Multiproduct Quality Competition: Fighting Brands and Product Line Pruning

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Firms selling multiple quality-differentiated products frequently alter their product lines when a competitor enters the market. We present a model of multiproduct monopoly and duopoly using a general "upgrades" approach that yields a powerful analytical framework. We provide an explanation for the common strategies of using "fighting brands" and of product line "pruning." The optimal strategy depends on whether entry prompts an incumbent to expand or contract its total output. We also present a general condition that guarantees that a monopolist will sell but a single product. Our model addresses other issues, including intertemporal price discrimination and "damaged goods." (*JEL* D40, L10, M31)

Incumbent firms often adjust their product lines in response to competition. Sometimes they remove products from the market, thereby "pruning" their product lines. This was one response of Procter & Gamble to private label brands in the early 1990's.[1] At other times, an incumbent responds to competition by expanding its product line, often to include a lower-quality good called a "fighting brand."[2] This happened following AMD's entry into the market for 386DX microprocessors, when Intel released the 486SX as a companion to its higher-quality 486DX processor.[3]

Fighting brands are extremely widespread. For instance, AT&T launched Lucky Dog Telephone to help compete against lower-priced "dial around" phone carriers.[4] Brian Adamik of Yankee Group[5] commented, "They've introduced a fighting brand in the market that goes after price-sensitive consumers, while allowing AT&T to be their premier brand in the market."[6] BPL announced that it would introduce a fighting brand in the color television market to take on competition from Chinese and local brands. The head of corporate brand management at BPL noted that its fighting brand would be "a separate brand for the price-sensitive low-end CTV market."[7] And in the Indian market for electric fans, the growth in fringe competi-

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[1] "And in what amounts to virtual apostasy at a company that never gave up on struggling brands, P&G is consolidating some weak products with stronger siblings, while dumping others." *Business Week,* July 19, 1993.

[2] The "fighting brand" terminology is used in the management literature (Michael E. Porter, 1980; Kevin L. Keller, 1998) and by industry participants. In response to new entrants in the credit card business, American Express introduced the Optima card in 1991. Chairman James D. Robinson said, "Expect to see Optima as a fighting brand ... I think that we've got initiatives going in all the areas where there is competitive pressure" (*The Wall Street Journal,* July 18, 1991).

[3] We assess the AMD-Intel case (first highlighted by Raymond J. Deneckere and R. Preston McAfee, 1996) in Section V.

[4] Dial-around carriers allow consumers to bypass their home carriers by dialing a special access code.

[5] Yankee Group provides industry research and consulting services.

[6] "AT&T Launches Lucky Dog Telephone in Response to 'Dial-Around' Carriers." *The Wall Street Journal Europe,* October 8, 1998.

[7] "BPL Adopts Multi-Brand Strategy for CTVs." *The Economic Times,* May 9, 2000.

tion led Usha to introduce a discount fan called the Racer.[8]

Product line pruning is also common.[9] Timex recently announced that it would be removing a number of its lower-priced watches from the Indian market in response to growing competition in the low end from Titan,[10] while Mitsubishi announced the phasing out of low-end versions of its Trium mobile phones in response to a supply glut in that segment.[11] And after more than a century in the piano business, Kimball International in 1996 discontinued all but its prestigious Bosendorfer model following the capture of the low and middle markets by Japanese and Korean firms.[12]

One feature common to these (and many other) examples is that the increase in competition manifested itself in the low end of the market. Since the incumbent's decision to either expand or contract its product line amounts to a decision about how to segment the market, these examples suggest that understanding how price discrimination opportunities change with the presence of low-end competition might explain the prevalence of both fighting brands and product line pruning. Our main results imply that this is indeed the case.

Consider for instance the IBM LaserPrinter.[13] A single version was initially sold, capable of printing ten pages per minute. The absence of a lower-quality version suggests that IBM's gains from serving the low end of the market were not large enough to justify introducing a substitute product for its high-quality unit (which would have limited IBM's ability to extract surplus from high-value users). However, following Hewlett-Packard's entry into the market with its LaserJet IIP, a lower-quality substitute for IBM's LaserPrinter, IBM needed to reevaluate its product line strategy. On one hand, Hewlett-Packard's entry meant that a substitute for IBM's LaserPrinter was already on the market. Inasmuch as a desire to avoid offering such a substitute restrained IBM in the first place, it might have been sensible to introduce one following Hewlett-Packard's move. On the other hand, even though a lower-quality substitute would be on the market regardless of IBM's decision, introducing its own such product would have exacerbated (from IBM's perspective) the substitution possibilities for consumers. In fact, IBM decided to introduce a fighting brand, the LaserPrinter E, which was identical to its original LaserPrinter except for the fact that its software limited its printing to five rather than ten pages per minute. This suggests that the desire to compete in the newly opened low-end market was strong enough that IBM was willing to bear the additional erosion on its high-end profits resulting from its own entry into that market.

However, as noted above, not all firms mimic IBM's strategy of expanding its product line in response to low-end competition; some firms choose to prune their product lines when facing such competition. Therefore, the full explanation of the influence of competition on product line choices is more complicated than that suggested by the IBM example. Consider DEC, which until 1996 sold a range of PCs to both home and business customers. By offering lower-quality PCs targeted at home users, DEC provided a potential substitute to its business clients, but clearly felt this downside was worth bearing for the opportunity to serve the low-end market. In 1996, however, DEC faced increasing competition in the home computer market and responded by exiting that market to focus on its high-end desktops and servers.[14] Despite the contrast with IBM's decision, DEC's reaction also can be understood in terms of changing opportunities for market segmentation. In particular, DEC's response suggests that competition reduced the available profits in the low-end market enough that it became more important to attempt to preserve profits on its high-end

[8] "Summer of Discontent." *The Business Standard,* July 3, 2001.

[9] Philip Kotler (1965) and John A. Quelch and David Kenny (1994) emphasize the benefits of regular pruning of product lines. Kotler writes, "As the pace of competition quickens and as consumer tastes become surfeited, the need for pruning company product lines of casualties becomes as great as that for finding replacements."

[10] "Timex to Exit Mass Market for Watches." *The Economic Times,* April 30, 2001.

[11] "Mitsubishi to Phase Out Low-End Mobile Hand Sets." *The Economic Times,* November 24, 2001.

[12] "Piano Industry Off-Key." *Baltimore Sun,* February 18, 1996.

[13] For a more detailed review of this case, see Section V.

[14] "Digital Equipment to Exit Home-Computing Sector." *The Asian Wall Street Journal,* January 31, 1996.

products by not contributing to a mass of low-priced substitutes. Pruning its product line was the means to accomplish this.

We argue in this paper that it is indeed true that competition's influence on market segmentation opportunities is crucial to understanding the changing product line decisions of multiproduct firms. Moreover, we show that it is straightforward to understand when either expansion or contraction of a product line will occur. More precisely, we identify demand and competitor characteristics that make either fighting brands or pruning optimal responses to heightened competition. This is important since, while much is known about the optimal product line choice of multiproduct monopolists, relatively little is known about competition between multiproduct firms, and virtually nothing is known about the decision of a firm to alter its vertically differentiated product line (by pruning it or introducing a fighting brand) as a response to the arrival of new competition. (We relate our work to the existing literature in Section I.)

In our analysis we presume that entry by a single firm has occurred in a market originally dominated by a monopolist. The duopolists compete in quantities, each potentially offering a range of quality-differentiated products. Whether the incumbent will choose to extend or contract its product line depends closely on the shape of the marginal revenue curves in the market. When marginal revenue is everywhere decreasing, the incumbent never responds to the entrant by expanding its product line. Rather, entry tends to be associated with the pruning of lower-quality products from the incumbent's menu, meaning that it chooses to "focus on quality." However, when marginal revenue is increasing in some regions, an incumbent may find it optimal to respond to entry through the introduction of a lower-quality product.

It might seem that marginal revenue that is increasing in some regions represents an unimportant case. However, there are two important points to keep in mind when evaluating this assumption. First, firms with constant marginal cost certainly do not choose output in regions where marginal revenue is increasing, i.e., in equilibrium firms operate in regions where marginal revenue is decreasing. We are not suggesting, therefore, that marginal revenue will be increasing in a region local to that observed in equilibrium. Second, while everywhere decreasing marginal revenue is a convenient technical assumption, it is in fact incompatible with some very plausible demand structures. For example, as we show in Section II, subsection B, the existence of a bimodal distribution of consumer preferences, corresponding perhaps to segments of home and business users, readily generates marginal revenue that is increasing in some regions.

The intuition for our results is straightforward. Suppose that marginal revenue is decreasing, so that firms face the typical Cournot incentives to reduce their own output as the output of a rival increases. Consider a simple example in which the incumbent originally marketed both a low- and a high-quality product, and suppose that the entrant is able to offer only a low-quality good. When confronted with positive output by the entrant, the incumbent restricts output in the low-quality market. Furthermore, since the total production in the low-quality market also adversely effects the price of the high-quality good, the incumbent faces additional pressure to lower its own output in the low-quality market. If the entrant finds it optimal to produce beyond a certain level, the best course of action for the incumbent is to cede that market to the entrant in an effort to preserve margins on the high-quality good—it will prune its product line in order to focus on quality.

As noted above, marginal revenue that is increasing in some regions can be consistent with plausible demand structures, such as the existence of distinct "market segments." This leads to the possibility that a sufficiently large intrusion by a competitor may lead an incumbent to expand its supply. The intuition for this is simply that as a monopolist the firm might choose not to serve the low-end market segment in order to maintain high prices. Once a competitor enters on a large enough scale, however, the possible increase in marginal revenue may encourage the incumbent to expand into that segment along with the entrant.

Strikingly, such an incentive for the incumbent to increase its total supply is what drives the introduction of a fighting brand. To see this, suppose once again that the entrant is able to offer only a low-quality good, and also that the incumbent would choose to sell only the high-quality good as a monopolist. Following entry, the incumbent may wish to expand its output. But such pressure only applies at the quality level offered by the entrant, since that is the only market in which the entrant is active. Although the incumbent faces competition in the supply of a complete high-quality product, it (conceptually) maintains monopoly power in "upgrades" from low to high quality, and hence wishes to restrict their supply. The desires of the incumbent are manifested by the introduction of a low-quality fighting brand that allows its total output to increase while still exercising market power through the restriction of supply of the high-quality good.

Our analysis generates other predictions as well, which are potentially testable. First, fighting brands tend to emerge when the entrant offers only low-end products—that is, when there is some asymmetry between the technological capabilities of the incumbent and the entrant. Second, if the incumbent introduces a fighting brand, that brand will be of quality comparable to the lowest quality good of the entrant's. In other words, any price-discriminating behavior by the incumbent takes place at quality levels (weakly) above that of the entrant's worst product. This result is consistent with much observed behavior. For instance, the IBM LaserPrinter E was slightly faster than the Hewlett-Packard IIP.[15] Third, assuming that marginal revenue is decreasing, an increase in the maximum quality that an entrant can offer (whether due to the termination of a patent, reverse engineering, or some other factor) makes it more likely that the incumbent will choose to exit the lower markets.

Beyond our results on changing product lines, we also provide a more specialized technical result about price discrimination. We demonstrate in a very general model of monopoly nonlinear pricing that there exists a weak and plausible condition that is sufficient to ensure that a firm never offers more than a single product. In the special case of multiplicatively separable preferences, this condition reduces to increasing returns in the provision of quality. This condition might hold when there are production costs that must be incurred regardless of the final quality choice, as when an automobile manufacturer chooses the final trimline of a vehicle only after first building the basic platform. We believe this result is of some interest because, to our knowledge, such a straightforward condition for extreme "bunching" of consumers has been underexplored.

