain a stable price.

This is all underpinned by the assumption (otherwise maintained throughout) that firms are risk neutral. Suppose instead that we split each firm into two players: a manager, and an operational pricing agent. We define a game (of perfect information) with $2n$ players in which

$(t = 1)$ the firms' managers simultaneously choose regular price positions $p_i \in [0, v]$; and then
$(t = 2)$ the firms' agents simultaneously choose whether to offer special deals $\tilde{p}_i \in [0, p_i]$.

Agents' payoffs are simply profits, and so they are assumed to be risk neutral and maximize expected profit. The manager of firm i, however, has payoff $u_i(\pi_i)$, which is a smoothly increasing and concave function of the firm's profit. (The more general and key assumption here is that the manager is more risk averse than the pricing agent.) Equilibrium play in any subgame (this is a price-cutting game) is unaffected by the move to this "$2n$ player" environment. If firms' managers choose the regular prices p_i described in our results, then they obtain payoffs $u_i(\pi_i)$ where π_i is the corresponding profit of firm i under the relevant price profile. Any upward deviation leads to a subgame with the same expected profit, but a lower expected utility. This means that manager i's choice of p_i is the unique best reply to the regular (stable) prices, p_j, of managers $j \neq i$.

B.3. **Prominence.** In Section 6 we described a triopoly in which one firm is prominently considered. One industry-optimal undercut-proof profile is stable (Proposition 8). Profits for the firms (which we order so that $\phi_2 \geq \phi_3$) are

$$\pi_1 = v\phi_1, \quad \pi_2 = p_2\phi_2 = \frac{v\phi_1\phi_2}{\phi_1 + \phi_2 + \phi_3}, \quad \text{and} \quad \pi_3 = p_3\phi_3 = \frac{v\phi_1\phi_3}{\phi_1 + \phi_3}. \tag{B2}$$

Here we describe a Nash equilibrium from the play of the standard single-stage pricing game. In this equilibrium, firm 2 mixes over the interval $[p_2, p_3]$ according to the distribution

$$F_2(p) = \frac{\phi_1 + \phi_2 + \phi_3}{\phi_2} - \frac{v\phi_1}{\phi_2 p}, \tag{B3}$$

where $p_2 = v\phi_1/(\phi_1 + \phi_2 + \phi_3)$ and $p_3 = v\phi_1/(\phi_1 + \phi_3)$ are the stable prices from Proposition 8. Firm 3 then mixes over the interval $[p_3, v]$ according to

$$F_3(p) = \frac{\phi_1 + \phi_3}{\phi_3} - \frac{v\phi_1}{\phi_3 p}. \tag{B4}$$

Finally, the prominent firm 1 mixes over the entire interval $[p_2, v)$ with the distribution

$$F_1(p) = 1 - \frac{v\phi_1}{p(\phi_1 + \phi_2 + \phi_3)}, \tag{B5}$$

with remaining mass as an atom of size $\phi_1/(\phi_1+\phi_2+\phi_3)$ at v. It is straightforward to confirm that all firms are indifferent across all $p \in [p_2, v)$. In this equilibrium firms 1 and 2 earn the expected profits reported above in eq. (B2). However, the expected profit of firm 3 is

$$\tilde{\pi}_3 = \frac{v\phi_1\phi_3}{\phi_1 + \phi_2 + \phi_3} < \frac{v\phi_1\phi_3}{\phi_1 + \phi_3} = \pi_3 \tag{B6}$$

and so firm 3 is strictly worse off than it would be under our stable prices.

As noted in our concluding remarks, this is a setting in which our profit predictions do not coincide with those from a Nash equilibrium of the corresponding single-stage game. That game was studied by Inderst (2002, Section 3). We obtain equivalence by setting $\phi_2 = \phi_3$ (non-prominent firms are symmetric) and $\delta = 0$ in his paper (eliminating captives for non-prominent firms). Lemma 3 of Inderst (2002) suggests that the non-prominent firms mix over the same support, whereas we have an equilibrium in which their supports are non-overlapping.[40]

This setting can further illustrate how our pricing framework can be a component of a deeper model. Inspired by papers in which suppliers pay for prominence (Armstrong and Zhou, 2011; Chen and He, 2011), we introduce a prominence provider that sells that position to firms.

Suppose that all three firms begin with exclusive local customer bases, so that firm $i \in \{1, 2, 3\}$ would charge v to ϕ_i customers within its locality. A monopolist prominence provider, M, offers, in a preliminary (pre-pricing) stage, to bring one firm to national prominence. For example, a provider may be a department store that chooses a product to display in the window, or a website that shows a product on its home page or highlights it at the top of search results.

Specifically, M makes a take it or leave it offer to one firm, and commits to make a specified competitor prominent if the offer is refused. We assume firms have differently sized bases and label them so that $\phi_1 > \phi_2 > \phi_3$. Following the allocation of prominence, we assume that firms set stable prices in which the larger non-prominent firm is cheapest, as per Proposition 8.

Because firms' profits are increasing in the size of the prominent firm's base, and the largest firm is the cheapest when it is not prominent, M maximizes its fee (which is accepted in equilibrium) by offering prominence to the firm with the largest base, firm 1, while threatening to make their rival with the smallest base, firm 3, prominent if it refuses.

In essence, a small non-prominent firm has a threatening lean and hungry look, which strengthens the ability of M to extract a fee from a large firm. As such, in equilibrium, M bestows prominence upon firm 1. The prominence provider profits by exploiting the asymmetries between the largest and smallest firm. It then compounds this asymmetry by making firm 1 prominent. This is to the detriment of customers, for whom firm 1 is the worst choice.

B.4. **Advertising.** In Section 6 we sketched an extension to the independent awareness specification in which adverting influences consideration sets: risk-neutral profit-maximizing firms play a simultaneous-move game by choosing awareness $\alpha_i \in [0, 1]$. A firm's operating profit π_i is determined by stable prices. Its net profit $\pi_i - C_i(\alpha_i)$ deducts the cost of advertising. Firm i's

[40] It is possible that the source of the difference in predictions might lie within the derivation of the second displayed equation of the proof of Lemma 3 in the appendix of Inderst (2002).

advertising cost $C_i(\alpha_i)$ is smoothly increasing, convex, $C_i(0) = 0$, and $C_i'(0) < v$. When firms are asymmetric we index them so that $C_1'(\alpha) < \cdots < C_n'(\alpha)$ for all $\alpha \in (0, 1]$. This differs from McAfee (1994) by allowing asymmetric firms, while in Ireland (1993) advertising is free.[41] We write the firms' expected sales revenues as

$$\pi_i = \begin{cases} v\alpha_i \prod_{j \neq i}(1-\alpha_j) & \alpha_i > \max_{j\neq i}\{\alpha_j\} \text{ and} \\ v\alpha_i(1-\alpha_i)\prod_{j \notin \{i,k\}}(1-\alpha_j) & \alpha_i < \alpha_k \text{ where } \alpha_k = \max_{j \neq i}\{\alpha_j\}, \end{cases} \tag{B7}$$

and where both expressions apply when firm i ties to be the largest firm.

A firm's sales revenue reacts differently depending on whether it is the largest. The largest firm sets the highest (monopoly) price and so does not worry about undercutting, thus an increase in α_i raises expected revenue linearly. In contrast, smaller firms' prices must deter undercutting by larger firms. There are two competing effects: fixing prices, an increase in α_i scales up sales; however, it also forces a firm's price down. In fact,

$$\frac{\partial \pi_i}{\partial \alpha_i} = \begin{cases} v \prod_{j \neq i}(1-\alpha_j) & \alpha_i > \max_{j\neq i}\{\alpha_j\} \text{ and} \\ v(1-2\alpha_i)\prod_{j \notin \{i,k\}}(1-\alpha_j) & \alpha_i < \alpha_k \text{ where } \alpha_k = \max_{j \neq i}\{\alpha_j\}. \end{cases} \tag{B8}$$

For a smaller firm, revenue is decreasing in advertising exposure when a firm reaches a majority of customers, that is, when $\alpha_i > 1/2$. If not, then this revenue kinks upward as α_i passes through the maximum advertising exposure of competing firms. Specifically,

$$\frac{\lim_{\alpha_i \downarrow \max_{j\neq i}\alpha_j} \partial \pi_i/\partial \alpha_i}{\lim_{\alpha_i \uparrow \max_{j\neq i}\alpha_j} \partial \pi_i/\partial \alpha_i} = \frac{1-\max_{j\neq i}\alpha_j}{1-2\max_{j\neq i}\alpha_j} > 1, \tag{B9}$$

where the inequality is strict because (once dominated strategies have been eliminated) every firm chooses positive exposure. This implies that no firm chooses its advertising reach to be exactly equal to the maximum of others, and so there is a unique largest firm.

For smaller firms, advertising increases sales revenue only if $\alpha_i \leq 1/2$. This implies firms other than the largest restrict awareness to a minority of potential customers (no matter the cost).

The proofs of Lemma B1, and Propositions B1 and B2 can be found in Appendix C.

Lemma B1 (Properties of Advertising Choices). *In any Nash profile of advertising there is a unique largest firm, and all other firms advertise to a minority of customers.*

On the revenue side, the largest firm always faces an incentive to increase its exposure. Labeling this firm as k, it is straightforward to confirm that, in equilibrium, $\partial \pi_k/\partial \alpha_k \geq 1/2^{n-1}$. Hence, if $C'(1) < 1/2^{n-1}$ then firm k chooses $\alpha_k = 1$ and advertises to everyone.