Our paper is organized as follows. In Section I we relate our work to earlier literature. Our model is specified in Section II. We present the general monopoly results in Section III, and also construct the framework for the case of duopoly when consumers' preferences are separable. Duopolistic competition is our focus in Section IV. In Section V we discuss a number of illustrative case studies, prior to offering concluding remarks.

I. Related Literature

Our work is related to that on product design decisions and price discrimination by firms in imperfectly competitive markets. There are three main branches of interest: monopoly, price-setting competition, and quantity-setting competition. We discuss each of these below.

The incentive to engage in second-degree discriminatory behavior has long been recognized. In a classic contribution, Michael Mussa and Sherwin Rosen (1978) consider the product line decisions of a price-discriminating monopolist able to offer a range of products of different qualities. An important insight of this work is that a monopolist may offer inefficiently low qualities, in the sense that the quality level supplied to lower-value customers is distorted downward. Inducing such a distortion is optimal as it reduces the substitution possibilities of higher-value customers.

One situation in which this downward distortion does not occur is when a firm offers but a single product—the monopolist simply does not supply lower-value customers. Nancy Stokey (1979) shows that when a monopolist is able to discriminate on the delivery date of a single product, so that products delivered in the future are essentially of lower quality, it may choose to offer only products for sale today. Multiple dates of delivery are chosen only when costs fall

[15] Some reviews claim that the IBM printer had slightly inferior print quality, and hence we might conclude that the overall quality of the two products was approximately equal. See Section V for more details.

sufficiently quickly over time. Stephen W. Salant (1989) considers the conditions under which price discrimination occurs: The marginal production cost must be sufficiently convex in a product's quality—in a sense, there must be decreasing returns to quality. In contrast, when there are increasing returns to quality, the monopolist will sell a single product at the highest feasible quality level. We obtain similar results with our own analysis (Proposition 1).[16]

While the literature on monopoly price discrimination is obviously important, such work necessarily cannot address the matter of how firms with market power adjust their product lines in response to competition. There are several papers that explore multiproduct competition between firms. Perhaps the most important of these is that of Paul Champsaur and Jean-Charles Rochet (1989), who consider product line competition in prices between two firms in a general model. They allow firms to commit to producing in chosen intervals of quality before competing on prices.[17] They find that firms choose to offer nonoverlapping product lines, as this reduces the intensity of price competition. Hence the product line offered by a given firm need not match the product line offered by a monopolist capable of offering the entire range of goods. In particular, the product line of the firm offering high-quality goods can contain fewer products than a monopolist would offer. At first, this would appear to address the issue of product line pruning. However, we wish to ask how an incumbent firm with a *fixed* technology adjusts its line following entry. In the Champsaur and Rochet (1989) analysis, this corresponds to comparing the product line offering of the high-quality firm in monopoly versus duopoly, in each case for a fixed feasible quality interval. Importantly, they show there is no difference in the optimal product line in these two cases, i.e., no pruning occurs. As such, for fixed production opportunities product line pruning does not occur, and fighting brands never arise.

There is nonetheless a sense in which the equilibrium quality gap between the two firms is related to our analysis in Section IV, subsection B. We show that, when marginal revenue is decreasing, the incumbent never introduces a fighting brand. Introducing a fighting brand is not optimal because the best response to entry is for an incumbent to reduce its total supply, which leads to fewer distinct products. Expanding into lower-quality markets only negatively effects all prices for the incumbent. This is similar in spirit to the desire of firms that can precommit to qualities in order to avoid head-to-head competition.

In contrast to these price-setting analyses, we present a quantity-setting model, as others do. Esther Gal-Or (1983) assumes a symmetric Cournot equilibrium where each firm offers a range of qualities (and states appropriate sufficient conditions for a particular example), obtaining comparative statics as the number of firms increases. In the equilibria she considers, firms do not change their product lines as the level of competition changes.[18] Giovanni De Fraja (1996) offers a quantity-setting model with the income-effect utility functions of Gabszewicz and Thisse (1979). His main result is that any equilibrium is symmetric when firms have identical technologies—we offer a similar result as part of Proposition 6. In contrast to these contributions, we offer a more complete analysis of equilibrium product lines, and consider a more general specification where one firm potentially is limited in the qualities it can offer. Moreover, none of these papers considers the issue of fighting brands or product line pruning.

Our analysis, and that of many of the authors mentioned above, considers a single dimension of quality where all consumers agree on the

[16] Other authors offer similar results based upon slightly different specifications. For instance, Jean Jaskold Gabszewicz et al. (1986) describe a model in which consumers are distinguished by their income levels. They find that, as the income distribution narrows, a monopolist focuses its production on a single quality level.

[17] A number of other authors offer price-setting models of competition in which firms precommit to quality levels. Gabszewicz and Thisse (1979, 1980) and Avner Shaked and John Sutton (1982, 1983) allow firms to precommit to their quality levels, prior to the simultaneous choice of prices. In all of these papers, firms are restricted to a single product and hence they cannot address the issue of product ranges we consider.

[18] She moves on to combine her analysis with that of Gabszewicz and Thisse (1980) and Shaked and Sutton (1982) in Gal-Or (1985). In this later paper, however, she restricts to single product provision and decreasing returns to quality.

ranking of products. Alternative models combine quality provision with horizontal differentiation elements (Richard J. Gilbert and Carmen Matutes, 1993; Lars A. Stole, 1995; Frank Verboven, 1999). Others allow preferences to differ only along a horizontal dimension (B. Curtis Eaton and Richard G. Lipsey, 1979; Kenneth L. Judd, 1985) and consider the ability of firms to deter entry by "covering" the markets for certain brands. James A. Brander and Jonathon Eaton (1984) address product line choices by two firms, assuming that firms are able to commit to product lines prior to competing on prices. In particular, there are four products, split into "groups" of two. Two products in one group are close substitutes for one another, and less close substitutes for the other two goods, which are in turn close substitutes for one another. They show that an "interlace" equilibrium can exist in which each firm offers one product in each group. The reason is that if one firm believes the other will occupy one product space in each group, it is certainly wise to avoid competing in those exact product spots, as price competition will drive profits to zero. However, it might be profitable to occupy the "open" product spots within the two groups, if the resulting price competition is not too fierce (as it may not be because distinct products are somewhat differentiated). Note that, if there is no commitment stage in their model, each firm produces all the products and interlace is not an equilibrium. We show that, absent such commitment, there is no interlace equilibrium in a vertically differentiated industry. In particular, the entrant offers any product the incumbent does, subject only to being physically capable of producing it (we actually prove the contrapositive in Section IV). Heuristically, the entrant has less to lose by offering a product, since it has fewer high-quality products and is hence less concerned about negative price consequences associated with increasing the overall level of goods. Hence, if the incumbent wishes to offer a positive supply of some good, the entrant must also wish to do so.

In short, the existing literature has made great progress in understanding monopoly product design decisions. However, less progress has been made in understanding multiproduct competition, and no one has addressed the matter of changing product lines that we consider.

II. A Market for Quality-Differentiated Products

In this section we lay out the structure of demand and costs. In Section II, subsection A, and Section II, subsection B, we describe consumer preferences and the demand side of the market in general. In Section II, subsection C, we describe different cost structures that we will make use of later on.

A. *The Demand Side*

Demand is generated from a unit mass of consumers indexed by a type parameter θ. The cumulative distribution function is $F(\theta)$, which has support on $[0, \bar{\theta}]$. On its support, $F(\theta)$ is strictly increasing and continuously differentiable with density $f(\theta)$. A consumer of type θ who purchases a good of quality q at price p enjoys utility $\theta q - p$.[19] She faces a selection of n products indexed by i, where product i is of quality q_i and price p_i. We order the goods so that $q_n > q_{n-1} > \cdots > q_1 > 0$. Note that goods differ only with respect to quality, and that all consumers prefer goods of higher quality. Each consumer purchases a single unit of the good i that maximizes $\theta q_i - p_i$, unless this yields negative utility in which case she purchases nothing.

Before deriving the entire system of inverse demand functions under this multiplicative specification, first suppose that only a single good of quality $q > 0$ is being sold, at price $p > 0$. This good would be purchased by all consumers satisfying $\theta \geq p/q$, yielding demand of $z = 1 - F(p/q)$. Similarly, the inverse demand curve satisfies $p = qH(z)$ where:

$$(1) \qquad H(z) = \begin{cases} F^{-1}(1 - z) & z < 1 \\ 0 & z \geq 1. \end{cases}$$

When $z \leq 1$, $\theta = H(z)$ is the type with a mass of z consumers above her. At a price of

[19] This is equivalent to the more general formulation $u(\theta, q) = v(\theta)w(q)$, where $v(\cdot)$ and $w(\cdot)$ are both increasing. We simply redefine a consumer's type to be $v(\theta)$ and quality to be $w(q)$. When we refer to quality q, therefore, we are really considering the (scaled) monetary value of quality, since a consumer θ is willing to pay θq for quality q.

$p = \theta q$ it is exactly these z consumers who are willing to buy, with consumer θ being just indifferent between buying and not. Hence $p = qH(z)$ equilibrates supply and demand. For $z > 1$, supply exceeds the number of willing purchasers and so the only possible equilibrium price is $p = 0$.

As quantities will be the strategic choice variables of firms in our later analysis, and as products are differentiated only with respect to quality in the eyes of consumers, we can compute the market-clearing prices for the n products knowing only that industry supply of each product i is given by z_i. It turns out that the inverse demand function for a single good, based on $H(z)$ from equation (1), is central to the general analysis with n products.

Naturally, we must consider the possibility that $z_i = 0$ for some products. However, it is conceptually easier to first derive the inverse demands assuming that $z_i > 0$ for each i. We will then explain how to incorporate the possibility that some products are not supplied at all.

When $\sum_{i=1}^{n} z_i < 1$ there is partial market coverage: Not all consumers are able to purchase a good. Thus, given a set of supplies $\{z_i\}$, we require a set of positive prices $\{p_i\}$ such that exactly z_i consumers wish to purchase good i. If a lower-quality good were priced no lower than a higher-quality good, then it would attract no demand. There must, therefore, be a price premium for higher quality. Such higher-quality products must be purchased by consumers with higher types: If a consumer θ is willing to pay a premium for higher quality, then higher types will strictly wish to do so. Thus the highest z_n consumers purchase product q_n, and the next z_{n-1} purchase quality q_{n-1} and so on. The consumer with $\sum_{j=1}^{n} z_j$ others above her must be just indifferent between purchasing quality q_1 and not purchasing at all, so that $p_1 = q_1 H(\sum_{j=1}^{n} z_j)$. Similarly, the consumer with $\sum_{j=i}^{n} z_j$ consumers above her must be just indifferent between products i and $i - 1$, and so $p_i = p_{i-1} + (q_i - q_{i-1}) H(\sum_{j=i}^{n} z_j)$. Defining $q_0 = p_0 = 0$ for convenience, for $i \in \{1, 2, \ldots, n\}$ we obtain:

$$(2) \quad p_i - p_{i-1} = (q_i - q_{i-1}) H\left(\sum_{j=i}^{n} z_j\right).$$

Notice that $p_i - p_{i-1}$ represents the price of an upgrade from quality q_{i-1} to quality q_i. This observation leads us to consider the cumulative variables defined by $Z_i = \sum_{j=i}^{n} z_j$. Z_i is the total supply at quality q_i and above. We offer the following interpretation. We may suppose that the industry supplies Z_1 units of a "baseline" product of quality q_1. There are then supplies of successive upgrades to the baseline product in order to achieve qualities above this. For instance, a product of quality q_2 consists of a baseline product at price p_1, plus an upgrade $q_2 - q_1$ priced at $p_2 - p_1$. Continuing in this manner, equation (2) becomes:

$$(3) \quad p_i - p_{i-1} = (q_i - q_{i-1}) H(Z_i).$$

Hence the price of upgrade i depends only on its own supply, and not on the supply of any other upgrades. In contrast, the complete product i with quality q_i has a price p_i that depends on the supplies of all n products.

Equation (3) also applies when there is complete market coverage, so that $Z_1 \geq 1$.[20] If $Z_1 = 1$, then p_1 must equal zero, given our assumptions on $F(\theta)$. If $Z_1 > 1$, then $p_1 = 0$ as well, because there is strictly excess supply. Similarly, $p_i - p_{i-1} = 0$ for any upgrade i satisfying $Z_i \geq 1$. But of course, if this holds then $H(Z_i) = 0$ by definition [see equation (1)] and hence equation (3) continues to hold.[21] Note that, if $Z_{j+1} \geq 1$ then there can be no demand for product j. The reason is that the price of the upgrade to product $j + 1$ is zero, so that all consumers will purchase a product at least of quality $j + 1$.

It turns out that defining prices in the manner just described easily allows us to address the possibility that some products are in zero supply. To see how, suppose that $\{z_i\}$ satisfies only that $z_i \geq 0$, and define the prices $\{p_i\}$ and cumulative variables $\{Z_i\}$ exactly as above.

[20] Given our assumption that the lower bound of the support of $F(\theta)$ is zero, there will only be partial coverage in equilibrium. Nevertheless, we must consider the possibility of complete coverage.

[21] Our assumption that the lower bound of $F(\theta)$ is zero ensures that all prices will be positive in equilibrium, so that the market is not fully covered. We still must address the possibility that some prices are zero, as we do here. See also footnote 20 above.

Note that a product i is in positive supply if $Z_i - Z_{i+1} > 0$, at least if we define $Z_{n+1} = 0$, while a product is in zero supply if $Z_i - Z_{i+1} = 0$. If j is the first product in positive supply then there are a total of $Z_j = \sum_{i=j}^{n} z_i$ products on the market. If the prices defined above are correct in this case, it must be that consumer $\theta = H(Z_j)$ is just indifferent between buying good j and not. The price of good j can be obtained by adding up the incremental prices given by the right-hand side of equation (3). Making use of the fact that for $k < j$ we have $Z_k = Z_j$, we obtain

$$\begin{aligned} p_j = p_j - p_0 &= \sum_{i=1}^{j} (p_i - p_{i-1}) \\ &= \sum_{i=1}^{j} (q_i - q_{i-1}) H(Z_i) \\ &= H(Z_j) \sum_{i=1}^{j} (q_i - q_{i-1}) \\ &= q_j H(Z_j). \end{aligned}$$

Hence, a consumer with type $\theta = H(Z_j)$ is indeed indifferent between buying good j and not buying at all. The only subtlety is that she is also indifferent between buying goods $k < j$ and not at the prices defined, but these are not in positive supply. Hence, we adopt the convention that when a consumer is indifferent between several products she purchases the one of highest quality. We have only dealt with the first good in positive supply, but a similar process can be applied recursively. With some more work, it can be shown that the process just described can be extended to consistently define demand for all possible supply configurations. Hence, given the convention that a consumer purchases the highest quality good to which she is indifferent, the prices defined above are correct for all circumstances.

B. *Demand Segments and the Shape of Marginal Revenue*

Equation (3) reveals that the shape of inverse demand for all n products is tied to the function $H(z)$, which is itself the inverse demand curve for a single good of quality $q = 1$. A standard "textbook" assumption would be to suppose that $H(z)$ exhibits decreasing marginal revenue.[22]

Definition 1 (*Decreasing marginal revenue*): $H(x + y) + xH'(x + y)$ is decreasing in y for all x.

This states that if a firm is producing x units, then the marginal revenue of its final unit is decreasing in the output y of the rest of the industry. If marginal revenue satisfies this definition, then the marginal revenue of a firm also is decreasing in its own output.

Decreasing marginal revenue is a property exhibited by many demand curves. Nonetheless, there are some very plausible specifications that are inconsistent with marginal revenue being everywhere decreasing. We illustrate this with an example.

Example 1: θ is drawn from a mixture of two normal distributions with means $\theta_H > \theta_L$ and variances σ_H^2 and σ_L^2. The former is chosen with probability α. Types $\theta \leq 0$ do not purchase.

Example 1 is a stylized representation of a market where demand is drawn from two separate sources, as shown in Figure 1(a).[23] For instance, when consumers are drawn from home and business sectors, the specification of Example 1 may be appropriate.[24] Note that in this example, marginal revenue is *not* decreasing everywhere [Figure 1(b)].

[22] The assumption of decreasing marginal revenue is also a convenient sufficient condition for many existence and uniqueness results in oligopoly. For instance, combining decreasing marginal revenue with weakly convex marginal costs ensures that a single-product Cournot game has a unique and symmetric pure strategy Nash equilibrium—see, for instance, Xavier Vives (1999, Ch. 4).

[23] In our specification of the model above, we assumed that there was a finite upper bound to the support of θ. However, allowing the support to be unbounded above, as in this example, does not affect our results.

[24] Many firms explicitly direct these categories of consumers toward different product lines. The personal computer manufacturer Dell divides its web site into Consumer and Business products. In the former division it offers Dimension desktops and Inspiron laptops, whereas in the latter it offers Optiplex desktops and Latitude laptops.

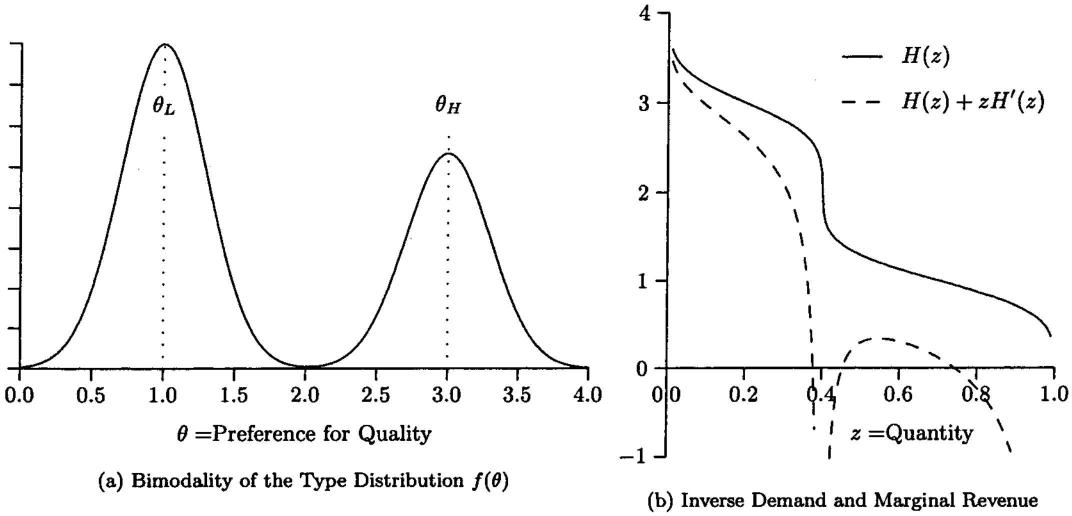


FIGURE 1. MULTIPLE MARKET SEGMENTS AND NONMONOTONIC MARGINAL REVENUE

Notes: Figure 1 illustrates the specification of Example 1. The parameters chosen are $\theta_H = 3$ and $\theta_L = 1$ for the centers of two "market segments," $\sigma_H^2 = \sigma_L^2 = 0.3$ for the variance in each segment and $\alpha = 0.4$ for the relative weight on the higher segment. The sharp drop in inverse demand (and corresponding negative marginal revenue) near $z = 0.4$ corresponds to the division between the two segments.

Our opening discussion of product lines and competition suggests that firms are able to identify different "segments" of market demand. Upon the introduction of Lucky Dog Telephone, AT&T vice-president Howard E. McNally commented that "[w]hat we want to do with this brand is attract a different group of people." In the context of a formal model, this comment suggests that the density $f(\theta)$ may well be multimodal, with each mode corresponding to a different segment of consumers.

In Section IV we show that the presence or absence of decreasing marginal revenue critically affects the product ranges offered by competing duopolists. As we wish to take both possibilities seriously, we make two more points here. First, even if marginal revenue is increasing in some regions, firms will always choose output in the region where it is decreasing, in the textbook manner.[25] Thus, incorporating marginal revenue that is increasing in *some* regions is simply a way to admit other demand structures, such as Example 1.

To build to our second point, note that the monotonicity of marginal revenue is also related to the price sensitivity of consumers. For a single good of quality $q = 1$, the price elasticity of demand $\varepsilon(z)$ satisfies:

$$\frac{1}{\varepsilon(z)} = -\frac{d \log H(z)}{d \log z} = -\frac{H'(z)z}{H(z)}$$

yielding marginal revenue:

$$H(z) + zH'(z) = H(z)\left(1 - \frac{1}{\varepsilon(z)}\right).$$

Since $H(z)$ is an inverse demand function, it is automatically decreasing in z. This means that if marginal revenue is both positive and increasing in a region (as in Example 1), then it must be the case that $\varepsilon(z)$ is increasing in z in that region. In other words, an expansion in z toward the "low end" of the market naturally results in an increase in the price elasticity of demand. This is consistent with our example of BPL, where an expansion to the low end of the market resulted in greater price sensitivity.[26]

[25] We assume constant marginal costs of production within a quality level.

[26] Both marginal revenue and the price elasticity of demand are closely related to the hazard rate of $F(\theta)$:

C. *The Cost Side: Increasing and Decreasing Returns to Quality*

We assume that any firm capable of producing a product of quality q_i has access to the same production technology. Precisely, within a particular quality level q_i, there are constant marginal costs of production $c_i > 0$. There are no fixed costs of production.[27] Costs may be related to quality in a number of different ways. For instance, we say that the production cost c_i exhibits "decreasing returns" if both the average cost of quality c_i/q_i and the marginal cost of quality $(c_{i+1} - c_i)/(q_{i+1} - q_i)$ are increasing for all i. Of course, if the marginal cost of quality exceeds the average cost, then the average cost of quality is clearly increasing. Hence, defining $c_0 = 0$,

$$\frac{c_1}{q_1} = \frac{c_1 - c_0}{q_1 - q_0} < \frac{c_2 - c_1}{q_2 - q_1} < \cdots < \frac{c_n - c_{n-1}}{q_n - q_{n-1}} \Rightarrow \frac{c_1}{q_1} < \frac{c_2}{q_2} < \cdots < \frac{c_n}{q_n}.$$

This simply says that if the marginal cost of quality is increasing for all i, it is necessarily true that the average cost of quality is increasing.

Likewise, the average cost of quality is decreasing when the marginal cost of quality is less than the average cost; that is, when $(c_{i+1} - c_i)/(q_{i+1} - q_i) < c_i/q_i$. When the average cost of quality is decreasing, we say there are "increasing returns" to quality.

A production technology may easily exhibit both increasing and decreasing returns. For instance, suppose that any product, irrespective of quality, has an unavoidable marginal "build cost" of $\underline{c} > 0$. In addition, the production cost increases with quality according to the strictly increasing and convex function $c(q)$ satisfying $c(0) = 0$. Hence:

$$c_i = \underline{c} + c(q_i) \Rightarrow \frac{c_i}{q_i} = \frac{\underline{c} + c(q_i)}{q_i}.$$

For $i > 1$, the convexity of $c(q)$ ensures that the marginal cost of quality is increasing. For small q, however, the average cost of quality c_i/q_i is decreasing, and exhibits the classic "U-shape" familiar from undergraduate textbooks: There are first increasing, and then decreasing returns to quality. Motivated by this example, we categorize different cases of interest as follows.