[41] McAfee (1994) also related his paper to that of Robert and Stahl (1993), who specified the simultaneous (rather than sequential) choice of advertising exposure and price.

An advertising equilibrium is characterized by the specification of a leading (and largest) firm k, and n advertising choices which satisfy the n first-order conditions

$$\frac{C_k'(\alpha_k)}{v} = \prod_{j\neq k}(1-\alpha_j) \quad \text{and} \quad \frac{C_i'(\alpha_i)}{v} = (1-2\alpha_i)\prod_{j\notin\{i,k\}}(1-\alpha_j)\ \forall i\neq k. \tag{B10}$$

Because payoffs can be written to rely on a product of all firms' advertising choices, we can (and do, in the proof of Proposition B1) treat this as an aggregative game and solve accordingly (see, e.g., Anderson, Erkal and Piccinin, 2020; Nocke and Schutz, 2018). To characterize fully an equilibrium we also need to check for any non-local deviations. For example, one of the smaller firms $i \neq k$ has the option to deviate and choose $\alpha_i > \alpha_k$, and become the largest firm. The proof of Proposition B1 checks such remaining details.

Proposition B1 (Endogenous Advertising). *One firm chooses a strictly higher advertising than, sets the monopoly price, and only sells to customers who are uniquely aware of it. Others advertise to at most half of potential customers and set lower prices.*

In equilibrium, one leading firm advertises distinctly more than others. Proposition B1 does not identify this firm. If the advertising cost functions are not too different then any firm can play this role.[42] If they are different then the leading firm is one with relatively low advertising costs.[43] The other minority-audience firms can, however, be ordered given the structure of the advertising cost functions. For example, if $k = 1$ then advertising choices satisfy $\alpha_1 > \cdots > \alpha_n$.

If firms are symmetric ($C_i(\alpha_i) = C(\alpha_i)$ for all i) then the first-order conditions simplify appreciably. Writing α for the common advertising choice of the smaller firms,[44]

$$\frac{C'(\alpha_k)}{v} = (1-\alpha)^{n-1} \quad \text{and} \quad \frac{C'(\alpha)}{v} = (1-2\alpha)(1-\alpha)^{n-2}. \tag{B11}$$

A special case is when advertising is free (Ireland, 1993): there is a pathological equilibrium in which multiple firms choose $\alpha_i = 1$ and profits are driven to zero. Putting this aside, the "free advertising" case yields $\alpha = 1/2$ for $n-1$ firms, and complete coverage, $\alpha_k = 1$, for one firm.

Another case of interest is the random mailbox postings technology suggested by Butters (1977).[45] Equivalently, this is what McAfee (1994) called constant returns to scale in the availability of a

[42] This is true for the specifications of Ireland (1993) and McAfee (1994), under which costs are symmetric.

[43] Formally: there is some $k^\star$ such that there is an equilibrium in which any $k \in \{1, \ldots, k^\star\}$ leads the industry.

[44] The expressions in (B11) are precisely the equilibrium conditions stated by McAfee (1994).

[45] Suppose that customers are divided into $1/\Delta$ segments each of size Δ. Each segment corresponds to a mailbox. An advertisement costs $\gamma_i\Delta$ for firm i, and randomly hits one of the segments. Hence, with a total spend of $C_i(\alpha_i)$, a firm is able to distribute $C_i(\alpha_i)/(\gamma_i\Delta)$ advertisements. It follows that $\alpha_i = 1-(1-\Delta)^{C_i(\alpha_i)/(\gamma_i\Delta)}$. Taking the limit as $\Delta \to 0$, we observe that $(1-\Delta)^{C_i(\alpha_i)/(\gamma_i\Delta)} \to \exp(-C(\alpha_i)/\gamma_i)$. Solving suggests a cost specification $C_i(\alpha_i) = \gamma_i \log[1/(1-\alpha_i)]$ where (for asymmetric firms) we assume that $\gamma_n > \cdots > \gamma_1 > 0$.

firm's price.[46] This is obtained by setting $C(\alpha) = \gamma \log[1/(1-\alpha)]$, so that the marginal cost of increased advertising satisfies $C'(\alpha) = \gamma/(1-\alpha)$. Setting $\gamma = 1$ without loss of generality (γ only matters relative to the valuation v) and requiring $v > 1$ (otherwise all firms choose zero advertising) the relevant first-order conditions become

$$\frac{1}{v(1-\alpha_k)} = (1-\alpha)^{n-1} \quad \text{and} \quad \frac{1}{v} = (1-2\alpha)(1-\alpha)^{n-1}. \tag{B12}$$

These equations solve recursively. Substituting the second into the first, we find that $\alpha_k = 2\alpha$: no matter what the level of cost, the large firm reaches twice as many customers as each smaller firm. The solution for α satisfies the natural comparative-static property that α is increasing in the product valuation v, and so is decreasing in the advertising cost parameter γ.

Proposition B2 (Equilibrium with Symmetric Advertising Costs). *If advertising is free, as it is for Ireland (1993), then, in an equilibrium in which firms earn positive profits, the largest firm chooses maximum advertising exposure, while others advertise to half of potential customers. The largest firm earns twice the profit of each smaller firm.*

If the cost of advertising reach is determined by a random mailbox postings technology, as it is under the constant returns case of McAfee (1994), so that $C(\alpha) = -\gamma \log(1-\alpha)$, then the largest firm chooses advertising awareness equal to double that of the competing small firms. Advertising is increasing in customers' willingness to pay.

In both cases, with firms' labels chosen appropriately, the ensuing stable prices satisfy $p_i = v/2^{i-1}$.

The "independent awareness" advertising technology and its endogenous selection are not new to this paper: Ireland (1993) and McAfee (1994) both report that the leading firm is twice the size (in terms of advertising reach) and earns twice the profit of other firms. Other authors have, more recently, studied versions of the single-stage model but with a pre-pricing stage in which firms determine their captive shares and have also found asymmetric equilibrium advertising outlays (Chioveanu, 2008).[47] In contrast to those papers, our result maintains the prediction of asymmetric advertising intensities while producing stable prices. We identify (as the final claim of Proposition B2) an interesting pricing sequence: the margin of each firm in the ladder of stable prices is half that of the firm immediately above it.

[46] Under "constant returns" two merging firms do not save advertising costs. The probability that a customer considers firm i or j is $1-(1-\alpha_i)(1-\alpha_j)$. There are constant returns if $C(\alpha_i) + C(\alpha_j) = C(1-(1-\alpha_i)(1-\alpha_j))$.

[47] A similar result, but in a setting with a comparison site that advertises alongside sellers for its captive base can be found in an earlier version of Ronayne and Taylor (2022): Ronayne and Taylor (2020, Appendix W.3).

Appendix C. Other Omitted Proofs and Results

C.1. **Proofs Main-Text Results Omitted from Appendix A.** The proof of Lemma 1 in Appendix A focused on claim (iii). Here we complete the proofs of claims (i) and (ii).

Proof of Lemma 1. We first prove claim (i). If strictly positive prices tie, then a firm in that tie recognizes there is a positive mass of customers who compare them to another firm in that tie (and no other). Such a firm strictly improves by undercutting. Thus, any strictly positive prices within a profile are distinct. A special case is claim (i), when all prices are strictly positive.

Turning to claim (ii), if prices are undercut-proof, then no higher priced firm $j < i$ wishes to undercut a cheaper competitor i. If $p_i = 0$ then this is trivially true. If $p_i > 0$, then the strictly positive prices p_i and p_j are distinct and so firm j earns p_j from any comparisons that exclude any higher indexed (strictly cheaper) firms. These are all the comparison sets $B \subseteq \{1, \ldots, j\}$, each of which has mass $\lambda(B)$, that include firm j, which is incorporated by the use of the indicator $B_j \in \{0, 1\}$. Hence $p_j \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B)$ is the profit of j. The same logic says that j can achieve (arbitrarily close to) a profit $p_i \sum_{B\subseteq\{1,\ldots,i\}} B_j\, \lambda\,(B)$ by undercutting p_i and so winning any comparisons amongst the first i firms. Thus, the no-undercutting constraint is

$$p_j \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B) \geq p_i \sum_{B\subseteq\{1,\ldots,i\}} B_j\, \lambda\,(B) \quad \Leftrightarrow \quad p_i \leq \frac{p_j \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B)}{\sum_{B\subseteq\{1,\ldots,i\}} B_j\, \lambda\,(B)}. \tag{C1}$$

This must hold for all $j < i$, giving eq. (2) in the lemma. Now suppose that we have a price profile that satisfies eq. (2). The inequality (C1) holds for any pair $j < i$. This inequality is the correct no-undercutting constraint so long as the positive prices involved are distinct. However, the inequalities holding imply that the prices are distinct. To see this, note that

$$p_i \leq \frac{p_j \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B)}{\sum_{B\subseteq\{1,\ldots,i\}} B_j\, \lambda\,(B)} \leq \frac{p_j \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B)}{\lambda\,(\{i,j\}) + \sum_{B\subseteq\{1,\ldots,j\}} B_j\, \lambda\,(B)} < p_j, \tag{C2}$$

where the final strict inequality follows from our maintained "twoness" assumption. □

Proposition 5 in the main text is concerned with a triopoly under exchangeable consideration.