Definition 2: There are *increasing returns to quality* when c_i/q_i is decreasing for all i. There are *decreasing returns to quality* when both c_i/q_i and $(c_{i+1} - c_i)/(q_{i+1} - q_i)$ are increasing for all i. The production technology is *U-shaped* if, for some k, the average cost of quality is decreasing for $i \leq k$ and the marginal and average costs of quality are increasing for $i > k$:

$$\frac{c_1}{q_1} > \frac{c_2}{q_2} > \cdots > \frac{c_{k-1}}{q_{k-1}} > \frac{c_k}{q_k} < \frac{c_{k+1} - c_k}{q_{k+1} - q_k} < \cdots < \frac{c_{n-1} - c_{n-2}}{q_{n-1} - q_{n-2}} < \frac{c_n - c_{n-1}}{q_n - q_{n-1}}.$$

$$H(z) + zH'(z) = \theta - \frac{1 - F(\theta)}{f(\theta)}$$

and:

$$\varepsilon(z) = \frac{\theta f(\theta)}{1 - F(\theta)}$$

where $\theta = H(z)$. If the hazard rate $f(\theta)/(1 - F(\theta))$ is increasing in θ, then both marginal revenue and the price elasticity of demand are decreasing in z. This is inconsistent with Example 1, and with greater price sensitivity upon an expansion to serve the low end of a market. Whereas the monotonicity of the hazard rate is a convenient assumption, and holds for many common unimodal distributions (including the uniform, normal, etc.) it cannot hold if we wish to model the presence of multiple demand segments as in Example 1.

[27] Thus the only asymmetry between firms will be in the range of qualities that they are able to produce. Allowing the marginal cost of production to vary across firms does not substantially change our analysis. With some straightforward modifications, the bulk of our results continue to hold, in particular those regarding product line pruning and fighting brands.

In this case k is the product that minimizes the average cost of quality.

Notice that U-shaped costs of quality incorporate increasing and decreasing returns as special cases. The former occurs when $k = n$ and the latter occurs when $k = 1$.

Inspection of the production technology in objective terms typically is not enough to determine the returns to quality; consumer preferences are part of this definition. For example, suppose that the product in question is a microprocessor, and that increases in clockspeed correspond to increased quality. A consumer with type θ is willing to pay up to θq for a processor of quality q. Thus q indexes the monetary value of the microprocessor, and not necessarily its physical clockspeed.[28] In some cases, however, we can immediately identify the structure of costs. Such is the case with "damaged goods" (Deneckere and McAfee, 1996), where a firm obtains a low-quality product by intentionally "crimping" a higher-quality variant. Such a low-quality product costs no less to produce, and hence the returns to quality are automatically increasing.

III. Monopoly

In this section we derive the optimal product lines for a monopolist. We first consider the case of multiplicative preferences, which serves as a point of comparison when we introduce competition in Section IV. Following that, we briefly consider the monopolist's product line choices with more general preferences.

A. *Monopoly with Multiplicative Preferences*

With n different goods available, the monopolist's profit on product i is simply $z_i(p_i - c_i)$. The monopolist then chooses $z_i \geq 0$ to maximize total profits $\sum_{i=1}^{n} z_i(p_i - c_i)$ across all products. It is equivalent, and much easier in the end, for the monopolist instead to choose a range of upgrade supplies $\{Z_i\}$. These must respect the constraint $Z_i \leq Z_{i-1}$—for instance, an upgrade to quality q_2 may only be sold to consumers who purchase the baseline product q_1. Using this formulation, the monopolist's problem becomes:

$$\max \sum_{i=1}^{n} Z_i[(q_i - q_{i-1})H(Z_i) - (c_i - c_{i-1})]$$

subject to $Z_i \leq Z_{i-1}$ for each $i > 1$. Observe that the ith element of the summation involves only the term Z_i. In fact:

$$(4) \quad Z_i[(q_i - q_{i-1})H(Z_i) - (c_i - c_{i-1})] \propto Z_i\left[H(Z_i) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}}\right].$$

The last term is equivalent to the profit from selling a single good of quality $q = 1$ with a marginal production cost of $(c_i - c_{i-1})/(q_i - q_{i-1})$. If it were not for the constraint $Z_i \leq Z_{i-1}$, then the monopolist could maximize the objective function termwise. Neglecting this monotonicity constraint, the solution $\{Z_i^*\}$ would satisfy

$$(5) \qquad H(Z_i^*) + Z_i^* H'(Z_i^*) = \frac{c_i - c_{i-1}}{q_i - q_{i-1}}.$$

The "upgrade" reformulation reveals the simple economics of second-degree price discrimination: The monopolist sets marginal cost (of increased quality) equal to marginal revenue in the market for each upgrade.

Alas, this approach ignores the monotonicity constraint $Z_i \leq Z_{i-1}$. Imposing this constraint sheds light on the product range offered by the monopolist. The simplest case is one of decreasing returns to q_i, so that $(c_i - c_{i-1})/(q_i - q_{i-1})$ is strictly increasing in i for all $i \geq 1$. The unconstrained solutions to equation (5) naturally satisfy $Z_i^* < Z_{i-1}^*$ and hence $z_i = Z_i - Z_{i-1} > 0$. Simply put, when there are decreasing returns to quality, the monopolist offers the full range of potential product qualities.

[28] With multiplicatively separable preferences, changes in q represent common changes in the proportional willingness to pay of all consumers. Under increasing returns to quality (when c_i/q_i is decreasing in i) an increase in quality raises a consumer's willingness to pay proportionally more than it raises the physical cost of producing the good. This notion of increasing returns to quality (and similarly for decreasing and constant returns) is invariant to the labeling of quality levels, since it deals solely with the willingness to pay for, and the production cost of, particular physical products.

In contrast, when there are increasing returns to quality provision, the monotonicity constraints bind. For simplicity, consider the case of $n = 2$. Increasing returns to quality implies that $c_2/q_2 < c_1/q_1$, or equivalently $(c_2 - c_1)/(q_2 - q_1) < c_1/q_1$. Suppose that the monopolist finds it optimal to offer two distinct products, so that $Z_1^* > Z_2^*$. Then it must be the case that:

$$Z_1^*H(Z_1^*) - Z_2^*H(Z_2^*) \geq \frac{c_1}{q_1}(Z_1^* - Z_2^*)$$

$$> \frac{c_2 - c_1}{q_2 - q_1}(Z_1^* - Z_2^*).$$

The first inequality makes use of equation (4), and says that the monopolist does not wish to lower supplies of the baseline product to Z_2^*, which it may do without violation of the monotonicity constraint. The second inequality follows from increasing returns to quality, keeping in mind that $c_0 = q_0 = 0$. Combining, the resulting strict inequality says that the monopolist would strictly benefit by raising the supply of the upgrade $q_2 - q_1$ from Z_2^* to Z_1^*. Thus the original supplies cannot have been optimal, and we have a contradiction. Thus, the firm must optimally sell a single quality level.

This argument naturally extends to the case of general n. In fact, if there are increasing returns to quality everywhere, then a monopolist will wish to set $Z_1^* = Z_2^* \cdots = Z_n^*$, and hence will offer only the highest quality product q_n. We summarize our results in the following proposition.[29]

PROPOSITION 1: *If the production technology is U-shaped (Definition 2) with minimum average cost for product k, then a monopolist offers in positive supply exactly the $n - k + 1$ products of highest quality. In terms of upgrades, $Z_1^* = Z_2^* = \cdots = Z_k^* > Z_{k+1}^* > \cdots > Z_n^*$.*

Proposition 1 demonstrates the crucial role that the shape of average and marginal costs of quality play in a monopolist's product selection decision. In the region where average cost is decreasing, it sells a single product. It is optimal to segment the market with multiple products exactly in the region where average cost and marginal cost are increasing.

B. *Monopoly with General Payoffs*

Here we briefly consider more general preferences, before returning to the multiplicative specification for the remainder of the paper. We ask what properties of preferences and costs in this more general setting lead to price discrimination using multiple products. To answer this question, we suppose that a consumer of type θ purchasing a good of quality q at price p enjoys utility $u(\theta, q) - p$, where $u(\theta, q)$ is strictly increasing in its arguments, continuously differentiable, and satisfies $u(\theta, 0) = u(0, q) = 0$. Consumers purchase a single unit of the utility-maximizing product so long as this yields nonnegative utility. A first property we need is for this function to satisfy the familiar sorting condition: For $q_2 > q_1$, $u(\theta, q_2) - u(\theta, q_1)$ must be increasing in θ. Equivalently, the function $u(\theta, q)$ is *supermodular* in θ and q.[30] This ensures that higher types will purchase higher qualities. This in turn implies the upgrade formulation of the inverse demand functions continues to hold. For $n = 2$:

$$p_1 = u(H(Z_1), q_1)$$

$$p_2 = p_1 + [u(H(Z_2), q_2) - u(H(Z_2), q_1)].$$

The sorting condition ensures that price discrimination is *feasible.* It does not, however, imply that such discrimination is *optimal.* To elicit an appropriate condition for optimality, suppose that the monopolist optimally supplies two distinct products, so that $Z_1^* > Z_2^*$. For simplicity of exposition, we suppose that $c_1 = c_2 = 0$, so that there are increasing returns to quality.[31] It must be the case that:

[29] The proofs to this and subsequent results are given in the Appendix.

[30] A differentiable function $u(\theta, q)$ is supermodular if $\partial^2 u/\partial\theta\partial q \geq 0$, and log supermodular (when positive) if $\partial^2 \log u/\partial\theta\partial q \geq 0$.

[31] In the formal specification of our model we assume that marginal production costs are strictly positive. When marginal production costs are zero all of our monopoly

$$Z_1^* u(H(Z_1^*), q_1) \geq Z_2^* u(H(Z_2^*), q_1)$$

and:

$$Z_1^*[u(H(Z_1^*), q_2) - u(H(Z_1^*), q_1)]$$
$$\leq Z_2^*[u(H(Z_2^*), q_2) - u(H(Z_2^*), q_1)].$$

The first inequality says that the monopolist does not wish to reduce supplies of the baseline product. The second inequality says that the monopolist does not wish to expand supplies of the upgrade. Dividing the second inequality by the first we obtain:

$$\frac{u(H(Z_1^*), q_2)}{u(H(Z_1^*), q_1)} \leq \frac{u(H(Z_2^*), q_2)}{u(H(Z_2^*), q_1)}.$$

Since $H(Z_2^*) > H(Z_1^*)$, this says that $u(\theta, q_2)/u(\theta, q_1)$ must be increasing in θ, so that higher types prefer higher qualities *proportionally more* than lower types. Mathematically, the utility function $u(\theta, q)$ must be log supermodular in the θ and q, at least over the range of price discrimination. Of course, this means that log submodularity [which means that $u(\theta, q_2)/u(\theta, q_1)$ is decreasing in θ] is sufficient to ensure that no price discrimination takes place. Incorporating costs, and allowing for n products, we obtain the following.

PROPOSITION 2: *Suppose that a monopolist finds it optimal to offer distinct products. Then the surplus $u(\theta, q) - c(q)$ must be log supermodular over some range.*

To apply this proposition, suppose that all qualities may be produced at an identical constant marginal cost of $c > 0$. This seems to be an appropriate specification for the case of the IBM LaserPrinter, since the two versions differed only in their controller card. For multiplicative utility, the surplus function satisfies $u(\theta, q) - c(q) = \theta q - c$. By inspection, $(\theta q_2 - c)/(\theta q_1 - c)$ is strictly decreasing in θ, and hence the surplus is log submodular. The monopolist will offer only the higher-quality product. Moving back to the general specification, $u(\theta, q)$ must be *strictly* log supermodular to overcome this.

IV. Duopoly

Here we consider a simple quantity-setting duopoly model. Broadly, our goal is to characterize the equilibrium product lines under a variety of conditions. We allow for both decreasing and nonmonotone marginal revenue, and also consider equilibria for environments in which the entrant and incumbent have either symmetric or asymmetric technological capabilities.