Proof of Proposition 5. Consider the profile from eq. (13). If firm 2 deviates upward to $\hat{p}_2 > p_2$ then we construct an equilibrium of the price-cutting game in which firm 3 charges p_3 (earning the profit matching that from stable prices) while 1 and 2 mix using the distributions

$$F_2(p) = \frac{(\lambda_1 + X_2)(p - p_2)}{pX_2} \quad \text{and} \quad F_1(p) = \frac{(\lambda_2 + X_2)(p - p_2)}{pX_2} \tag{C3}$$

over the interval $[p_2, \hat{p}_2)$ with (if $\hat{p}_2 < p_1$) both firms placing remaining mass at their initial prices. If $\hat{p}_2 = p_1 = v$ then the solutions above yield $F_2(p_1) = F_2(v) = 1$ and so only firm 1 plays an atom at its initial price $p_1 = v$. These strategies generate the required payoffs for both firms across the support of their mixed strategies, and they have no incentive to deviate elsewhere.

As noted in the text, the more difficult case involves firm 3 deviating upward to $\hat{p}_3 > p_3$. We construct an equilibrium in which firm 1 sets $\tilde{p}_1 = p_1$. We then (as explained in the main text) build mixed strategies for firms 2 and 3 over $[p_3, \min\{\hat{p}_3, p_2\})$ with distributions

$$F_2(p) = \frac{(\lambda_3 + X_3)(p - p_3)}{p(X_3 - X_2)} \quad \text{and} \quad F_3(p) = \frac{(\lambda_2 + X_3)(p - p_3)}{p(X_3 - X_2)}, \tag{C4}$$

where both firms place remaining mass at their initial prices. These distributions give both firms their pre-deviation expected profits across the support. As noted in the text, $F_3(p_2) = 1$. This means that if $\hat{p}_3 > p_2$ then firm 3 cannot play any price $\tilde{p}_3 \in (p_2, \hat{p}_3]$. We need to check that firm 3 does not wish to play such a price. By the argument in the text, that is true if and only if $\hat{p}_3(\lambda_3+X_2) \leq p_3(\lambda_3+X_3)$, which is satisfied for all $\hat{p}_3 \leq v$ if and only if $v(\lambda_3+X_2) \leq p_3(\lambda_3+X_3)$. Rearranging this gives the inequality (16) stated in the proposition.

So far we have shown that, if eq. (16) holds, there is a strategy profile in which firms 2 and 3 mix and obtain their required profits, and where they have no incentive to deviate anywhere else. However, we need to check that firm 1 does not wish to deviate from charging $\tilde{p}_1 = p_1 = v$. If $\hat{p}_3 < p_2$ then any deviation $p_1 \in (\hat{p}_3, p_2)$ should be to just below p_2 to capture the atom of firm 2. However, this is not profitable owing to the no-undercutting constraint. This means that we need to check firm 1's expected profit from deviating to some price $\tilde{p}_1 \in [p_3, \min\{\hat{p}_3, p_2\})$ which is (in essence) the "dance floor" across which firms 2 and 3 tango. By Lemma A1, that expected profit, $\pi_1(p_1)$, is quasi-convex in p_1 over the interval $[p_3, p_2)$, which means that

$$\begin{aligned}\pi_1(p_1) \leq \max\left\{\pi_1(p_3), \lim_{p_1 \uparrow p_2} \pi_1(p_1)\right\} &= \max\left\{p_3(\lambda_1 + X_3), p_2(\lambda_1 + X_2(1 - \lim_{p_1 \uparrow p_2} F_2(p_1))\right\} \\ &< \max\{v\lambda_1, p_2(\lambda_1 + X_2)\} = v\lambda_1.\end{aligned} \tag{C5}$$

The strict inequality holds for both of the components over which the maximum is taken. Specifically, $p_2(\lambda_1 + X_2(1 - \lim_{p_1 \uparrow p_2} F_2(p_1))) < p_2(\lambda_1 + X_2)$ because firm 2 places an atom at p_2. Also $p_3(\lambda_1 + X_3) < v\lambda_1$ because firm 1 strictly prefers not to undercut firm 3. Explicitly:

$$\begin{aligned}p_3(\lambda_1 + X_3) =& v\lambda_1 \frac{\lambda_1 + X_3}{\lambda_1 + X_2} \frac{\lambda_2 + X_2}{\lambda_2 + X_3} \\ =& v\lambda_1 \frac{\lambda_1\lambda_2 + X_2X_3 + (\lambda_1 + \lambda_2)X_2 + \lambda_2(X_3 - X_2)}{\lambda_1\lambda_2 + X_2X_3 + (\lambda_1 + \lambda_2)X_2 + \lambda_1(X_3 - X_2)} < v\lambda_1.\end{aligned} \tag{C6}$$

From this we conclude that firm 1 does not wish to step onto the dance floor.

In summary, we have constructed an equilibrium in relevant price-cutting games with expected profits equal to those from the claimed stable prices so long as $\hat{p}_3(\lambda_3 + X_2) \leq p_3(\lambda_3 + X_3)$, which is necessarily true if (16) holds. Now suppose that this fails, which means that a deviation $\hat{p}_3(\lambda_3 + X_2) > p_3(\lambda_3 + X_3)$ is possible. Our construction no longer works, as we now explain.

We know that firm 1 can achieve at least $v\lambda_1$ by charging $p_1 = v$. This means that the price p_3, and prices just above it, are strictly dominated for 1. It follows that the support of any mixed strategy for firm 1 lies strictly above p_3. If the support for the mixed strategy of firm 2 were to lie strictly above p_3, then firm 3 could achieve strictly more than its on path expected profit. (There would be a price $\tilde{p}_3 > p_3$ below the support of the competitors which would allow firm 3 to win all comparisons and so earn $\tilde{p}_3(\lambda_3 + X_3) > p_3(\lambda_3 + X_3)$.)

We conclude that firm 2 must mix down to p_3 or below. Suppose that p_3 is indeed the lower bound. (We can make the same argument for a strictly lower lower bound.) Firms 2 and 3 must mix continuously as we move up from that bound. Given that their expected profits are determined by capturing all comparisons at the lower bound, we can solve for their mixed strategies as before. As we move up the price range, we can evaluate $\pi_1(p)$ from firm 1 joining in at any price p. We have already showed that this is strictly less than $v\lambda_1$. We conclude that firm 1 never joins the dance floor as we move up through the prices. Eventually we reach the same conclusion as before: firm 3 has a strict incentive to set $p_3 = \hat{p}_3$, and our construction fails. □

Proposition 8 in the main text concerns the stability of prices in a "prominence" triopoly.

Proof of Proposition 8. The two industry-optimal undercut-proof profiles are derived in the main text. To prove the remainder of claim (i), consider price-cutting games following local deviations in the prices p_1, p_i, p_j, where $p_1 > p_i > p_j$. Specifically, consider firm i setting some $\hat{p}_i \in (p_i, p_1]$. The following strategies constitute a Nash equilibrium. Firms 1 and i mix over $[p_i, \hat{p}_i)$ via

$$F_1 = 1 - \frac{p_i}{p}, \quad F_i = 1 - \frac{(v-p)\phi_1}{p\phi_i}, \tag{C7}$$

with residual mass at p_1 and $\hat{p}_i$ respectively. Firm j sets $p_j = p_j$. Next consider j setting $\hat{p}_j \in (p_j, p_i]$. The following strategies constitute a Nash equilibrium. Firms 1 and j mix over $[p_j, \hat{p}_j)$ via

$$F_1 = 1 - \frac{p_j}{p}, \quad F_j = 1 - \frac{(v-p)\phi_1 - p\phi_i}{p\phi_j}, \tag{C8}$$

with residual mass placed at p_1 and $\hat{p}_j$, respectively; firm i sets $p_i = p_i$, earning $p_i\phi_i(1 - F_1(\hat{p}_j)) = p_i\phi_i(p_j/\hat{p}_j)$. We confirm i does not have an incentive to deviate to some $p \in [p_j, \hat{p}_j)$:

$$p\phi_i(1 - F_1(p)) = p_j\phi_i \leq p_j\phi_i(p_i/\hat{p}_j). \tag{C9}$$

For claim (ii) we address "non-local" deviations. The first case is that when the smaller non-prominent firm is cheaper: $i = 2$ and $j = 3$, and $\phi_2 > \phi_3$, with $p_1 > p_2 > p_3$ as stated in Proposition 8. We now prove that in any Nash equilibrium of the price-cutting game following prices p_1, p_2, and $\hat{p}_3 \in (p_2, v\phi_1/(\phi_1 + \phi_3))$, firm 3 gets a strictly greater profit than $p_3\phi_3$.

(1) No firm places an atom strictly below its initial price: if a firm did, then no competitor would ever price at or just above this atom, and so the firm could safely move the atom upward.

(2) The prominent firm uses a mixed strategy: if pure, each firm simply does not cut their price, and the prominent firm would find it profitable to undercut p_2 and capture all customers.