Our leading results are as follows. When marginal revenue is decreasing, fighting brands never emerge as optimal weapons in response to entry. To the contrary, we show that increases in the entrant's technological prowess tend to induce product line pruning of the lower-quality products from the incumbent's line.

However, moving away from the assumption of decreasing marginal revenue reveals that fighting brands can emerge as effective tools to maintain profitability for the incumbent. This tends to happen when the total output of the incumbent expands following entry. Moreover, the incumbent never offers goods of lower quality than the entrant. Thus, when competition does lead the incumbent to expand its product line, it chooses to maintain at least a weak quality advantage.

Our analysis is arranged as follows. First, we set up the framework for analysis and present several results that hold irrespective of the shape of marginal revenue. Next, we address the case in which marginal revenue is everywhere decreasing. Finally, we expand our analysis to incorporate the possibility of nonmonotonic marginal revenue, and consider again the bimodal type distribution described in Example 1.

A. *Duopoly Framework and General Results*

The n products are supplied by two firms. Our interpretation is that one is the incumbent and the other an entrant. They supply x_i and y_i units of good i respectively, yielding a total supply of $x_i + y_i$. The two firms simultaneously choose quantities. The incumbent is able to pro-

results continue to hold. Furthermore, all of our duopoly results continue to hold when we exclude equilibria where a firm offers a supply that floods an entire upgrade market, forcing the price of that upgrade to zero.

duce the entire range of qualities. The entrant, however, is limited to products of quality q_m and below for some $m \leq n$, and so $y_i = 0$ for i satisfying $m < i \leq n$.[32] As in Section II, we define the upgrade quantities as follows:

$$X_i = \sum_{j=i}^{n} x_j, \quad Y_i = \sum_{j=i}^{m} y_j$$

where of course $Y_i = 0$ for $i > m$. The upgrade supplies satisfy $X_i \leq X_{i-1}$ and $Y_i \leq Y_{i-1}$. Employing equation (3), these yield profits for the incumbent and entrant of:

$$\pi_I = \sum_{i=1}^{n} (q_i - q_{i-1}) \times X_i\left(H(X_i + Y_i) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}}\right)$$

$$\pi_E = \sum_{i=1}^{m} (q_i - q_{i-1}) \times Y_i\left(H(X_i + Y_i) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}}\right).$$

As in the monopoly case, these are convenient forms. The ith term of each objective function depends only on X_i and Y_i. Each firm chooses its supplies to maximize its objective function, subject to the monotonicity constraints on $\{X_i\}$ and $\{Y_i\}$. We seek pure strategy Nash equilibria in these quantities and use $\{X_i^*\}$ and $\{Y_i^*\}$ to denote the equilibrium supplies. Notice that absent the monotonicity constraints we would be able to seek separate Cournot equilibria in the supply of each upgrade. The monotonicity constraints have to be satisfied, however, and they allow us to ascertain the relationships between the product lines of the two firms.

Since the incumbent is active in all the upgrade markets while the entrant may not be, we might expect that the total production of the incumbent exceeds that of the entrant at and above any quality level. This is indeed the case.

We only consider pure strategy Nash equilibria throughout our entire paper.

PROPOSITION 3: *Any (pure strategy) equilibrium entails $X_i^* \geq Y_i^*$ for each i. If $m = n$ then $X_i^* = Y_i^*$ for each i.*

Note that the incumbent need not produce more of any single quality level. Rather, the total supply at and above any particular quality level is greater for the incumbent. Furthermore, in the absence of any strict quality advantage any (pure strategy) equilibrium must be symmetric. We can push this analysis slightly further using the techniques of Proposition 3 to obtain the following.

PROPOSITION 4: *A product $i \leq m$ is supplied by the incumbent only if it is supplied by the entrant.*

This says that the incumbent will not be active in any market that the entrant is not active in, save potentially for those that the entrant is not technologically capable of serving. One implication of this is that the incumbent will never choose to offer products that are of quality inferior to that of the entrant's lowest-quality product.

The results so far are useful, but do not succeed in characterizing the precise nature of the equilibrium product lines. To proceed further, we consider separately the cases of decreasing (Section IV, subsection B) and nonmonotonic marginal revenue (Section IV, subsection C).

B. *Decreasing Marginal Revenue and Product Line Pruning*

When marginal revenue is decreasing and returns to quality are increasing, a monopolist supplies only the highest feasible quality (Proposition 1). The entry of a competitor does not alter this.

PROPOSITION 5: *With increasing returns to quality and decreasing marginal revenue (Definitions 1–2), both firms offer a single, highest*

[32] We could allow for a more general specification of technological capabilities, so that, for example, the entrant is able to produce some goods that the incumbent cannot. With some straightforward modifications, the bulk of our results continue to hold, in particular those regarding product line pruning and fighting brands.

available quality product. In terms of the upgrade supplies: $X_1^* = \cdots = X_n^* = X^*$ *and* $Y_1^* = \cdots = Y_m^* = Y^*$.

Proposition 5 says that the incumbent offers only product n and the entrant offers only product m. If $m = n$, we have a pair of symmetric duopolists selling a single product. In contrast, if $m < n$, the incumbent has a technological advantage. The profit equations reduce to:

$$\text{(6)} \quad \pi_I = X(q_m H(X+Y) - c_m) + X((q_n - q_m)H(X) - (c_n - c_m))$$

$$\text{(7)} \quad \pi_E = Y(q_m H(X+Y) - c_m).$$

Heuristically, the incumbent has stronger incentives to produce because it is selling a higher-quality product that is produced at lower average cost. The equilibrium therefore should exhibit $X^* > Y^*$.

PROPOSITION 6: *Suppose marginal revenue is decreasing and there are increasing returns to quality. If* $m = n$, *the firms operate as symmetric duopolists. If* $m < n$, *the output level of the incumbent exceeds that of the entrant, so that* $X^* > Y^*$. *In this case,* X^* *is increasing in* q_n *and decreasing in* q_m, *whereas* Y^* *is decreasing in* q_n *and increasing in* q_m.

Proposition 5 implies that when marginal revenue is decreasing and returns to quality are increasing, the emergence of competition cannot provide an explanation for the introduction of an additional product. The reason is that we know from Proposition 1 that a monopolist facing increasing returns to quality also would sell a single product of quality q_n. The optimal strategy for the incumbent is to accept the decline in prices and continue selling only the high-quality good. We might suspect that, had the monopolist been selling multiple products, entry would induce the incumbent to remove certain products in an attempt to maintain margins on the higher-quality goods.

To see that pruning can occur in this case, suppose that the average cost of quality is U-shaped (Definition 2). We know from Proposition 1 that the monopolist would offer distinct products in the range where average cost is increasing. If entry occurs in this case, the incumbent will tend to (weakly) reduce the number of products offered. The following lemma is a first step in proving this.

LEMMA 1: *If the production technology is U-shaped (Definition 2) with minimum average cost for product* k, *and marginal revenue is decreasing, then the incumbent produces no more distinct goods than it did as a monopolist. If* $m \leq k$ *then (i) the entrant offers only product* m, *and (ii) for some* $h \leq n - k + 1$ *the incumbent sells the* h *highest quality products that it can produce.*

There are no fighting brands, even with a general U-shaped cost structure. This result places an upper bound on how many products an incumbent will offer. It is possible to say more, however, and characterize precisely the product lines of firms in equilibrium, at least when the entrant is constrained to offer products of relatively low quality (i.e., products in the range where the average cost of quality is decreasing).[33] As will be shown, it is sufficient to determine the lowest quality product offered by the incumbent. This is straightforward since, in equilibrium, it is as if the incumbent were competing with a single product against the entrant's single product and also selling certain upgrades in independent markets. Conceptually, the only question is which product the incumbent wields against the entrant.

Suppose that the incumbent's lowest quality product is r in equilibrium and that $m \leq k$. From Lemma 1, we know that $r \geq k$ and that the entrant supplies only product m. To satisfy the appropriate monotonicity constraint, it must be the case that the monopoly supply of upgrade $r + 1$ is below the incumbent's duopoly supply of product r. Define $X_{rm}^{\dagger}$ and $Y_{rm}^{\dagger}$ to be the equilibrium outputs of the incumbent and en-

[33] The entrant is constrained to offer products of relatively low quality when $m \leq k$. Johnson and Myatt (2003) consider a related model of quantity competition between multiproduct oligopolists allowing for more general preferences and arbitrary technological limitations. They focus on the equilibrium structure of firms' product lines as opposed to the product line response of an incumbent firm to entry.

trant when they are forced to offer single products of quality q_r and q_m respectively. The objective functions for this restricted game are given by equations (6) and (7), changing the n to r for the incumbent. Given that marginal revenue is decreasing, it turns out that there is a unique equilibrium to such a game (Vives, 1999, p. 47). Now, we can also consider the unrestricted monopoly supplies of upgrade products $Z_i^\dagger$, for $i \geq k$. Increasing marginal costs of quality for $i > k$ ensure that this sequence is decreasing, and satisfies $Z_i^\dagger = Z_i^*$, where Z_i^* is the output of upgrade i that a monopolist would choose, for $i > k$ (from the Proof of Proposition 1). The following lemma indicates that the sequences $\{Z_r^\dagger\}_{r=k}^n$ and $\{X_{rm}^\dagger\}_{r=k}^n$ satisfy a single crossing property.

LEMMA 2: *Suppose that marginal revenue is decreasing. Fixing m, define $\mathcal{R} = \{r : Z_r^\dagger \geq X_{rm}^\dagger > Z_{r+1}^\dagger, r \geq k\}$. If there is no such r then define $\mathcal{R} = \{n\}$. The set $\mathcal{R}$ contains a single element $\bar{r}$ for each $m \leq k$.*

We will now show that there is a real sense in which $\bar{r}$ determines the state of competition in the industry. In equilibrium, $\bar{r}$ is the only product of the incumbent that is in direct competition with the entrant, and beyond this quality level the incumbent exercises unrestricted monopoly power over upgrades to quality.[34] The following result demonstrates this, and shows how $\bar{r}$ determines the equilibrium outputs for both firms for all quality levels.

PROPOSITION 7: *Suppose that the production technology is U-shaped (Definition 2) with minimum average cost for product k, and that marginal revenue is decreasing. Also suppose that the entrant's maximal quality is $m \leq k$. In equilibrium the incumbent produces distinct products $\bar{r}$, $\bar{r} + 1, \ldots, n$, while the entrant produces only product m. Furthermore, $X_{\bar{r}m}^\dagger = X_1^* = \cdots = X_{\bar{r}}^*$, and for $i > \bar{r}$, the incumbent produces the same quantity of upgrades as if it were a monopolist, so that $X_i^* = Z_i^* = Z_i^\dagger$. Finally, the output of the entrant is $Y_{\bar{r}m}^\dagger$.*

[34] Notice that the incumbent does not hold monopoly power over the complete high-quality product.

When unrestricted duopoly levels of low-quality upgrades fall beneath monopoly levels of higher-quality upgrades, the equilibrium reaction is for the incumbent to remove its lower-quality goods from the market. The equilibrium in the industry can therefore be determined by identifying the product $\bar{r}$ such that, if the incumbent wields only this product against the entrant, the resulting duopoly supply for the incumbent is just large enough for the monopoly supply of upgrade $\bar{r} + 1$ to be strictly feasible.

Even though the incumbent is a monopolist in all markets for quality upgrades greater than q_m (the entrant's highest quality), the monotonicity constraint on upgrade outputs must still be obeyed. Hence, in equilibrium, the monopoly power of the incumbent is potentially more restricted. In particular, it is restricted to upgrades to quality levels greater than that of product $\bar{r}$.

This result also suggests that entrants who are able to produce goods of higher quality will capture more of the market for low-quality goods, forcing the incumbent out of those markets. As we now show, this is indeed the case. So long as the entrant is constrained to offer goods of quality less than q_k, increases in the quality of the entrant's product tend to lead to the further elimination of lower-quality products by the incumbent.

PROPOSITION 8: *When marginal revenue is decreasing, the equilibrium number of products offered by the incumbent is decreasing in m, so long as $m \leq k$. More precisely, $\bar{r}$ is a (weakly) increasing function of m in the region $m \leq k$.*

When $m \leq k$ the entrant will offer only a single product. In equilibrium, it is as if the incumbent were competing only with a single product itself. This result says that the incumbent finds it optimal to use a higher-quality product in response to a higher-quality entrant.

Note that in the region $m > k$ the logic of Proposition 8 does not hold. There are two reasons. First, even if the entrant continued to sell but a single good, the effective marginal cost of that product would be increasing with $m > k$. This would tend to lower the entrant's output and raise the incumbent's, which might well lead to the reintroduction of a previously removed brand. Second, the entrant itself will likely not choose to offer only a single product. Increasing average cost means that choosing to sell a second product, for example, raises the cost of that good and therefore further lowers the entrant's output.

Thus, while entry itself cannot lead the incumbent to expand its product line under the assumption of decreasing marginal revenue, pronounced increases in the entrant's technological possibilities can have this effect. In particular, only when the maximal quality of the entrant passes the threshold k where average cost begins increasing can the incumbent possibly introduce new products. However, these are always goods that were sold as a monopolist.

In contrast, when marginal revenue is nonmonotonic, the incumbent might choose to sell products in the face of competition that it would not sell as a monopolist. We now examine this case.

C. *Nonmonotonic Marginal Revenue and Fighting Brands*

In Section II, subsection B, we argued that the presence of multiple sources of demand might generate a multimodal distribution for θ and nonmonotonic marginal revenue. This leads to the possibility that a sufficiently large incursion by an entrant may raise marginal revenue, and hence lead an incumbent firm to expand its own output.

The existence of such expansionary pressure on an incumbent's output leads to a novel explanation for the introduction of fighting brands. When an incumbent firm holds a quality advantage, an entrant is unable to offer upgrades to higher-quality levels, and so the expansionary pressure is only felt at lower-quality levels. The incumbent expands output at such lower levels while continuing to restrict its output of higher-quality products. This output expansion manifests itself as a fighting brand, whereby the incumbent can compete for new customers (whom it would not have served as a monopolist) while protecting the markup on its high-quality products.

To explore this intuition in more depth, suppose that the incumbent holds a strict quality advantage over the entrant, so that $n > m$, and that as a monopolist it would not sell product m. In terms of upgrades, this means that the incumbent's output satisfies $X_m = X_{m+1}$, absent competition.

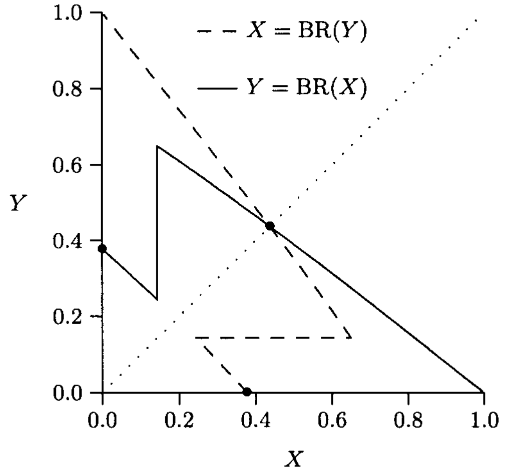
Following entry, the incumbent faces competition in the upgrade market to quality m. That is, the incumbent will choose some $Y_m > 0$, which in equilibrium will lead the incumbent to adjust its own upgrade level in that market. As already noted, and as we show by example below, it is possible that the equilibrium response is for the incumbent to increase its output in that market relative to pre-entry levels, so that its final choice is some $X_m^* > X_m$.

It is important to note, however, that while the incumbent faces competition in upgrade market m, it faces no competition for the supply of upgrades from q_m to q_{m+1}. More precisely, the optimal choice of upgrade $m + 1$ for the incumbent, neglecting the monotonicity constraint $X_m \geq X_{m+1}$, is unaffected by the competitor's presence. If this optimal choice lies below the duopoly supply of the product of quality q_m, then $X_m^* > X_{m+1}^*$ in equilibrium. That is, a fighting brand of quality q_m emerges following entry.

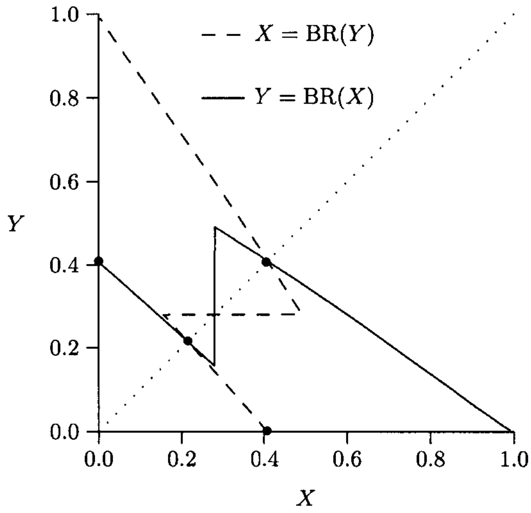
It may be helpful to contrast this logic with that from Section IV, subsection B. For Propositions 7 and 8 we argued that, even when the incumbent faces no direct competition in upgrade markets above m, its optimal choices of such upgrades are still potentially influenced by the entrant through the need to obey the monotonicity constraints on the upgrades. This is particularly important when marginal revenue is decreasing, since the presence of competition will lead the incumbent to restrict X_m, thereby potentially causing more constraints to bind, which is equivalent to the elimination of lower-quality goods. Abandoning the decreasing marginal revenue assumption, the key conceptual difference is that entry may lead to an expansion in X_m by the incumbent, thereby causing fewer upgrade constraints to bind. This leads naturally to the introduction of new lower-quality products.

We now present two concrete examples of fighting brands. In both cases, the intuition that we wish to convey is as described above: When entry leads an incumbent to expand total output, it may be optimal for it to introduce a new product of lower quality.

We begin by exhibiting a nonmonotonicity in



(a) $\theta_H = 3$, $\sigma_H = \sigma_L = 0.3$ and $\alpha = 0.4$.



(b) $\theta_H = 4$, $\sigma_H = 0.6$, $\sigma_L = 0.4$ and $\alpha = 0.45$.

FIGURE 2. COURNOT REACTION FUNCTIONS FOR EXAMPLE 1

Notes: These are reaction functions for the Cournot duopoly selling a single good manufactured at zero marginal cost. Both configurations involve $\theta_L = 1$. The demand specification is taken from Example 1, and for Figure 2(a) is identical to that used in Figures 1(a) and 1(b).

the Cournot reaction functions for a single product of quality $q = 1$ and marginal cost $c = 0$ generated by Example 1 [see Figure 2(a)]. As a monopolist, a firm restricts output to serve only the upper market segment, centered at θ_H. That is, the monopoly output is $Z^* = 0.377$, and we can see from Figure 1(b) that this corresponds to serving only the higher segment of the underlying distribution of consumers. A small incursion by a competitor results in a contraction of output in the standard way—the reaction function is initially decreasing. If the incursion is sufficiently large, however, then the incumbent will wish to expand output and serve the lower market segment (the one around θ_L). This results in a single jump upward in the reaction function, which decreases smoothly again thereafter. Inspecting Figure 2(a), notice that the symmetric duopoly quantity satisfies $X^* = Y^* = 0.437$. In other words, the incursion of a competitor prompts the incumbent to expand its supply beyond the monopoly quantity.

We now show how a fighting brand can arise under this demand structure. To this end, assume that the incumbent has two products, both of which can be produced at zero cost, with qualities $q_1 = 1$ and $q_2 = 2$. From Figure 2(a) note that unconstrained monopoly supply of each upgrade is given by the best response of the incumbent to a zero supply by the entrant, and is equal to 0.377 for each upgrade. Hence, a monopolist would set $Z_1^* = Z_2^* = 0.377$, in effect selling only the high-quality good.

Now suppose that entry occurs by an entrant only capable of producing good 1. Figure 2(a) reveals that, if each firm were to only sell product 1, there would be an equilibrium in which each firm produces 0.437. When the incumbent can offer both products, and since $Z_2^* = 0.377 < 0.437$, there is an equilibrium in which the incumbent responds to entry by raising its output of upgrade 1, resulting in an increase in total output. In particular, the output of product 2 remains constant, but the incumbent raises its output of product 1 from 0 to $0.06 = 0.437 - 0.377$. Equivalently, the incumbent serves 6 percent of consumers with a low-quality fighting brand.

Our second example is given by a discrete version of the bimodal Example 1, which corresponds to letting the standard deviations of the two normal distributions fall to zero, so that $\sigma_H = \sigma_L = 0$.

Example 2: There is a mass α of types θ_H and a mass $1 - \alpha$ of types θ_L, with $\theta_H > \theta_L$. Two qualities $q_2 > q_1$ are both produced at zero cost.

Proposition 1 is valid in this discrete model, as inspection of its proof reveals. Since the production cost for both quality levels is zero, there are increasing returns to quality,[35] and so a monopolist will wish to offer only the high-quality product. It is optimal to restrict supply and sell only to the high types when $\alpha > \theta_L/\theta_H$.

Consider next a pair of duopolists each offering a single product q_1. In a symmetric equilibrium, they serve either the entire market at a price of $\theta_L q_1$ or restrict to the fraction α of high types at a price $\theta_H q_1$. When $\alpha < \frac{1}{2}$, there is a symmetric equilibrium in which the entire market is served.[36] The reason is that, if one firm is selling 0.5 units, the other firm cannot raise the price even it were to drop its output to zero. Hence it would be best off also selling 0.5 units.

Therefore, when $\theta_L/\theta_H < \alpha < \frac{1}{2}$ is satisfied, we can find an equilibrium in which the incumbent releases a fighting brand. In particular, entry leads the incumbent to expand its output of upgrade X_1 from α to 0.5, while keeping its output of upgrade X_2 constant at α. As a result, the incumbent introduces the low-quality product as a fighting brand, in supply $0.5 - \alpha$.

The two examples of fighting brands just presented are representative of a more general result. Formally, we have the following proposition.

PROPOSITION 9: *Consider the restricted duopoly game in which each firm can offer only up to quality level q_m, for some $m < n$. Fix an equilibrium of this game in which the incumbent's production of upgrade m is X_m^*. Suppose that:*

$$X_m^* > \max_{m < i \leq n} \left\{ \arg\max_X \left[X \left(H(X) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}} \right) \right] \right\}.$$

Then there is an equilibrium of the unrestricted duopoly game in which the incumbent offers multiple products. In particular, the incumbent sells product m, so that $X_{m+1}^* < X_m^*$.

To understand how this proposition relates to fighting brands, suppose that in the absence of competition the incumbent would not sell product m. We know from Proposition 1, for example, that this would be the case if the average cost of quality were decreasing for product m, as in Definition 2. When the hypotheses of Proposition 9 are satisfied, however, product m emerges as a fighting brand.[37]

An alternative viewpoint is as follows. We could expand our model to allow for the possibility that a firm could innovate, allowing it to produce goods of higher qualities. We might ask whether the innovating firm would continue to sell its old product. To address this possibility, suppose that each firm is initially able to offer only quality levels at or below q_m—that is, even the incumbent cannot produce the highest quality products. Proposition 6 ensures that any pure strategy equilibrium is symmetric, and hence $X_m^* = Y_m^*$. Imagine now that the incumbent firm benefits from a successful product innovation, which permits it to offer all products up to quality q_n. If the condition of Proposition 9 is satisfied, then the unconstrained supplies of upgrades $m + 1, \ldots, n$ lie below X_m^*. In this case, the incumbent will sell not only products that it has newly developed, but also its older, lower-quality products. In contrast, if marginal revenue is everywhere decreasing, then the lower-quality products would be removed following the innovation.