(3) For the prominent firm, prices $p < p_3$ are strictly dominated, as are $p \in (p_2, v\phi_1/(\phi_1 + \phi_3))$. A firm $k \in \{2, 3\}$ can secure all the relevant customers by charging p_3 and so can guarantee an expected profit of $p_3\phi_k > 0$. Take the highest price charged by any non-prominent firm. This wins customers with strictly positive probability (as it must to generate a positive expected profit) only if the prominent firm prices above it with strictly positive probability. Thus the prominent firm places an atom at $p_1 = v$, which implies its expected profit is $v\phi_1$.

(4) Excluding the atom at $p_1 = v$, consider the support of the prominent firm's (continuous) mixed strategy. This lies within the union of the competitors' supports: any other price can be safely raised (that is, without losing sales) which strictly raises profit. The support of any competitor lies within the support of the prominent firm, and for the same reason. It follows that the two supports (the prominent firm's, and the union of the competitors') coincide. At the lower bound of that support, the prominent firm sells to everyone, a mass $\phi_1 + \phi_2 + \phi_3$. This firm's profit is $v\phi_1$, and so that lower-bound price must equal $p_3 = v\phi_1/(\phi_1 + \phi_2 + \phi_3)$.

(5) Consider the interval $[p_3, p_2)$. Price p_3 is the lower bound of firm 1's support and therefore also for some $h \in \{2, 3\}$. There cannot be any gaps in the union of all firms' supports in $[p_3, p_2)$. For all other prices in that interval that h plays, h must be indifferent: $p_3\phi_h = p\phi_h(1 - F_1(p)) \Leftrightarrow F_1(p) = 1 - p_3/p$. For any $p \in [p_3, p_2)$ charged by $k \neq h$, k must be indifferent to p and the infimum of those, x, implying k's expected profit is $x\phi_k(1 - F_1(x)) = p_3\phi_k$ and so 1 must again price by the same CDF for p charged by k: $F_1(p) = 1 - p_3/p$ over all $p \in [p_3, p_2)$.

(6) No price in the initial profile is in $[p_3, p_2)$, and so there are no atoms. Within this interval there is no gap within the support of the prominent firm: if so, then there would be a gap in the support of the competitors' strategies, and so the prominent firm could safely (i.e., without losing sales) move a price from the bottom of the gap upward, and so strictly gain. Similarly, there is no gap with the union of opponents' supports. Given that, at least one $h \in \{2, 3\}$ is willing to set p_2, earning an expected profit of least $p_3\phi_h$. Because $F_1(p)$ does not depend on which firm has p in their support, the two non-prominent firms face the same expected profit when pricing against the prominent firm, and so 3 can guarantee at least $p_3\phi_3$ by setting p_2.

(7) Firm 3 earns $p_3\phi_3$ without deviating and at least that much by playing $p_3 = p_2$ in the deviant price-cutting game. Recall firm 1 never prices just above p_2. Hence, prices p slightly above p_2 earn $p\phi_3(1 - F_1(p_2)) = p_3\phi_3(p/p_2) > p_3\phi_3$. We conclude that any equilibrium renders the deviation profitable, so that the profile with firm 3 as the cheapest is not stable.

The second case is that when the larger non-prominent firm is cheaper, i.e., $i = 3$ and $j = 2$, with $p_1 > p_3 > p_2$ as stated in Proposition 8. Consider the price-cutting game following a deviation of firm 2 to some $\hat{p}_2 \in (p_3, p_1]$. The following strategy profile constitutes a Nash equilibrium.

All firms mix: firm 1 over $[p_2, \hat{p}_2)$, 2 over $[v\phi_1/(\phi_1 + \phi_2), \hat{p}_2)$ and 3 over $[p_2, v\phi_1/(\phi_1 + \phi_2))$ via

$$F_1(p) = 1 - \frac{p_2}{p}, \quad F_2 = 1 - \frac{(v-p)\phi_1}{p\phi_2}, \quad \text{and} \quad F_3 = 1 - \frac{(v-p)\phi_1 - p\phi_2}{p\phi_3}, \tag{C10}$$

with any residual mass for firms 1 and 2 placed at p_1 and $\hat{p}_2$, respectively. Firm 2 earns $\phi_2 p_2$, the same as without the deviation, when all three firms choose the stated regular prices.

Claim (iii) concerns n firms and $\phi_1 = \cdots = \phi_n \equiv \phi$. Without loss of generality, label the firms inversely to price so that $p_1 > \cdots > p_n > 0$ where firm 1 is the prominent firm. As usual, $p_1 = v$ in any industry-optimal undercut-proof profile. Now consider p_i for $i > 1$. The prominent firm's no-undercutting constraints (one for each local firm) are

$$v\phi_1 \geq p_i \left(\sum_{j=1}^{i} \phi_j\right) \quad \Leftrightarrow \quad p_i \leq \frac{v\phi_1}{\sum_{j=1}^{i} \phi_j} = \frac{v}{i}. \tag{C11}$$

These bind, and so $p_i = v/i$. It remains to check upward price deviations.

Suppose that firm $i > 1$ raises its price to $\hat{p}_i > p_i$. Consider this strategy profile in the associated price-cutting game. Firm 1 mixes over $[p_i, \hat{p}_i)$ using the distribution function

$$F_1(p) = 1 - \frac{p_i}{p}, \tag{C12}$$

and places all remaining mass at $p_1 = v$. Any cheaper firm, $j > i$ sets $p_j = p_j$ as a pure strategy. Any firm $j < i$, which satisfies $p_j \geq \hat{p}_i$ also plays a pure strategy, $p_j = p_j$. Any other firm $j \neq i$ satisfies $p_i < p_j < \hat{p}_i$. Such a firm mixes over $[p_{j+1}, p_j)$ using the distribution function

$$F_j(p) = 1 - \frac{v - jp}{p}. \tag{C13}$$

Finally, consider the deviant firm i. Take the lowest index k (and so highest price in the initial profile, p_k) which satisfies $p_k < \hat{p}_i$. Firm i mixes over $[p_k, \hat{p}_i)$ using the distribution function

$$F_i(p) = 1 - \frac{v - (k-1)p}{p}, \tag{C14}$$

and places all remaining mass at the deviant price $\hat{p}_i$.

This is a Nash equilibrium. Firm 1 earns $v\phi_1$, the same as without the deviation; by undercut-proofness of the initial prices, firm 1 does not do better with a price outside $[p_i, \hat{p}_i)$. For any local firm $j < i$, a price satisfying $p_i \leq p \leq \hat{p}_i$ is in firm 1's support and yields an expected profit of $p_i\phi$. If $p_j \leq \hat{p}_i$ then a firm can do no better than this, and optimally plays the prescribed strategy. If $p_j > \hat{p}_i$ then j is strictly better off with $\tilde{p}_j = p_j$, and so does so. □

C.2. **Proof of Additional Results in Appendix A.** Lemmas A2 to A4 are stated in Appendix A.

Proof of Lemma A2. Claim (1) holds trivially for $i = 1$. If it holds for all $j < i$ then

$$p_i = \min_{j<i} \left\{ p_j \frac{\lambda_j + X_j}{\lambda_j + X_i} \right\} \leq p_{i-1} \frac{\lambda_{i-1} + X_{i-1}}{\lambda_{i-1} + X_i} \leq p^\ddagger_{i-1} \frac{\lambda_{i-1} + X_{i-1}}{\lambda_{i-1} + X_i} = p^\ddagger_i, \tag{C15}$$

and so it holds also for i, and, by the principle of induction, for all i.

Claim (2) can also be proved inductively. It holds for $i = 2$. If it holds for all $j < i$ then

$$\begin{aligned} p_i = \min_{j<i} \left\{ p_j \frac{\lambda_j + X_j}{\lambda_j + X_i} \right\} &= \min_{j<i} \left\{ p^\ddagger_j \frac{\lambda_j + X_j}{\lambda_j + X_i} \right\} \\ &= \min_{j<i} \left\{ p^\ddagger_j \frac{\lambda_j + X_j}{\lambda_j + X_i} \left(\prod_{k=j+1}^{i} \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}} \right) \left(\prod_{k=j+1}^{i} \frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k} \right) \right\} \\ &= p^\ddagger_i \min_{j<i} \left\{ \frac{\lambda_j + X_j}{\lambda_j + X_i} \left(\prod_{k=j+1}^{i} \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}} \right) \right\} \\ &= p^\ddagger_i \left\{ 1, \min_{j<i-1} \left\{ \frac{\lambda_j + X_j}{\lambda_j + X_i} \left(\prod_{k=j+1}^{i} \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}} \right) \right\} \right\} = p^\ddagger_i. \end{aligned} \tag{C16}$$

The first line holds by the inductive hypothesis. The second line introduces product terms which cancel each other. The third line recognizes that the second product term multiplied by $p^\ddagger_j$ is $p^\ddagger_i$. The fourth line is obtained by separating out the first term for $j = i - 1$ and the remaining terms for $j < i - 1$. The final line is obtained by noting that for each $j < i - 1$,

$$\frac{\lambda_j + X_j}{\lambda_j + X_i} \prod_{k=j+1}^{i} \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k-1}} \leq \frac{\lambda_j + X_j}{\lambda_j + X_i} \prod_{k=j+1}^{i} \frac{\lambda_j + X_k}{\lambda_j + X_{k-1}} = \frac{\lambda_j + X_j}{\lambda_j + X_i} \frac{\lambda_j + X_i}{\lambda_j + X_j} = 1. \tag{C17}$$

The inequality in the chain holds because $X_k \geq X_{k-1}$ in each of the ratio terms, which means that such terms are each decreasing in λ_{k-1}. An upper bound for each term is obtained by replacing λ_{k-1} with $\lambda_j \leq \lambda_{k-1}$, where this inequality holds because $j \leq k - 1 \leq i - 1$ and (by assumption) firms below i are in size order. The claim holds by the principle of induction.