We close with a technical remark regarding fighting brands. Proposition 9 assumes that the

[35] We define increasing returns to quality as decreasing c_i/q_i, which clearly is constant if $c_i = 0$ for all i. However, inspecting the proof of Proposition 1 reveals that in fact a monopolist would also sell only the highest quality good when all costs are zero.

[36] There are multiple equilibria as a result of the discrete types. We focus on the symmetric one, since in the continuous-type version of this example, equilibrium outputs would be symmetric. We are also ignoring equilibria in which there is strictly excess supply, even though zero marginal costs imply that equilibria in which each firm is flooding the market with its own output may exist (albeit in weakly dominated strategies).

[37] In fact, it is possible that other products of qualities exceeding q_m also emerge as fighting brands.

incumbent has a quality advantage over the entrant. In fact, however, fighting brands may arise when the incumbent and entrant have identical technological capabilities. When marginal revenue is nonmonotonic, a Cournot quantity-setting game may well have multiple equilibria. It is possible, therefore, for two firms to coordinate on one equilibrium in the market for a baseline product and a second equilibrium in the market for an upgrade. This idea is illustrated in Figure 2(b). When costs are zero, the payoff functions in the baseline and upgrade market for two quality levels are proportional to each other. Hence it is possible for the duopolists to play the low-output equilibrium in the upgrade market and the high-output equilibrium in the baseline market. However, note that this multiplicity of equilibria is not required for fighting brands to emerge. In fact, neither example presented above leans upon the presence of multiple equilibria.

V. Discussion

We point to price-discrimination-based reasons for a firm to expand or contract its product line in the face of entry. Thus we find novel theoretical support for both the use of fighting brands and the practice of pruning product lines in order to focus on quality, two commonly observed strategies. Moreover, we show how the shape of marginal revenue plays a critical role.

We now turn to a number of empirical examples that help to illustrate our ideas.

A. *Computer Hardware*

The Intel 80486SX microprocessor and the IBM LaserPrinter 4019E are examples of fighting brands in the computer hardware industry.[38] Both of these examples feature prominently in the work of Deneckere and McAfee (1996). They analyze the phenomenon of damaged goods where a monopolist intentionally (and perhaps at some cost) damages a high-quality good, hence enabling price discrimination. They offer two models. The first employs multiple type dimensions, and generates relatively weak conditions for the damaged goods phenomenon. The second considers a single type dimension, and the authors show that a condition that is equivalent to log supermodularity of preferences is needed to generate the damaging of goods. They comment on the strict nature of their condition.

Reviewing the examples in more detail is worthwhile. In early 1991 Intel released the 80486SX microprocessor. This chip was a modified version of the earlier 80486, subsequently renamed 80486DX. The sole difference was the omission of an internal floating point mathematics coprocessor, yielding an initial pricing point of \$258 relative to the 486DX price of \$588. Interestingly, the industry literature recognized that the 486SX was a damaged version of the 486DX:

> In a move aimed at replacing the 386DX as the midrange processor of choice, Intel has launched the 486SX ... the 486SX is simply a 486 without the floating-point unit. In fact, the initial silicon includes the FPU, but it has been disabled (Michael Slater, 1991, p. 1).

Interestingly, the release of the 486SX followed the entry of Advanced Micro Devices (AMD) into the 386DX market.[39,40] The 486SX was therefore a fighting brand. Slater (1991) agreed, predicting that "AMD ... may be forced to lower 386DX prices significantly to maintain momentum in that market." It would appear that the presence of competition may have influenced Intel's decision to expand its product line.

[38] There is a growing empirical marketing literature on the product line decisions of firms. For example, William P. Putsis, Jr. and Barry L. Bayus (2001) investigate product line expansions and contractions in the personal computer industry in the years 1981–1992. They jointly consider the empirical determinants both of the decision to change a product line, and the magnitudes of such changes. Among other results, they find that a firm is more likely to prune its product line when the number of new products in the industry is higher.

[39] "AMD's 486 Clone Ready, But Will It Sell?" *Computer World,* March 1991.

[40] The 80386DX/SX series was an earlier generation of x86 architecture microprocessors that omitted the internal cache feature of the 486 series. AMD previously manufactured 386 generation processors under licence. Following lengthy litigation they were able to continue to produce the design independently ("The Chips Are Down." *PC User,* January 1991).

The release of the IBM LaserPrinter 4019E provides a similar example. This device was a slower version of the IBM 4019 (5 ppm versus 10 ppm). The only reason it was slower was that the controller card made it so. Deneckere and McAfee (1996, p. 170) view this as a second example of a relationship between product damaging and the presence of competition:

> At least two of the damaged goods, the Intel 486SX and the IBM LaserPrinter E, appear to have been introduced in response to competition by another producer. This is difficult to explain, particularly in the LaserPrinter case.

In both of these cases, entry by a lower-quality competitor induced the introduction (by the incumbent) of a new product that was inferior to the incumbent's but (at least weakly) superior to the entrant's. This exactly reflects our results in Section IV. We arrive at the following explanation for, say, the IBM case. Initially, IBM was content to serve only high-value business customers with the LaserPrinter. The entry of the HP LaserJet IIP forced it to serve home and low-value business users. Nevertheless, it retained monopoly power on a quality increase from 5 ppm to 10 ppm, and wished to deliver this quality premium only to high-value customers. Hence it introduced the crimped LaserPrinter E to execute the strategy of serving a broader market while still maintaining a markup on its premium product.

B. *Airlines*

Air travel is a canonical example of second-degree price discrimination. The U.K. carrier British Airways (BA) provides examples of expansion and contraction in its product line. BA's former CEO Robert Ayling engaged in an explicit strategy of reducing the economy class capacity on long-haul routes, and expanding the business class and first class provision. In other words, BA chose to focus on quality:

> Now the focus has moved to presenting products that Mr. Ayling hopes will attract higher numbers of premium customers, the executives and travellers willing to pay higher prices for premium services and facilities. If the plan works there will be little space left for the passengers wanting to travel on deeply discounted economy class tickets.[41]

Interestingly, however, BA adopted a different strategy in the European short-haul market, including the market for U.K. domestic flights. It introduced a fighting brand: the "no-frills" subsidiary Go.[42,43]

A possible explanation for this stems from March 1995 with the incorporation of the easyJet airline by Stelios Haji-Ioannou, the son of a Greek shipping magnate. easyJet operates from airports in the United Kingdom on a no-frills basis, similar to that of Southwest Airlines in the United States. In common with other operators of this kind, it offers a ticketless service devoid of complimentary in-flight meals and other nonessential features.[44]

Media reports at the time suggested that BA had predatory intentions.[45] Was the entry of Go a predatory move by British Airways? If so, it has failed, as easyJet has continued to expand in the no-frills market. Our analysis offers an alternative explanation for this event. It is possible that BA was initially reluctant to enter this market segment, due to the anticipated negative effect on its core operations. However, following the creation of easyJet, this segment was opened up and BA thus found it profitable to enter.

[41] "BA Aims at Business Traveller with Revamp of Aircraft Fleet." *Financial Times* (*London*), February 1, 2000.

[42] "BA's New Cut-Price Airline Hots Up Sky Wars." *The Independent* (*London*), October 1997.

[43] This behavior has also been seen in other markets. David Haugh and Tim Hazledine (1999) describe events in the Trans-Tasman air travel market. Following the entry of a former charter airline Kiwi International, Air New Zealand launched a no-frills subsidiary Freedom Air as a fighting brand in this market.

[44] The no-frills market has also seen the success of Ryanair. These carriers tend to use secondary airports, such as Luton and Stansted for London flights, rather than the mainstream Heathrow and Gatwick. Arguably, the better connection opportunities at these latter airports are the main component of increased quality for full-service carriers.

[45] "BA Sets Up No-Frills Airline: Budget Flight Rivals Fear Service Aims to Wipe Out Competition." *Financial Times* (*London*), November 18, 1997.

C. *The Market for Watches*

In 1998 Timex and Titan ended a joint venture in the Indian market for watches. The original purpose of the joint venture was to capture economies of scale in retail and distribution, and also to split the market for watches. Titan was to serve the high end of the market, and Timex the low end.[46]

Timex and Titan together were able to dominate much of the market in India.[47] However, the alliance proved unstable in part because each felt the other was cannibalizing sales by offering products not in line with the original demarcation of the market. With the termination of the joint venture, each firm moved aggressively. Timex launched the high-end Vista brand, while Titan introduced the Sonata brand in the under Rs 1,000 price range.

The Sonata was immensely successful, quickly becoming the second best-selling watch brand in India, following the parent brand Titan.[48] In 2001, Timex announced that it would exit the low-end watch market in India by phasing out most of its under Rs 1,000 watches. Its new strategy would be to focus on becoming a trendier "sports and technology brand," targeting its watches at the Rs 1,000–5,000 range.[49]

Since both Timex and Titan are capable of producing watches across a very broad quality range, it might seem that the exit of Timex from the low-end market is inconsistent with Proposition 3, which states that firms with identical technologies should offer products in the same range. Even if we imagine that Titan has an advantage in the high end, our Proposition 4 seemingly implies that Titan should not sell products of lower quality than Timex. However, in this situation it seems likely that, in fact, Titan was at a variable cost advantage in the low-end segment. Under the terms of the original joint venture, Titan (which is based in India) was in charge of all distribution for both Timex and Titan watches. Following the termination of the relationship, Timex had to establish its own channels. It initially fought to maintain its position in the low-end market, but this was complicated by the fact that the low-end market is concentrated in many smaller urban and rural areas, which might have made it more costly for Timex to distribute there compared to Titan, which already had an established network.

The possibility that Titan has an edge in distribution is attested to by Titan vice-chairman and managing director Xerxes Desai, who said that "a strong grip on distribution" has contributed to the success of Titan.[50] If true, this potentially reconciles the asymmetric product offerings of Timex and Titan, since in the market for basic low-end watches Titan would produce more baseline upgrades in equilibrium than Timex. Hence, it could well be the case that Timex would prune its product line following the entry of Titan in that segment, and that Titan would maintain a presence there following the end of the joint venture and exit of Timex from the segment.

D. *Concluding Remarks*

Fighting brands and product line pruning are pervasive and widespread responses to intensified competition. We have proposed a model of multiproduct quality competition, analysis of which provides an integrated explanation for both phenomena. In particular, we considered the product line response of an incumbent to entry by another multiproduct firm.

The equilibrium product lines of firms were derived under a variety of conditions. Whether an incumbent will choose to expand or contract its product line depends on whether entry prompts it to expand or contract its total output. This in turn is connected closely to whether marginal revenue is everywhere decreasing or not.

When marginal revenue is everywhere decreasing, entry induces a restriction in the

[46] "Stitch in Timex." *The Economic Times,* June 28, 2000.

[47] "The joint venture did extremely well, leveraging the strengths of both partners. While Titan chalked up a market share of 60 percent in the premium segment, Timex literally monopolised the low end of the market." *The Economic Times,* June 28, 2000.

[48] "The Changing Face of Titan." *The Strategist,* August 24, 1999.

[49] "Timex Corp to Hike Stake in Indian Arm." *The Economic Times,* January 14, 2002.

[50] "Clash of the Titans." *The Times of India,* December 13, 2000.

output of the incumbent's low-quality products. Thus, entry can lead naturally to the incumbent's exit from the lower markets, thereby pruning its line of products. In this case, there is a real sense in which the incumbent uses but a single product to compete against the entrant, while enjoying unrestricted monopoly power in the markets for upgrades to higher quality. Identifying which product the incumbent will use against the entrant in equilibrium essentially determines the entire equilibrium structure of the industry.

When marginal revenue is increasing in some regions, it may be optimal for the incumbent to expand output in response to entry. If this is the case, the incumbent may decide to expand into a lower-market segment by using a low-quality fighting brand, thereby allowing it to be competitive in that segment while preserving margins on its high-quality good. This is possible when the incumbent maintains a technological advantage over the entrant, since it therefore continues to hold market power over higher-quality upgrades. We showed that if the incumbent does offer new products it will never operate at quality levels inferior to those of the entrant. We also argued in Section II that demand structures exhibiting increasing marginal revenue in *some* regions are not pathological. In particular, simple bimodal consumer-type distributions naturally can generate nonmonotonicity in marginal revenue.

Introducing fighting brands may therefore be the optimal strategy when the incumbent wishes to expand its total output following entry. Product line pruning may be the optimal strategy when the incumbent wishes to restrict its total output following entry.

From a technical standpoint, a general upgrades approach has been presented that yields a powerful analytical framework. We also provided a new condition on costs and preferences that determines when a monopolist would choose to pursue market segmentation opportunities.

While we have performed no substantive empirical analysis, we have presented examples from numerous industries that seem to resonate well with our theory. Our theory of fighting brands and product line pruning provides one explanation for the observed product line expansions and contractions in these industries.

Appendix

Elements of the proofs to Lemma 1 and Propositions 2, 4, 5, and 9 are contained in a further technical Appendix, available from the AER web site (http://www.acaweb.org/aer/contents/) or from the authors on request.

PROOF OF PROPOSITION 1:

Suppose that the lowest quality product i supplied by the monopolist satisfies $i < k$. It must be the case that $Z_1^* = Z_2^* = \cdots = Z_i^* > Z_{i+1}^*$. Product i is therefore the effective "baseline" product. Following the argument given in the text:

$$Z_i^* H(Z_i^*) - Z_{i+1}^* H(Z_{i+1}^*) \geq \frac{c_i}{q_i}(Z_i^* - Z_{i+1}^*)$$

$$> \frac{c_{i+1} - c_i}{q_{i+1} - q_i}(Z_i^* - Z_{i+1}^*).$$

The first inequality says that the monopolist does not wish to lower supplies of the baseline product to Z_{i+1}^*. The second inequality follows from the fact that $i < k$. But this means that it would be better to raise the supply of the upgrade $i + 1$ to Z_i^*, and hence the original configuration was not optimal. We conclude that $Z_1^* = Z_2^* = \cdots = Z_k^*$. Consider the monopolist's problem with the relaxed constraint that $Z_1^* = Z_2^* = \cdots = Z_k^*$. Increasing average and marginal costs imply that the quality-normalized costs [i.e., $(c_{i+1} - c_i)/(q_{i+1} - q_i)$] are increasing for $i \geq k$. Hence, ignoring the monotonicity condition we immediately see that $Z_k^* > Z_{k+1}^* > \cdots > Z_n^*$ occurs naturally. Since this relaxed solution satisfies the monotonicity constraint, it solves the maximization problem.

PROOF OF PROPOSITION 2:

Suppose that the monopolist offers distinct products and that product i is the lowest-quality product offered, so that $Z_1^* = Z_2^* = \cdots = Z_i^* > Z_{i+1}^*$. As in the proof of Proposition 1, product i is effectively the baseline product, and hence we may relabel it as $i = 1$. The argument given in the text, upon the inclusion of costs, yields the required proof.

PROOF OF PROPOSITION 3:

If $n > m$ then this proposition is true by assumption for all $i > m$. For $i \leq m$ such that $X_i^* = X_{i+1}^*$ and $Y_i^* = Y_{i+1}^*$, amalgamate neighboring upgrades by viewing upgrades i and $i + 1$ as a combined upgrade. Following this, suppose that the proposition does not hold for some $i \leq m$. Take the lowest such i, so that $Y_i^* > X_i^*$. Either $i = 1$, or $i > 1$ and $X_{i-1}^* \geq Y_{i-1}^* \geq Y_i^* > X_i^*$. In either case, the upward monotonicity constraint on the incumbent is not locally binding. The incumbent cannot have a strict incentive to locally raise X_i, and thus has a weak incentive to lower it. This implies that the entrant must have a strict incentive to lower Y_i:

$$H(X_i^* + Y_i^*) + Y_i^* H'(X_i^* + Y_i^*) < H(X_i^* + Y_i^*) + X_i^* H'(X_i^* + Y_i^*)$$

$$\leq \frac{c_i - c_{i-1}}{q_i - q_{i-1}}.$$

The first inequality follows from $Y_i^* > X_i^*$ and $H'(X_i^* + Y_i^*) < 0$, and the second from the weak incentive for the incumbent to lower X_i. Since there is an incentive for the entrant to lower Y_i, the monotonicity constraint must bind and hence $Y_i^* = Y_{i+1}^* > X_{i+1}^*$. We have amalgamated all identically supplied neighboring upgrades and therefore we know that $X_i^* > X_{i+1}^*$. Turning to upgrade $i + 1$, the upward monotonicity constraint on the incumbent is not locally binding, and we may repeat our argument until we conclude that the entrant has a strict incentive to lower Y_m. But there is no constraint on downward movement of Y_m, and thus we have a contradiction.

PROOF OF PROPOSITION 4:

Use an identical approach to Proposition 3.

PROOF OF PROPOSITION 5:

Use an identical approach to Proposition 1.

PROOF OF PROPOSITION 6:

When $m = n$ the result is obvious. For $m < n$, and when both marginal revenue and returns to quality are decreasing, the reaction functions are continuously decreasing with absolute slope of less than 1. Under these standard conditions the incumbent's reaction function intersects the entrant's from above and there is a unique equilibrium (Vives, 1999, p. 47). The incumbent's reaction function is an outward shift of the inverse of the entrant's reaction function. This ensures that $X^* > Y^*$. Finally, note that an increase in q_n pushes the incumbent's reaction function outward. To see this, note that we may replace π_I from equation (6) by $\tilde{\pi}_I$ where:

$$\tilde{\pi}_I = X\left[\left(1 - \frac{q_m}{q_n}\right) H(X) + \frac{q_m}{q_n} H(X + Y) - \frac{c_n}{q_n}\right].$$

Hence a marginal expansion in quantity yields:

$$\frac{\partial \tilde{\pi}_I}{\partial X} = \left(1 - \frac{q_m}{q_n}\right)(H(X) + XH'(X)) + \frac{q_m}{q_n}(H(X+Y) + XH'(X+Y)) - \frac{c_n}{q_n}.$$

This is increasing in q_n (due to decreasing marginal revenue and returns to quality), pushing the incumbent's reaction function outwards. Similar operations using q_m yield the desired results.

PROOF OF LEMMA 1:

Use an identical approach to Propositions 1 and 5.

PROOF OF LEMMA 2:

Suppose not, so that both s and r are members of the set $\mathcal{R}$ and satisfy $s > r \geq k \geq m$. It follows that $X^\dagger_{rm} > Z^\dagger_{r+1} \geq Z^\dagger_s \geq X^\dagger_{sm}$, so that $X^\dagger_{rm} > X^\dagger_{sm}$. In other words, an increase in the incumbent's quality level results in a drop in its output. Write $\pi_{Ir}(X, Y)$ for the incumbent's profit with quality r, and similarly for quality s. Observe that:

$$\begin{aligned}\frac{\partial \pi_{Is}(X^\dagger_{sm}, Y^\dagger_{sm})}{\partial X} &= \frac{\partial \pi_{Ir}(X^\dagger_{sm}, Y^\dagger_{sm})}{\partial X} + (q_s - q_r)\left[H(X^\dagger_{sm}) - \frac{c_s - c_r}{q_s - q_r} + X^\dagger_{sm}H'(X^\dagger_{sm})\right] \\ &\geq \frac{\partial \pi_{Ir}(X^\dagger_{sm}, Y^\dagger_{sm})}{\partial X} + (q_s - q_r)\left[H(X^\dagger_{sm}) - \frac{c_s - c_{s-1}}{q_s - q_{s-1}} + X^\dagger_{sm}H'(X^\dagger_{sm})\right] \\ &\geq \frac{\partial \pi_{Ir}(X^\dagger_{sm}, Y^\dagger_{sm})}{\partial X}.\end{aligned}$$

The first inequality follows from the assumption that marginal cost is increasing beyond k. The second follows from the fact that $Z^\dagger_s \geq X^\dagger_{sm}$, and hence $X^\dagger_{sm}$ is below the profit-maximizing supply of upgrade s. This means that the reaction function with quality q_r must lie weakly below that for quality q_s, evaluated at $X^\dagger_{sm}$. But this of course means that $X^\dagger_{rm} \leq X^\dagger_{sm}$, and we have reached a contradiction.

PROOF OF PROPOSITION 7:

By assumption $m \leq k$, and hence $Y^*_1 = \cdots = Y^*_m$ and $X^*_1 = \cdots = X^*_k$ from Lemma 1. Suppose that for $r \geq k$ we have $X^*_1 = \cdots = X^*_r > X^*_{r+1}$. A local increase in X_{r+1} must (weakly) lower profits on upgrade $r + 1$. The upgrade profit functions are concave, and hence $X^*_{r+1} \geq Z^\dagger_{r+1}$ where $Z^\dagger_{r+1}$ is the unrestricted monopoly supply of this upgrade. Of course, if this inequality holds then it must be optimal to set $X^*_i = Z^\dagger_i$ for all $i > r$. We have $X^*_1 = \cdots = X^*_r$, and hence it must be the case that the incumbent produces the duopoly supply of product r, and hence $X^*_r = X^\dagger_{rm}$. We need to check that it cannot do better by varying X^*_r downwards. For this we need $X^*_r \leq Z^\dagger_r$. Hence r must satisfy $Z^*_r \geq X^\dagger_{rm} > Z^*_{r+1}$, or equivalently $r = \bar{r}$.

PROOF OF PROPOSITION 8:

Take $\bar{r}$ and observe that by definition $X^\dagger_{im} - Z^\dagger_{i+1} \leq 0$ for all i such that $k \leq i \leq \bar{r} - 1$. Now suppose the entrant can offer good $m + 1 \leq k$. The effective marginal cost of the entrant is lower in the hypothetical game where it sells only product $m + 1$ and the incumbent sells only some good i. Now, decreasing marginal revenue together with constant marginal costs is sufficient to imply that the resulting hypothetical outputs of the incumbent are lower when the entrant sells good $m + 1$, given that the incumbent's reaction curves intersect the entrant's curve from above. That is, $X^\dagger_{i(m+1)} \leq X^\dagger_{im}$ for all i. This obviously implies that $X^\dagger_{i(m+1)} - Z^\dagger_{i+1} \leq 0$ for all i such that $k \leq i \leq \bar{r} - 1$, which proves the result.

PROOF OF PROPOSITION 9:

Throughout we assume that all of our maximization problems have a unique solution. Generically this will be the case, but the proof can be easily modified to allow multiple maxima. For $m < i \leq n$ suppose that the set of upgrade supplies $\{X_j^*\}$ solve the problem:

$$\max \sum_{i=m+1}^{n} (q_i - q_{i-1}) X_i \left(H(X_i) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}} \right) \text{ subject to } X_{m+1} \geq \cdots \geq X_n.$$

If $X^*_{m+1} < X^*_m$ then this yields an equilibrium of the unrestricted games. The incumbent achieves the unconstrained maximum profits on upgrades above product m. Its profits on upgrades m and below are maximized, since we began with an equilibrium of the original restricted duopoly game. Similarly, the entrant is maximizing given the supplies of the incumbent. To verify the proposition we need only check:

$$X^*_{m+1} \leq \bar{X} = \max_{m < i \leq n} \left\{ \arg\max_X \left[X \left(H(X) - \frac{c_i - c_{i-1}}{q_i - q_{i-1}} \right) \right] \right\}.$$

The remainder of the proof follows a similar approach to our other results.

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