For Claim (3), suppose that firms are not in size order. Consider the first firm k that is out of order: $\lambda_1 \geq \cdots \geq \lambda_{k-1}$ but $\lambda_k > \lambda_{k-1}$. We know that $p_i = p_i^\ddagger$ for all $i \leq k$. This means that

$$p_k = p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k}, \tag{C18}$$

which (given that $X_{k-1} < X_k$) is strictly increasing in λ_{k-1}. The next price is

$$\begin{aligned} p_{k+1} &= \min\left\{p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_{k+1}}, p_k\frac{\lambda_k + X_k}{\lambda_k + X_{k+1}}\right\} \\ &= p_{k-1}\min\left\{\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_{k+1}}, \frac{\lambda_k + X_k}{\lambda_k + X_{k+1}}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k}\right\} \\ &= p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k}\min\left\{\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}}, \frac{\lambda_k + X_k}{\lambda_k + X_{k+1}}\right\} = p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_{k+1}}. \end{aligned} \tag{C19}$$

Suppose that we interchange the two firms; we swap λ_k and λ_{k-1}. Prices p_i for $i < k$ remain unchanged. We write $p_k^\diamond$ for the remaining maximal undercut-proof prices. Clearly,

$$p_k^\diamond = p_{k-1}\frac{\lambda_k + X_{k-1}}{\lambda_k + X_k} > p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k} = p_k, \tag{C20}$$

where the inequality holds because $\lambda_{k-1} < \lambda_k$. Next,

$$\begin{aligned} p_{k+1}^\diamond = \min&\left\{p_{k-1}\frac{\lambda_k + X_{k-1}}{\lambda_k + X_{k+1}}, p_k^\diamond\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}}\right\} \\ &= p_{k-1}\min\left\{\frac{\lambda_k + X_{k-1}}{\lambda_k + X_{k+1}}, \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}}\frac{\lambda_k + X_{k-1}}{\lambda_k + X_k}\right\} \\ &= p_{k-1}\frac{\lambda_k + X_{k-1}}{\lambda_k + X_k}\min\left\{\frac{\lambda_k + X_k}{\lambda_k + X_{k+1}}, \frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}}\right\} = p_{k-1}\frac{\lambda_k + X_{k-1}}{\lambda_k + X_k}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}} \\ &> p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k}\frac{\lambda_{k-1} + X_k}{\lambda_{k-1} + X_{k+1}} = p_{k-1}\frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_{k+1}} = p_{k+1}. \end{aligned} \tag{C21}$$

Straightforwardly all other prices for $i > k+1$ must also (at least weakly) rise.

For Claim (4), from Claim (3) prices are highest by placing firms in order. This maximizes the industry profit $\sum_{i=1}^n p_i X_i$ from comparator customers. The profit from captives is maximized when the largest firms charge the highest prices, and this is so when firms are in size order.

This claim also holds when we correct a misstep in the first group of firms: if $\lambda_1 \geq \cdots \geq \lambda_{k-1}$ but $\lambda_k < \lambda_{k-1}$, then switching $k-1$ and k raises the profits earned by the first k firms.

For Claim (5), consider again the procedure above of switching $k-1$ and k into the correct order. Firm $k-1$ benefits: this firm was previously indifferent to charging p_{k-1} and charging p_k, but now gains strictly because $p_k^\diamond > p_k$. Firm k benefits from this switch if

$$p_k(\lambda_k + X_k) \geq p_{k-1}(\lambda_k + X_{k-1}) \quad \Leftrightarrow \quad \frac{\lambda_{k-1} + X_{k-1}}{\lambda_{k-1} + X_k} > \frac{\lambda_k + X_{k-1}}{\lambda_k + X_k}, \tag{C22}$$

where this last inequality holds because $\lambda_k < \lambda_{k-1}$. This means that the first pair of misordered firms both gain by "correcting" their order, as well as raising the profits of all firms $i > k$. □

Proof of Lemma A3. Suppose that firm k (where necessarily $k > 1$) deviates upward to $\hat{p}_k > p_k$. There is some $i < k$ such that $p_{i+1} < \hat{p}_k \leq p_i$. For example, one case is where $i = k-1$, so that firm k deviates without crossing another price. An extreme case is when $i = 1$ and $\hat{p}_k = p_1 = v$. We build a mixed-strategy equilibrium (illustrated in Figure 1) in which all firms earn their (common) industry-optimal undercut-proof (or "regular") profits, $v\lambda$. Firms $j \in \{i, \ldots, k\}$ mix (with atoms and gaps) over the interval $[p_k, p_i]$. Others maintain their prices: $\tilde{p}_j = p_j$ for $j \notin \{i, \ldots, k\}$.

Given that firms $\{1, \ldots, k\}$ price (by construction below) at or above p_k, any firm $l \in \{k + 1, \ldots, n\}$ has no profitable deviation downward, and is constrained upward. Firms $j \in \{i, \ldots, k\}$ will (again by construction below) earn their regular profits, and this implies firms $\{1, \ldots, i-1\}$ cannot profitably deviate to within $[p_k, p_i]$. This is because an upper bound to the expected profit a lower-indexed firm can achieve by doing so is that from "throwing some $j \in \{i \ldots, k\}$ off the dance floor" and charging one of the prices j used to. Given the symmetry of firms, this gives the deviator the same expected profit j had before their ejection, $v\lambda$.

We now build the strategies used by the actively mixing firms $\{i, \ldots, k\}$. This group consists of the deviant firm k and all lower-indexed firms up to the firm i with the lowest initial price that weakly exceeds the deviant's new price. We consider three cases.

Case (i): a deviation that does not cross another price.

If $i = k - 1$, so that $\hat{p}_k \in (p_k, p_{k-1}]$, then firms k and $k-1$ mix continuously over the single interval of prices $[p_k, \hat{p}_k)$, and then place atoms (these are strictly positive if and only if $\hat{p}_k < p_{k-1}$) at their respective initial prices $\hat{p}_k$ and p_{k-1}. They mix using the same distribution $F(p)$. Taking

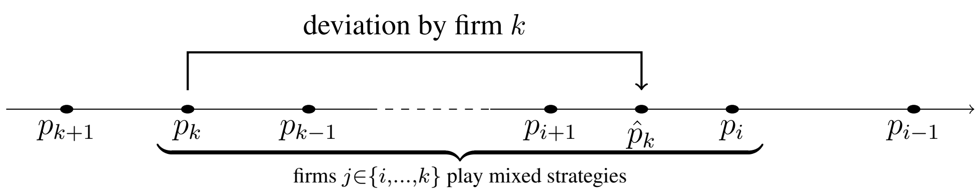

FIGURE 1. An Upward Deviation in Initial Price

the indifference condition for firm k (the same condition holds for firm $k-1$), $F(p)$ satisfies

$$\lambda v = p \sum_{B \subseteq \{1,\ldots,k-2\}} [\lambda(B \cup \{k\}) + (1-F(p))\lambda(B \cup \{k, k-1\})]$$
$$= p \sum_{x=0}^{k-2} \binom{k-2}{x} \left[\frac{I_{1+x}}{\binom{n}{1+x}} + \frac{[1-F(p)]I_{2+x}}{\binom{n}{2+x}} \right], \tag{C23}$$

where we define $I_1 \equiv n\lambda$.[48] The left-hand side is the expected profit of firm k. The right-hand side is the price p multiplied by the probability that firm k wins any comparisons. Firm k wins from comparisons which group it with any subset of $\{1, \ldots, k-2\}$ (these are the higher priced firms). Additionally, it wins comparisons that also include $k-1$ so long as $k-1$ prices above p, which happens with probability $1 - F(p)$. The second line computes the sizes of the relevant comparison sets. The summation over x ranges over the possible sizes of $B \subseteq \{1, \ldots, k-2\}$, noting that for each x there are $\binom{k-2}{x}$ relevant sets. Bringing in firm k, these comparison sets are of size $1+x$. The total mass of comparison sets of this size is I_{1+x}, and there are $\binom{n}{1+x}$ such sets. Hence $I_{1+x}/\binom{n}{1+x}$ is the size of each comparison set. Similar calculations apply when firm $k-1$ is added, where this time the combined mass of the relevant comparison sets is multiplied by $1 - F(p)$. The solution for $F(p)$ is strictly increasing in p, $F(p_k) = 0$, and $F(p_{k-1}) = 1$.

Case (ii): a deviation into the upper part of a higher price interval.

A second case is when firm k crosses the another price, so that $i < k-1$ or equivalently $\hat{p}_k > p_{k-1}$, and when that deviant price is sufficiently high in $(p_{i+1}, p_i]$. We consider $\hat{p}_k \in [p_i^\diamond, p_i]$ where $p_i^\diamond \in (p_{i+1}, p_i)$ is a threshold to be determined below.

We build profile in which there is a threshold $p_j^\diamond \in (p_{j+1}, p_j)$ for each $j \in \{i, \ldots, k-2\}$ such that the interval $(p_{j+1}, p_j^\diamond)$, which is the lower part of the interval between the regular prices of firms $j+1$ and j, is a gap in the mixing distributions of all firms. (This gap must exist because, for any price in that interval, a firm would prefer to undercut the price p_{j+1} in order to capture an atom which will be played by firm $j+1$.) For $j > i$, over the upper part of the interval $[p_j^\diamond, p_j)$ firms in $\{i, \ldots, j\} \cup \{k\}$ will mix continuously. Firm j then places an atom at p_j. Turning to the top interval between the prices of $i+1$ and i, firms i and k mix continuously over $[p_i^\diamond, \hat{p}_k)$ and then place remaining mass at their respective regular prices. Across the lowest interval $[p_k, p_{k-1})$ all firms mix continuously, with firm $k-1$ placing an atom at p_{k-1}. For $k = 4$ and $i = 1$ the basic plan of the equilibrium support of the firms' mixed strategies is illustrated in Figure 2.

For each $j \in \{i, \ldots, k-1\}$ (these are firms tempted to undercut following k's deviation), consider the interval of prices $[p_{j+1}, p_j)$. Firms $\{i, \ldots, j\} \cup \{k\}$ will actively used mixed strategies within

[48] Note that captive customers are included in the first line, with $\lambda(\emptyset \cup \{k\})$.

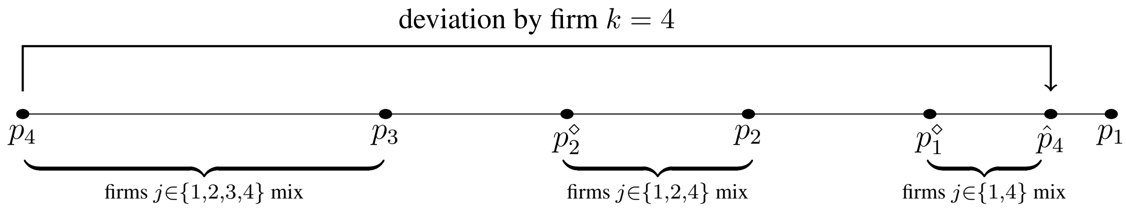

FIGURE 2. Mixing Supports for an Equilibrium of Type Case (ii)

this interval, where this is a strict subset for $j < k-1$. Note that there are $j-(i-1)+1$ such firms. Specify the cumulative distribution function, $F_j(p)$, to satisfy

$$\begin{aligned}\lambda v &= p \sum_{B\subseteq\{1,\ldots,i-1\}} \sum_{\widetilde{B}\subseteq\{i,\ldots,j\}} \lambda\left(B\cup\widetilde{B}\cup\{k\}\right)[1-F_j(p)]^{|\widetilde{B}|} \\ &= p\sum_{x=0}^{i-1}\sum_{y=0}^{j-i+1}\binom{i-1}{x}\binom{j-i+1}{y}\frac{I_{1+x+y}}{\binom{n}{1+x+y}}[1-F_j(p)]^y \end{aligned} \tag{C24}$$

The left-hand side is the (common) equilibrium expected profit of each firm. The right-hand side is k's expected profit when, at price p, all firms in $\{i,\ldots,j\}$ mix according to $F_j(p)$. The first summation collects together subsets of lower-indexed firms who always lose any comparisons with price p. The second summation deals with those who actively mix. For any set $\widetilde{B}$ there are $|\widetilde{B}|$ such firms, and so the price p wins comparisons against them all with probability $[1-F_j(p)]^{|\widetilde{B}|}$. The second line follows from the various masses of consideration sets. This is an indifference condition for firm k. The same condition also holds for other firms in $\{i,\ldots,j\}$. The solution for $F_j(p)$ satisfies $F_j(p_{j+1}) = 0$ and is strictly increasing. Defining $F_j(p_j) = \lim_{p\uparrow p_j} F_j(p)$, the $k-i$ solutions satisfy $F_{k-1}(p_{k-1}) < F_{k-2}(p_{k-2}) < \cdots < F_i(p_i) = 1$.

Looking across the whole interval $[p_k, p_i)$, we might aim to join the $k-1$ functions to form a single distribution. However, such a function would jump downward at each initial price (to zero), and so would not be a valid distribution function. We "smooth out" these jumps as follows. For each $j \in \{i,\ldots,k-2\}$ we define $p_j^\diamond \in (p_{j+1}, p_j)$ to be the unique solution to $F_{j+1}(p_{j+1}) = F_j(p_j^\diamond)$. We now stitch together a full cumulative distribution as follows. First, we define $F(p) = F_{k-1}(p)$ for $p \in [p_k, p_{k-1}]$. For all other $j \in \{i,\ldots,k-2\}$ we define

$$F(p) = \begin{cases} F_{j+1}(p_{j+1}) & p \in (p_{j+1}, p_j^\diamond] \\ F_j(p) & p \in (p_j^\diamond, p_j] \end{cases} \tag{C25}$$

This distribution function continuously increases from $F(p_k) = 0$ to $F(p_i) = 1$. It is constant for each interval $[p_{j+1}, p_j^\diamond]$ for each $j \in \{i,\ldots,k-2\}$, but otherwise is strictly increasing.

We are finally ready to build our strategy profile. Firm k (the deviant) mixes according to $F(p)$ across $p \in [p_k, \hat{p}_k)$ and places any remaining mass (if $\hat{p}_k < p_i$) at $\hat{p}_k$, and so plays an atom of size $1 - F(\hat{p}_k)$ there. Firm i also mixes according to $F(p)$ for $p \in [p_k, \hat{p}_k)$ and then places its remaining mass $1 - F(\hat{p}_k)$ at p_i. (This means that firms i and k behave symmetrically save for the location of their atoms.) A firm $j \in \{i+1, \ldots, k-1\}$ mixes according to $F(p)$ across $p \in [p_k, p_j)$ and then places its remaining mass $1 - F(p_j)$ at p_j. This construction yields a mixed-strategy Nash equilibrium profile so long as the deviant initial price satisfies $\hat{p}_k \geq p_i^{\diamond}$.

We note that the constructed distribution function $F(p)$ is used by all firms below their respective regular prices. At any point in the support of a firm's strategy (so that $F(p)$ is strictly increasing) the function is constructed so that each mixing firm earns the profit $v\lambda$. Any price within a gap (where $F(p)$ is constant) generates an expected profit strictly below $v\lambda$. (At such prices a firm performs strictly better by undercutting the next price below and capturing the atom of a rival.)

The strategy profile constructed requires k to place an atom at its deviant price $\hat{p}_k$. If $\hat{p}_k \in (p_{i+1}, p_i^{\diamond})$, however, the deviant price lies strictly within an interval across which $F(p)$ is constant and so generates an expected profit strictly below $v\lambda$. We adapt to cover that case next.

Case (iii): a deviation into the lower part of a higher price interval.

We now consider $\hat{p}_k \in (p_{i+1}, p_i^{\diamond})$. Here we construct an equilibrium in which firms follow the previous strategy profile up to some critical price $p^\star$, at which point firm i ceases to participate (in essence, this firm "leaves the dance floor") and places remaining mass at its initial price. Specifically, we define $p^\star$ to be the lowest price which satisfies $F(p^\star) = F_i(\hat{p}_k)$. Necessarily this critical price satisfies $p^\star < p_{i+1}$. We retain our definition of $F(p)$ for $p \leq p^\star$.

We now change firm i's strategy so that it mixes according to $F(p)$ for $p \in [p_k, p^\star]$ but then places remaining mass at its initial price, so that it has an atom at p_i of size $1 - F(p^\star) = 1 - F_i(\hat{p}_k)$.

This construction means that firm k earns the same profit as without the deviation, $v\lambda$, from playing the price $\hat{p}_k$. For $p > p^\star$ firm i no longer actively mixes, and so we modify the behavior of other firms to maintain appropriate indifferences for each $j \in \{i+1, k-1\}$, and prices in the interval $[p_{j+1}, p_j)$ that are at or above $p^\star$ we specify $F_j^\star(p)$ to satisfy

$$\lambda v = p \sum_{B \subseteq \{1,\ldots,i-1\}} \sum_{\widetilde{B} \subseteq \{i+1,\ldots,j\}} \left[1 - F_j^\star(p)\right]^{|\widetilde{B}|} \left[\lambda(B \cup \widetilde{B} \cup \{k\}) + (1 - F(p^\star))\lambda(B \cup \widetilde{B} \cup \{i,k\})\right]$$

$$= p \sum_{x=0}^{i-1} \sum_{y=0}^{j-i} \binom{i-1}{x} \binom{j-i}{y} [1 - F_j^\star(p)]^y \left[\frac{I_{1+x+y}}{\binom{n}{1+x+y}} + \frac{I_{2+x+y}[1 - F(p^\star)]}{\binom{n}{2+x+y}} \right]. \tag{C26}$$

This is an indifference condition for k, but also applies to other relevant firms. It adjusts eq. (C24) to treat i separately, as i prices above p with (constant) probability $1 - F(p^\star)$ for $p \in (p^\star, p_i)$. The solution satisfies $F_j^\star(p) > F_j(p)$ for $p > p^\star$ ($F_j^\star(p) = F_j(p)$ for $p = p^\star$). To proceed, we replace $F_j(p)$ with $F_j^\star(p)$ for $p > p^\star$. We then redefine $F(p)$ and the thresholds $p_j^\diamond$ appropriately. This modification ensures k is indifferent between $\hat{p}_k$ and slightly undercutting p_{i+1}. □

Proof of Lemma A4. If firm k (where necessarily $k > 1$) deviates upward to $\hat{p}_k > p_k$, then there is some $i < k$ such that $\hat{p}_k \in (p_{i+1}, p_i]$. Just as in the proof of Lemma A3, we build a mixed-strategy equilibrium (illustrated in Figure 1) in which all firms $j \in \{i, \dots, k\}$ mix (with atoms and gaps) over the interval $[p_k, p_i]$. Other firms maintain their initial prices: $\tilde{p}_j = p_j$ for $j \notin \{i, \dots, k\}$.

Each firm $l \in \{k + 1, \dots, n\}$ has no profitable deviation, for the usual reasons. A firm $l \in \{1, \dots, i - 1\}$ cannot profitably deviate to within $[p_k, p_i]$. An upper bound on its profit from doing so is what it would get by "stealing" the position of some $j \in \{i \dots, k\}$. Specifically, suppose l sets a price in $[p_k, p_i]$ and could arrange for j to price above it. Under independent awareness, l's expected profit from a price position in competition with other firms is the same as it was for j, save for the fact that their expected profits are scaled by α_l and α_j, respectively. However, those scalings also apply to profits obtained from regular prices. This means that l does not profitably gain by "stepping on to the dancefloor" with higher-indexed firms.

We now build the strategies used by the actively mixing firms $\{i, \dots, k\}$.

Case (i): a local deviation upward to $\hat{p}_k \in (p_k, p_{k-1}]$.

Consider a strategy profile in which any firm $j \notin \{k - 1, k\}$ maintains its initial price, while firms $j \in \{k - 1, k\}$ continuously mix over $[p_k, \hat{p}_k)$ according to distribution functions

$$F_j(p) = \frac{1}{\alpha_j}\left(1 - \frac{p_k}{p}\right), \tag{C27}$$

and place remaining mass at their initial prices. These CDFs satisfy $F_j(p_k) = 0$. Because $\alpha_k \leq \alpha_{k-1}$ implies $F_{k-1}(p) \leq F_k(p)$, we need only check that $F_k(p)$ is a valid CDF:

$$F_k(p) \leq 1 \quad \Leftrightarrow \quad p \leq \frac{p_k}{1 - \alpha_k} = p_{k-1}, \tag{C28}$$

which holds because $\hat{p}_k \leq p_{k-1}$. Prices within this interval generate the expected profit

$$\pi_k(p) = p\alpha_k \left(1 - \alpha_{k-1}F_{k-1}(p)\right) \prod_{j>k}(1 - \alpha_j) = p_k\alpha_k \prod_{j>k}(1 - \alpha_j), \tag{C29}$$

which is the profit of firm k from regular prices, π_k. A similar calculation holds for $k - 1$.

For the remaining cases, firm k deviates to $\hat{p}_k \in (p_{i+1}, p_i]$ where $i < k - 1$.

Case (ii): a deviation to the upper part of a higher price interval, so that $\hat{p}_k \in ((1-\alpha_k)^{1/2} p_i, p_i]$.

We write $F_j(p)$ for the mixed strategy of j. In the lowest interval of prices $[p_k, p_{k-1})$ we set

$$F_j(p) = \frac{1}{\alpha_j}\left(1 - \left(\frac{p_k}{p}\right)^{1/(k-i)}\right), \tag{C30}$$

for each firm $j \in \{i, \ldots, k\}$. These are well-defined continuously increasing CDFs. Note that

$$\lim_{p \uparrow p_{k-1}} F_j(p) = \frac{1}{\alpha_j}\left(1 - (1-\alpha_k)^{1/(k-i)}\right) \leq \frac{1}{\alpha_k}\left(1 - (1-\alpha_k)^{1/(k-i)}\right) < 1, \tag{C31}$$

and so these solutions require $k-1$ (this firm faces the constraint $p_{k-1} \leq p_{k-1}$) to place an atom at its initial price p_{k-1}. The expected profit for j from any price within this interval is,

$$p\alpha_j \prod_{l \neq j}(1 - \alpha_l F_l(p)) = p_k \alpha_j \prod_{l>k}(1-\alpha_l) = v\alpha_j \prod_{l>1}(1-\alpha_l), \tag{C32}$$

which is equal to the profit from regular prices for firm j, π_j.

Next, for each $j \in \{i+1, \ldots, k-2\}$ consider the price interval $[p_{j+1}, p_j)$. This interval lies above the regular price of any firm $l \in \{j+1, \ldots, k-1, k+1, \ldots, n\}$, and so $F_l(p) = 1$ for all such firms. The firms $l \in \{i, \ldots, j\} \cup \{k\}$ (there are $j-i+2$ such firms) all actively mix via

$$F_l(p) = \begin{cases} \dfrac{1}{\alpha_l}\left(1 - (1-\alpha_k)^{1/(j-i+2)}\right) & p \in \left[p_{j+1}, (1-\alpha_k)^{1/(j-i+2)} p_j\right) \\ \dfrac{1}{\alpha_l}\left(1 - \left(\dfrac{p_j(1-\alpha_k)}{p}\right)^{1/(j-i+1)}\right) & p \in \left[(1-\alpha_k)^{1/(j-i+2)} p_j, p_j\right) \end{cases} \tag{C33}$$

$$= \max\left\{\lim_{p^\diamond \uparrow p_{j+1}} F_l(p^\diamond), \frac{1}{\alpha_l}\left(1 - \left(\frac{p_j(1-\alpha_k)}{p}\right)^{1/(j-i+1)}\right)\right\}. \tag{C34}$$

This means that the CDF remains flat (there is a gap in the support) across the lower part of the interval $[p_{j+1}, p_j)$. For any price in such a gap, a firm would prefer to deviate and undercut the initial price p_{j+1} given that firm $j+1$ places an atom there. Indeed,

$$\lim_{p \uparrow p_j} F_l(p) = \frac{1}{\alpha_l}\left(1 - (1-\alpha_k)^{1/(j-i+1)}\right) \leq \frac{1}{\alpha_k}\left(1 - (1-\alpha_k)^{1/(j-i+1)}\right) < 1, \tag{C35}$$

and so firm j places an atom at its regular price position. Any price $p \in \left[(1-\alpha_k)^{1/(j-i+2)} p_j, p_j\right)$ generates the industry-optimal profit for any mixing firm. For example, firm k gets

$$p\alpha_k \left[\prod_{h \in \{j+1, \ldots, k-1, k+1, \ldots, n\}} (1-\alpha_h)\right]\left[\prod_{l \in \{i, \ldots, j\}} (1 - \alpha_l F_l(p))\right]$$

$$= p_j\alpha_k \left[\prod_{h\in\{j+1,\dots,k-1,k+1,\dots,n\}} (1-\alpha_h) \right] (1-\alpha_k) = p_n\alpha_k = \pi_k. \quad \text{(C36)}$$

For the top price interval (this is for $j = i$), the same formulae apply up to $\hat{p}_k$. That is,

$$F_l(p) = \begin{cases} \frac{1}{\alpha_l}\left(1-(1-\alpha_k)^{1/2}\right) & p \in [p_{i+1}, p_i(1-\alpha_k)^{1/2}) \\ \frac{1}{\alpha_l}\left(1-\left(\frac{p_i(1-\alpha_k)}{p}\right)\right) & p \in [p_i(1-\alpha_k)^{1/2}, p_i). \end{cases} \quad \text{(C37)}$$

The two firms $l \in \{i, k\}$ then place their remaining mass on their initial prices. (If $\hat{p}_k = p_i$ then the CDFs described above specify $F_k(p_i) = 1$ and so firm k has no atom.)

Case (iii): a deviation to the lower part of a higher price interval, so that $\hat{p}_k \in (p_{i+1}, p_i(1-\alpha_k)]$.

For this case we build the same strategy profile that we would use if $\hat{p}_k = p_{i+1}$. There, firm i does not participate, and always chooses its initial price so that $p_i = p_i$. If $i = k-2$ then we build the strategy profile described in case (i), and if $i < k-2$ then we use the strategy profile from case (ii). In both cases, for prices just below p_{i+1}, the two firms k and $i+1$ mix. Specifically, for $p \in [p_{i+1}(1-\alpha_k)^{1/2}, p_{i+1})$ and $l \in \{k, i+1\}$,

$$F_l(p) = \frac{1}{\alpha_l}\left(1-\left(\frac{p_{i+1}(1-\alpha_k)}{p}\right)\right). \quad \text{(C38)}$$

We know $\alpha_k \leq \alpha_l$ and so $F_l(p) \leq F_k(p)$. Moreover, $\lim_{p\uparrow p_{i+1}} F_k(p) = 1$. This means that k places all mass continuously below p_{i+1}, and so does not use any prices within $(p_{i+1}, \hat{p}_k]$. However, for $\alpha_k < \alpha_l$, l places an atom at p_{i+1}. Firms earn their industry-optimal expected profits.

Notice that firm k places all mass below p_{i+1}, which captures the atom of firm $i+1$. We need to check that k does not get more than its profit without its deviation, π_k, by charging $\hat{p}_k$:

$$\hat{p}_k\alpha_k \prod_{j\in\{i+1,\dots,k-1,k+1,\dots,n\}} (1-\alpha_j) = \frac{\hat{p}_k\alpha_k}{1-\alpha_k}\prod_{j=i+1}^{n}(1-\alpha_j) \leq \pi_k \quad \Leftrightarrow \quad \hat{p}_k \leq p_i(1-\alpha_k), \quad \text{(C39)}$$

where this inequality holds by assumption in this case.

Case (iv): a deviation to an intermediate range, so that $\hat{p}_k \in (p_i(1-\alpha_k), p_i(1-\alpha_k)^{1/2}]$.

Case (iii) of Lemma A3's proof is similar in nature. For $p \in [p_k, p_{k-1})$, the lowest interval, define:

$$F_l^+(p) = \frac{1}{\alpha_l}\left(1-\left(\frac{p_k}{p}\right)^{1/(k-i)}\right) \quad l \in \{i, \dots, k\} \quad \text{(C40)}$$

$$F_l^-(p) = \frac{1}{\alpha_l}\left(1-\left(\frac{p_k}{p}\frac{\hat{p}_k}{p_i(1-\alpha_k)}\right)^{1/(k-i-1)}\right) \quad l \in \{i+1, \dots, k\}. \quad \text{(C41)}$$

Next, for each $j \in \{i+1, \ldots, k-2\}$ and the corresponding price interval $[p_{j+1}, p_j)$, define

$$F_l^+(p) = \frac{1}{\alpha_l}\left(1 - \min\left\{(1-\alpha_k)^{\frac{1}{j-i+2}}, \left(\frac{p_j(1-\alpha_k)}{p}\right)^{\frac{1}{j-i+1}}\right\}\right) \quad l \in \{i, \ldots, j\} \cup \{k\} \quad \text{(C42)}$$

$$F_l^-(p) = \frac{1}{\alpha_l}\left(1 - \min\left\{\left(\frac{\hat{p}_k}{p_i}\right)^{\frac{1}{j-i+1}}, \left(\frac{p_j}{p}\frac{\hat{p}_k}{p_i}\right)^{\frac{1}{j-i}}\right\}\right) \quad l \in \{i+1, \ldots, j\} \cup \{k\}. \quad \text{(C43)}$$

For the largest-awareness firm i and $p \in [p_k, p_i)$ define

$$F_i(p) = \min\left\{F_i^+(p), \frac{1}{\alpha_i}\left(1 - \frac{p_i(1-\alpha_k)}{\hat{p}_k}\right)\right\}, \quad \text{(C44)}$$

and let i place its remaining mass at the initial price p_i.

For other firms $l \in \{i+1, \ldots, k\}$ and prices $p < p_l$ define

$$F_l(p) = \begin{cases} F_l^+(p) & F_i(p) = F_i^+(p) \\ F_l^-(p) & \text{otherwise,} \end{cases} \quad \text{(C45)}$$

with remaining mass at the firm's initial price (so that $F_l(p) = 1$ for $p \geq p_l$). □

C.3. **Proof of Additional Results in Appendix B.**

Proof of Lemma B1. The first claim follows from the argument in the text. The second claim holds because if a firm is not the largest then the derivative of its profit π_i (ignoring awareness costs) with respect to α_i has the same sign as $1 - 2\alpha_i$, which is strictly negative if $\alpha_i > \frac{1}{2}$. □

Proof of Proposition B1. We seek an equilibrium of the advertising game, where firm i's payoff is $\pi_i - C_i(\alpha_i)$. We write $\alpha_i^\star$ for the (pure strategy) equilibrium choice of firm i. Recalling that we ordered firms according to $C_i'(\cdot)$, we will show there is an equilibrium in which firm 1 (the firm with the lowest (marginal) cost of advertising) chooses $\alpha_1^\star > \max_{i\neq 1}\{\alpha_i^\star\}$. Advertising choices (for $\alpha_i \in (0,1)$) satisfy the first-order conditions (B10). With $k = 1$, those become

$$\frac{C_1'(\alpha_1)}{v} = \prod_{j>1}(1-\alpha_j) \quad \text{and} \quad \frac{C_i'(\alpha_i)}{v} = \frac{1-2\alpha_i}{1-\alpha_i}\prod_{j>1}(1-\alpha_j) \ \forall i > 1, \quad \text{(C46)}$$

where for the set of $n-1$ first-order conditions for $i > 1$ we have divided by $1 - \alpha_i$ knowing that the equilibrium must satisfy $\alpha_i^\star \leq 1/2$. Define $R = \prod_{j>1}(1-\alpha_j)$. If $Rv > C_i'(0)$, then we define $A_i(R)$ to be the $\alpha_i \in (0, 1/2)$ that satisfies firm i's first-order condition. That is,

$$\frac{C_i'(A_i(R))}{v} = \frac{1-2A_i(R)}{1-A_i(R)}R. \quad \text{(C47)}$$

This is uniquely defined because the left-hand side is continuously increasing in $A_i(R)$ and the right-hand side is continuously decreasing (beginning from 1 and decreasing to zero at $A_i(R) = 1/2$). Furthermore, this solution is strictly increasing in R. If $Rv \leq C_i'(0)$ (the right-hand side lies everywhere below the left-hand side) then we set $A_i(R) = 0$. To find R we seek a solution to $R = \prod_{j>1}(1 - A_j(R))$. The right-hand side lies within $[0, 1]$, begins above zero, and is decreasing in R, and so we can find a unique solution $R^\star$. We then set $\alpha_i^\star = A_i(R^\star)$ for $i > 1$. Finally, we can find $\alpha_1^\star$, where $\alpha_1^\star = 1$ if $C_1'(1) < vR^\star$, but otherwise $\alpha_1^\star$ is the unique positive solution (which satisfies $\alpha_1^\star > \alpha_i^\star$ for $i > 1$) to the condition $C_1'(\alpha_1) = vR^\star$. The remaining checks are non-local:

(i) Firm 1 deviates to $\hat{\alpha}_1 \leq \alpha_j^\star$ where $j\colon \alpha_j^\star = \max_{i>1}\{\alpha_i^\star\}$. Firm j satisfies a first-order condition at $\alpha_j^\star$. Therefore, the best such deviation for 1 is to $\hat{\alpha}_1 = \alpha_j^\star$ (1's revenue (cost) curve is the same (flatter) for $\hat{\alpha}_1 \in [0, \alpha_j^\star]$ than j's over the same interval when $\alpha_i = \alpha_i^\star$ for $i \neq j$). By continuity and 1's first-order condition, 1's profit at any $\hat{\alpha}_1 \geq \alpha_j^\star$ is less than at $\alpha_1^\star$.

(ii) Firm $i > 1$ deviates to $\hat{\alpha}_i \geq \alpha_1^\star$. Firm 1 satisfies their first-order condition at $\alpha_1^\star$. Therefore, the best such deviation for $i > 1$ is to $\hat{\alpha}_i = \alpha_1^\star$ (i's revenue (cost) curve is flatter (steeper) for $\hat{\alpha}_i \in [\alpha_1^\star, 1]$ than 1's over the same interval when $\alpha_i = \alpha_i^\star$ for $i > 1$). But by continuity and i's first-order condition, i's profit at any $\hat{\alpha}_i \leq \alpha_1^\star$ is less than at $\alpha_i^\star$.

The other claims of the proposition follow from Lemma B1. □

Proof of Proposition B2. When all firms have zero costs, we put aside trivial equilibria where more than one firm chooses $\alpha_i = 1$. From symmetry it follows that although the profile of equilibrium advertising we report is unique, the assignment of firms is not. Subject to this disclaimer, one firm will advertise with the outright highest intensity, and we label this firm 1.

By eq. (B7), the profit of firm 1 is strictly (and linearly) increasing in α_1 for any $\alpha_i < 1$ for $i > 1$, hence $\alpha_1^\star = 1$. Given $\alpha_1^\star = 1$, eq. (B7) shows that the profit of the non-largest firms is maximized at $\alpha_i = 1/2$ for any $\alpha_j < 1$ where $j \neq 1, i$, hence $\alpha_i^\star = 1/2$ for $i > 1$.

For positive costs, firms $i, j > 1$ must satisfy their first-order conditions given in eq. (B10) but with $C_i = C$. Taking the ratio of i's and j's condition yields

$$\frac{C'(\alpha_i)}{C'(\alpha_j)} = \frac{(1 - 2\alpha_i)(1 - \alpha_j)}{(1 - 2\alpha_j)(1 - \alpha_i)}. \tag{C48}$$

If $\alpha_i > (<)\alpha_j$ the left-hand side $> 1(< 1)$ but the right-hand side $< 1(> 1)$. However, if $\alpha_i = \alpha_j$, eq. (C48) is satisfied. Hence $\alpha_i^\star = \alpha_j^\star$. Letting $C(\alpha) = -\log(1 - \alpha)$ gives eq. (B12), the solution to which gives the values of $\alpha_1^\star$ and $\alpha_i^\star$ for $i > 1$, and that $\alpha_1^\star = 2\alpha_i^\star$. Similar reasoning to that in the proof of Proposition B1 rules out profitable non-local deviations. □